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Numerical algorithm of cast steel latent heat identification

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Analysis and modelling

ABSTRACT

Purpose: The inverse problem consisting in the identification of volumetric latent heat of steel cast is presented. Additionally it is assumed that the courses of temperature at the points from the casting or mould are known. The substitute thermal capacity of steel cast (this parameter is directly connected with the latent heat) is approximated by the fourth-degree polynomial. In order to solve the inverse problem the least squares criterion containing the sensitivity coefficients is applied. In the final part of the paper the results of computations are shown.

Design/methodology/approach: In this work numerical modelling of solidification process is applied. A steel cast solidifying in a sand mould is analyzed. The information concerning the course of cooling curve at the selected point from the casting domain is used in order to identify the latent heat of steel cast.

Findings: It is shown that only one sensor located at the optional point from casting or mould domain gives the information sufficient for successful identification of unknown parameter.

Research limitations/implications: Further work requires an introducing of real temperature measurements to the model presented.

Practical implications: The paper shows the possibilities of thermophysical parameters identification on the basis of temperature measurements. In the case considered the latent heat of steel cast has been identified.

Originality/value: The algorithm presented allows to identify the latent heat in the case of the complex, temperature dependent function determining the substitute thermal capacity of metallic alloy.

Keywords: Numerical techniques; Inverse problem; Parameter identification

1. Introduction

The thermal processes proceeding in the system casting-mould-environment are described by the system of partial differential equations (energy equations) supplemented by the adequate boundary, initial, physical and geometrical conditions [1, 2, 3, 4, 5, 16]. The solidification process in domain of casting is described using the one domain approach and the substitute thermal capacity is introduced in order to take into account the evolution of latent heat Q [1, 3, 6].

If the parameters appearing in governing equations are known then the direct problem is considered, while if part of them is unknown then the inverse problem should be formulated [7, 8, 9, 10, 11]. In order to solve the inverse problem the additional information concerning the time-dependent temperature field at selected set of points from the domain considered must be used.

In the paper the problem of latent heat identification is discussed. The inverse problem formulated is solved using the least squares criterion in which the sensitivity coefficients appear [12, 13, 14, 15]. In order to determine these coefficients, the additional boundary initial problem resulting from the differentiation of basic equations with respect to the latent heat must be solved. The final equation allows to determine the value of Q using the iterative procedure. On the stage of numerical computations the finite difference method is used [1]. In the final part of the paper the results of computations are shown.

2. Direct problem

A casting-mould-environment system is considered. A transient temperature field in casting sub-domain determines the following energy equation

$$x \in \Omega: \quad C(T)\frac{\partial T(x,t)}{\partial t} = \lambda \nabla^2 T(x,t) \tag{1}$$

where $C(T)$ is the substitute thermal capacity [1, 6], λ is the thermal conductivity, T is the temperature, x are the spatial co-ordinates and t is the time.

A temperature field in mould sub-domain describes the equation of the form

$$x \in \Omega_m: \quad c_m \frac{\partial T_m(x,t)}{\partial t} = \lambda_m \nabla^2 T_m(x,t) \tag{2}$$

where λ_m is the thermal conductivity, c_m is the volumetric specific heat of mould.

On the contact surface between casting and mould the continuity condition

$$x \in \Gamma_c: \quad \begin{cases} -\lambda n \cdot \nabla T(x,t) = -\lambda_m n \cdot \nabla T_m(x,t) \\ T(x,t) = T_m(x,t) \end{cases} \tag{3}$$

is assumed.

On the outer surface of the system the no-flux condition can be accepted, namely

$$x \in \Gamma_0: \quad q_m(x,t) = -\lambda_m n \cdot \nabla T_m(x,t) = 0 \tag{4}$$

For the moment $t=0$ the initial temperature distribution is given

$$T(x,0) = T_0(x) \qquad T_m(x,0) = T_{m0}(x) \tag{5}$$

The substitute thermal capacity for cast steel is defined as follows [1]

$$C(T) = \begin{cases} c_L, & T > T_L \\ \dfrac{c_L + c_S}{2} - Q\dfrac{\mathrm{d}S(T)}{\mathrm{d}T}, & T_S \le T \le T_L \\ c_S, & T < T_S \end{cases} \tag{6}$$

where the temperatures T_L, T_S correspond to the beginning and the end of the solidification process, respectively, c_L, c_S are the constant volumetric specific heats of liquid and solid state, Q is the volumetric latent heat, $S(T)$ is the solid state fraction at the point considered.

