Mechanical systems vibrating longitudinally with the transportation effect

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Analysis and modelling

ABSTRACT

Purpose: High work speeds of mechanisms, using materials with high flexibility, high precision of work, etc. are the cause of searching of the new ways of modelling. One of these ways is presented in this thesis. The main purpose of this thesis is the dynamical analysis with taking into consideration the interaction between main motion and local vibrations during the model is loaded by longitudinal forces.

Design/methodology/approach: Derived equations of motion were made by classical methods, with generalized coordinates and generalized velocities assumed as orthogonal projections of individual coordinates and velocities of the rod and manipulators to axes of the global inertial frame.

Findings: Mathematical model of the longitudinally vibrating systems in terms of plane motion can be put to use to derivation of the dynamical flexibility of these systems, and also those equations are the starting point to the analysis of complex systems, especially we can use those equations to derivation of the substitute dynamical flexibility of n-linked systems in transportation.

Research limitations/implications: In the thesis were considered mechanical systems vibrating longitudinally in terms of rotation. Next problem of dynamical analysis is the analysis of systems in non-planar transportation and systems loaded by transversal forces.

Practical implications: Results of this thesis can be put to use into machines and mechanisms in transportation such as: wind power plant, high speed turbines, rotors, manipulators and in aerodynamics issues, etc.

Originality/value: Up to now there were analyzed beams and rods in a separate way, first main motion of the system and after that the local vibrations. The new approach of modelling were presented by authors of this thesis, a new modelling took into consideration the interaction between those two displacement. There was defined the transportation effect for models vibrating longitudinally in this thesis.

Keywords: Applied mechanics; Longitudinal vibrations; Multibody systems; Transportation effect

1. Introduction

The contemporary technical applications of mechanical and mechatronic systems reveal a maintaining tendency to optimization of parameters of work of machines and mechanisms. The cause of optimization of these parameters has been constantly growing requirements such as maximal precision of work of mechanical systems and positioning of manipulators and robots and also high quality assurance and reliability. There is assumed the discrepancy between main motion and amplitudes of vibrations in applied solutions at present. Apart from a commonly using method of superposition more often we search the more accurate ways of modelling. There is considered the model of the free rod vibrating longitudinally, the rod fixed in the origin of the global reference system and manipulators in transportation in this thesis. The mathematical model takes into consideration the transportation effect such as centrifugal forces and Coriolis

forces. The main motion of the rod was bounded to plane motion in one of the global reference system plane, which is independent from the analyzing rod. The rod was loaded down by the longitudinal harmonic axial force with the amplitude of the force equals one, that is consistent to the definition of the dynamical flexibility. Analyzing of systems, whilst they move with low speed or while they vibrate only locally is a well-known problem in scientific literature [7-15,18]. The model of analyzing systems takes into consideration an issue has not considered up to now, such as the relations between local vibrations and main motion [1-6,16-18]. Main motion in the thesis is considered as transportation. Today in problems of controlling mechanical systems and generally in their analysis, much often there are considered models much more adequate to actual phenomena. Solutions have done by consider main motion (in this thesis transportation) and local vibrations separately, up to now. That assumption has essential sense because vibrations from flexibility of elements of the mechanical composition are much smaller than main dislocation of this composition. There was increased scope of velocities and accelerations after there was used more efficient drives. In order to constraint power output of drives needful for motion of mechanical systems began using materials with lower mass density such as aluminums alloys, lower than mass density of materials using up to now. All those arguments are a cause of creating new models of designing systems, that take into consideration the flexibility of mechanism's links.

2. Modelling of the mechanical system vibrating longitudinally in transportation

This section considers the models of the free rod and the rod that is fixed in the origin of the global reference system and also manipulators, the two-linked manipulator. The analyzing systems are consisted by the homogenous rods made from an elastic materials with a full cross-section.

2.1. The longitudinally vibrating and rotating rod fixed in the origin

The rod in transportation was modeled. The rod is being rotated all round the origin of the global reference system, whereas it is fixed in that center of rotation (fig.1). The rod was loaded by the longitudinal force. The motion was described by one component of the instantaneous angular velocity vector with respect to the axis of the global inertial frame XY. Vibrations of the rod were considered with taking into consideration rod's deformation ability in transportation. There is considered the homogenous elastic rod with a full cross-section A that is constant on the whole length of the rod l. The rod was made from a material with longitudinal modulus of elasticity E and mass density ρ. There are known initial conditions: preliminary deflection of the rod and initial velocity of vibrations. There is not considered transverse deformation of the rod that is caused by longitudinal vibrations based on the principle of plane cross-