It is assumed that for $T \in [T_S, T_L]$ the function $C(T)$ is determined by the formula

$$C(T) = c_1 + c_2 T + c_3 T^2 + c_4 T^3 + c_5 T^4 \tag{7}$$

where c_e, $e = 1, 2, ..., 5$ are the coefficients and they have been found on the basis of conditions assuring the continuity of C^1 class and physical correctness of approximation, namely

$$\begin{aligned} & C(T_L) = c_L \\ & C(T_S) = c_S \\ & \left.\frac{\mathrm{d}C(T)}{\mathrm{d}T}\right|_{T=T_L} = 0 \\ & \left.\frac{\mathrm{d}C(T)}{\mathrm{d}T}\right|_{T=T_S} = 0 \\ & \int_{T_S}^{T_L} C(T)\,\mathrm{d}T = \frac{c_S + c_L}{2}(T_L - T_S) + Q \end{aligned} \tag{8}$$

and then

$$\begin{aligned} c_1 &= \frac{c_S T_L - c_L T_S}{T_L - T_S} + \frac{(c_L - c_S) T_L T_S (T_L + T_S)}{(T_L - T_S)^3} + \frac{30 T_L^2 T_S^2 Q}{(T_L - T_S)^5} \\ c_2 &= -\frac{6(c_L - c_S) T_L T_S}{(T_L - T_S)^3} - \frac{60 T_L T_S (T_L + T_S) Q}{(T_L - T_S)^5} \\ c_3 &= \frac{3(c_L - c_S)(T_L + T_S)}{(T_L - T_S)^3} + \frac{30(T_L^2 + 4 T_L T_S + T_S^2) Q}{(T_L - T_S)^5} \\ c_4 &= -\frac{2(c_L - c_S)}{(T_L - T_S)^3} - \frac{60(T_L + T_S) Q}{(T_L - T_S)^5} \\ c_5 &= \frac{30 Q}{(T_L - T_S)^5} \end{aligned} \tag{9}$$

3. Inverse problem

As was mentioned before, if the parameters appearing in governing equations are known then the direct problem is considered, while if part of them is unknown then the inverse problem should be formulated. In this paper we assume that the volumetric latent heat Q is unknown. So, in order to identify the value of Q, the additional information connected with the course of the process analyzed is necessary.

We assume that the values T_{di}^f at the selected set of points x_i from the domain considered for times t^f are known, namely

$$T_{di}^f = T_d(x_i, t^f), \quad i = 1, 2, ..., M, \quad f = 1, 2, ..., F \tag{10}$$

In order to solve the inverse problem, the least squares criterion is applied [7, 10]

$$S(Q)=\frac{1}{MF}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f-T_{di}^f\right)^2 \tag{11}$$

where $T_i^f = T\left(x_i, t^f\right)$ is the calculated temperature at the point x_i for time t^f for arbitrary assumed value of Q.

The criterion (11) is differentiated with respect to the unknown volumetric latent heat Q and next the necessary condition of optimum is applied

$$\frac{\mathrm{d}S}{\mathrm{d}Q}=\frac{2}{MF}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f-T_{di}^f\right)\left.\frac{\partial T_i^f}{\partial Q}\right|_{Q=Q^k}=0 \tag{12}$$

where k is the iteration number, Q^k for $k=0$ is the arbitrary assumed value of Q, while Q^k for $k>0$ results from the previous iteration. Next function T_i^f is expanded in a Taylor series about known value of Q^k, this means

$$T_i^f=\left(T_i^f\right)^k+\left.\frac{\partial T_i^f}{\partial Q}\right|_{Q=Q^k}\left(Q^{k+1}-Q^k\right) \tag{13}$$

Putting (13) into (12) one has

$$\sum_{i=1}^{M}\sum_{f=1}^{F}\left[\left(Z_i^f\right)^k\right]^2\left(Q^{k+1}-Q^k\right)=\sum_{i=1}^{M}\sum_{f=1}^{F}\left(Z_i^f\right)^k\left[T_{di}^f-\left(T_i^f\right)^k\right] \tag{14}$$

or

$$Q^{k+1}=Q^k+\frac{\sum_{i=1}^{M}\sum_{f=1}^{F}\left(Z_i^f\right)^k\left[T_{di}^f-\left(T_i^f\right)^k\right]}{\sum_{i=1}^{M}\sum_{f=1}^{F}\left[\left(Z_i^f\right)^k\right]^2} \tag{15}$$

where

$$\left(Z_i^f\right)^k=\left.\frac{\partial T_i^f}{\partial Q}\right|_{Q=Q^k} \tag{16}$$

are the sensitivity coefficients and $k=0, 1, ..., K$.