sections up to [12]. The solution is searched in the global reference system that is self-contained from the rod.
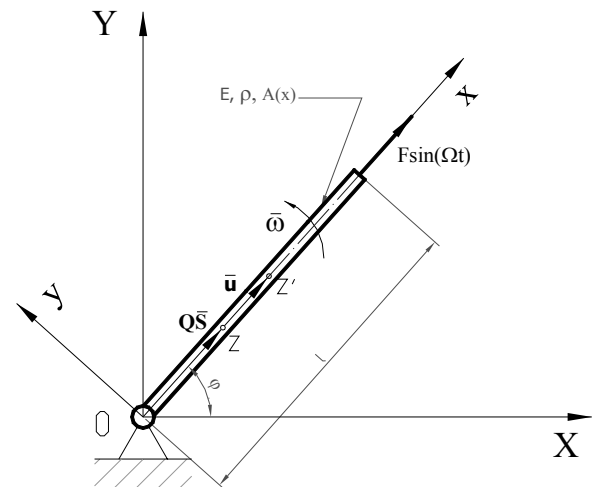

Fig. 1. The model of the rotating rod loaded by a longitudinal force

2.2. The model of the vibrating longitudinally free rod in transportation

The considered arrangement is the homogeneous flexible free rod with a symmetrical section. This paper's objective is also deriving of the dynamical model of the free rod vibrating with longitudinal vibrations and taking into consideration the transportation (fig. 2).
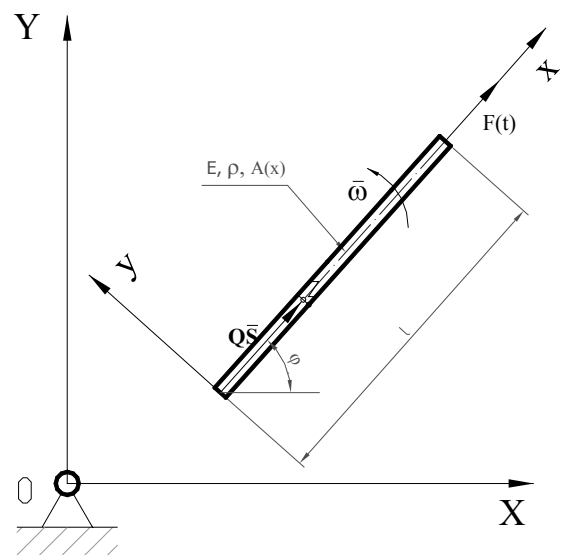

Fig. 2. The vibrating rod in terms of plane motion and loaded by a harmonic axial force

2.3. The model of the longitudinally vibrating two-linked manipulator

The two-link vibrating manipulator is considered. Rods from this system have cross sections suitably A_1 as the section of first link and A_2 as the section of second link which are constant on the whole length of rods appropriately for first link l_{01} and in second link l_{12} (fig. 3). Rods were made from materials with Young's modulus E_1 and E_2 and mass densities ρ_1 and ρ_2. Rods were loaded by a harmonic longitudinal force. The mathematical model was determined in global independent reference system in terms of plane motion.
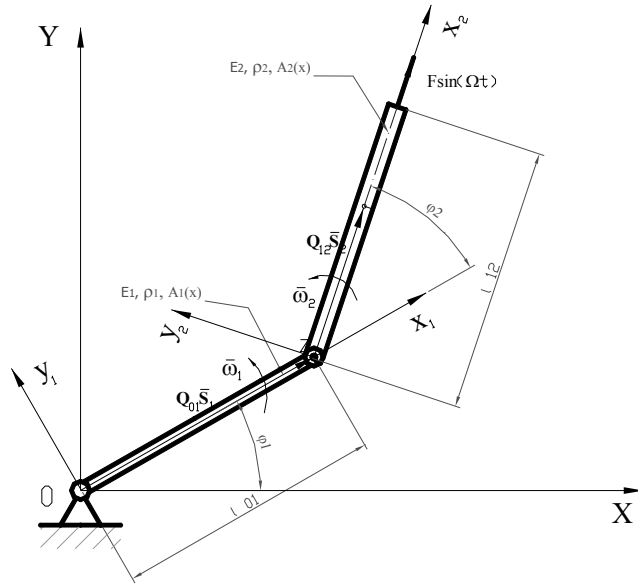

Fig. 3. The two-link manipulator loaded by a longitudinal force in transportation

3. Mathematical model

In this section the mathematical models of the analyzed systems were presented.