In order to determine the sensitivity coefficients appearing in equation (15), the governing equations (1), (2), (3), (4), (5) should be differentiated with respect to Q.

4. Results of computation

The casting-mould system shown in Figure 1 has been considered. The following input data have been introduced: $\lambda=30$ [W/(mK)], $\lambda_m=1$ [W/(mK)], $c_S=4.875$ [MJ/m^3 K], $c_L=5.904$ [MJ/m^3 K], $c_m=1.75$ [MJ/m^3], pouring temperature $T_0=1550$°C, liquidus temperature $T_L=1505$°C, solidus temperature $T_S=1470$ °C, initial mould temperature $T_{m0}=20$°C.

In order to identify the value of Q the temperatures (c.f. equation (10)) at the selected set of points from the casting-mould must be known. They result from the direct problem solution under the assumption that $Q=1984.5$ [MJ/m^3] (the identified parameter).

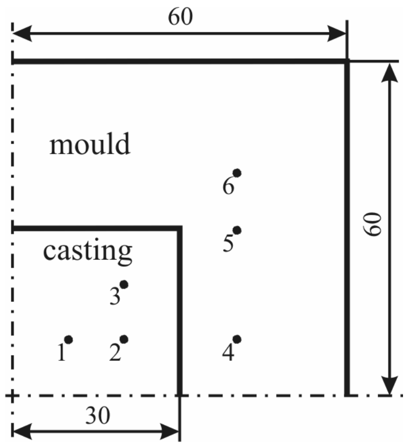

Fig. 1. Casting-mould system

The basic problem for the assumed value of Q and the additional one connected with the sensitivity function Z computations have been solved using the finite difference method. The regular mesh of dimensions 30×30 with constant step $h=0.002$ [m] has been introduced, time step $\Delta t=0.1$ [s].

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In Figure 2 the cooling curves at the selected set of points (see: Figure 1) from the domain considered are shown.

The testing computations show that it is possible to identify the unknown parameter Q only on the basis of one cooling curve knowledge. On the stage of numerical realization of iterative process it was assumed that the initial value $Q^0=0$. Figures 3 and 4 illustrate the course of iteration process. It is visible that the iteration process for the assumed initial value is convergent and the exact solution is obtained after 7 iterations.

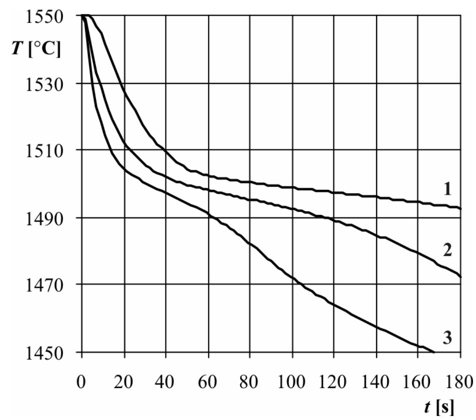

Fig. 2. Cooling curves at the points 1, 2, 3

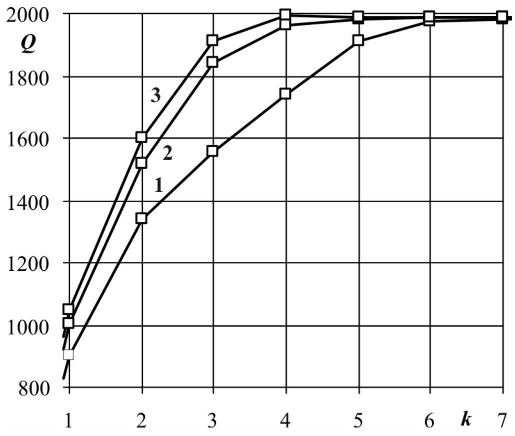

Fig. 3. Results of identification for sensors 1, 2, 3

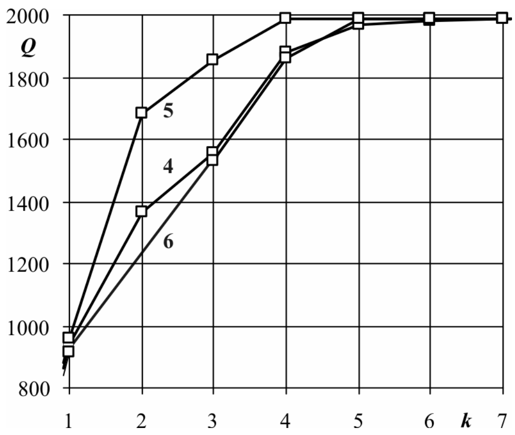

Fig. 4. Results of identification for sensors 4, 5, 6

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