3.1. Equations of motion of the rod

There were derived the equations of motion of the rod by using the classical methods such as the Lagrange's and were presented as projections into axes of the global reference frame. The projection into the X axis of the global reference system:

$$\frac{\partial^2 u_X}{\partial t^2} - \frac{E}{\rho} \cdot \frac{\partial^2 u_X}{\partial x^2} = \omega^2 \cdot (s \cdot \cos \varphi + u_X) + 2 \cdot \omega \cdot \frac{\partial u_Y}{\partial t} \quad (1)$$

The projection into the Y axis of the global reference system:

$$\frac{\partial^2 u_Y}{\partial t^2} - \frac{E}{\rho} \cdot \frac{\partial^2 u_Y}{\partial x^2} = \omega^2 \cdot (s \cdot \sin \varphi + u_Y) - 2 \cdot \omega \cdot \frac{\partial u_X}{\partial t} \quad (2)$$

3.2. Equations of motion of the two-linked manipulator

The mathematical model of the manipulator in form of the equations of motion is presented as system of equations, there are equations tied with first and second rod, projected into axes of the global reference system. The equations are not coupled each other. The projection into the X axis of the global reference system:

$$\begin{cases} \dfrac{\partial^2 u_{1X}}{\partial t^2} - \dfrac{E_1}{\rho_1} \cdot \dfrac{\partial^2 u_{1X}}{\partial x_1^2} = \\[2mm] = \omega_1^2 \cdot (s_{1X} + u_{1X}) + 2 \cdot \omega_1 \cdot \dfrac{\partial u_{1Y}}{\partial t}, \\[2mm] \dfrac{\partial^2 u_{2X}}{\partial t^2} - \dfrac{E_2}{\rho_2} \cdot \dfrac{\partial^2 u_{2X}}{\partial x_2^2} = \\[2mm] = (\omega_1 + \omega_2)^2 \cdot (s_{2X} + u_{2X}) + 2 \cdot (\omega_1 + \omega_2) \cdot \dfrac{\partial u_{2Y}}{\partial t}, \end{cases} \quad (3)$$

The projection into the X axis of the global reference system:

$$\begin{cases} \dfrac{\partial^2 u_{1Y}}{\partial t^2} - \dfrac{E_1}{\rho_1} \cdot \dfrac{\partial^2 u_{1Y}}{\partial x_1^2} = \\[2mm] = \omega_1^2 \cdot (s_{1Y} + u_{1Y}) - 2 \cdot \omega_1 \cdot \dfrac{\partial u_{1X}}{\partial t}, \\[2mm] \dfrac{\partial^2 u_{2Y}}{\partial t^2} - \dfrac{E_2}{\rho_2} \cdot \dfrac{\partial^2 u_{2Y}}{\partial x_2^2} = \\[2mm] = (\omega_1 + \omega_2)^2 \cdot (s_{2Y} + u_{2Y}) - 2 \cdot (\omega_1 + \omega_2) \cdot \dfrac{\partial u_{2X}}{\partial t}, \end{cases} \quad (4)$$

where:

$$\begin{aligned} u_{1X} &= u_1 \cdot \cos \varphi_1, \\ u_{1Y} &= u_1 \cdot \sin \varphi_1, \\ u_{2X} &= u_2 \cdot \cos (\varphi_1 + \varphi_2), \\ u_{2Y} &= u_2 \cdot \sin (\varphi_1 + \varphi_2). \end{aligned} \quad (5)$$

4. Conclusions

In this paper the equations of motion were presented. The systems were vibrating longitudinally in transportation. The systems move in terms of two-dimensional motion. The mathematical model is the beginning of further dynamical analysis of those type systems and it can be used for example for the derivation of dynamical flexibility of manipulator. The next thesis's objectives will be the derivation of attenuation-frequency characteristics and the stability analysis.

There were took into consideration occurrences of unbalanced forces bound with transportation in the mathematical model. There was emphasized the interactions between local displacements and transportation. There was took into account acting of the Coriolis' force and the centrifugal force, that was analyzed after projection of the forces components into the appropriate axes of the global reference system. The numerical examples were derived assumed that the material of rods was the aluminum alloy and the length of the beam was assumed as equal one meter. Equations of motion were derived by the Lagrange equations. A mathematical model of the longitudinally vibrating systems in terms of plane motion can be put to use to derivation of the dynamical flexibility of these systems, in particular we can use those equations to derivation of the substitute dynamical flexibility of n-linked systems. Derived mathematical model can be put to use into machines and mechanisms in transportation such as wind power plant, high speed turbines, rotors, manipulators and in aerodynamics issues, etc. Results should to be modified and adopted to appropriate physical models. Future problems of dynamical analysis are the analysis of systems in non-planar transportation and systems loaded by transversal forces.

Acknowledgements

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