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Piezoelectric layer modelling by equivalent circuit and graph method

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Methodology of research

ABSTRACT

Purpose: It is generally well know engineering applications of sandard materials for example aluminium, gold, steel etc. In recent years, piezoelectric materials beeing used to construct new mechatronical systems. In this paper equivalent circuit of piezoelectric was delivered and it was modeled by graph methods.This kind of modeling by grpah methods it is author's method.
Design/methodology/approach: Piezoelectric layer was modeled by Mason's equivalent circuit. Graph method for complex system was deivered. This method is based on the matrix method and the application of the graph aggregation.
Findings: After examination of piezoelectric layer, the matrix of impedance was delivered and complex system was analysis by graph methods. This matrix was author's idea for the impedance calculation of systems compound of many elements.
Research limitations/implications: Recurent formula and the analysis of other kinds of smart materials are proposed in the future research.
Practical implications: Piezoelectrics are widely applied as sensors and actuators. The advantages such as: little dimensions, simple structure, low noise factor at operating provide wide applications at vibration generating, damping, conversing of mechanical energy into electrical energy, elements of precise positioning and many others.
Originality/value: Graph method for piezoelectric layer for modeling complex system
Keywords: Smart materials; Piezoelement; Graph methods; Complex systems

1. Introduction

Many successfull engineering applications of sandard materials for example aluminium, gold, steel etc. In recent years, we are beginning to see smart materials beeing used to construct mechatronical systems. Increase of this applications what have properties scientists can manipulate. Each type of smart material has a different properties which can be changed, such as viscosity, volume, or shape. The properties that can be changed affect what types of applications the smart material can be used for. We can favour three main type of smart matherials:

- Piezoelectric materials applied as sensors and actuators.
- Electro-rheostatic and Magneto-rheostatic materials are fluids which can change their viscosity, there are used for damping vibration.
- Shape Memory Alloys are metals, which have two properties, pseudo-elasticity and the shape memory effect, there are used in gripping device.

In this article only piezoelectric effect is described. Piezoelectric are widely applied as sensors and actuators. Such materials show simple piezoelectrical phenomenon that consists in changing mechanical displacement into electrical voltage input signal, as well as opposite piezoelectrical phenomenon which generates mechanical displacement during voltage is applied to system plates (Fig. 1).

The advantages such as: little dimensions, simple structure, low noise factor at operating provide wide applications in vibration generating, damping, converse of mechanical energy into electrical energy and oppositely, elements of precise positioning and many others.

In this article the analysis of piezoelectricity phenomenon is presented on the grounds of Mason's equations. Matrix of dependency of input values on output values has been determined. In most cases the system considering merely external parameters is sufficient. Thus the system is limited to a few single four-terminal networks. There has been established the matrix determining the system from external terminals point of view. Next using the chain joint matrix of input-output dependences is provided.

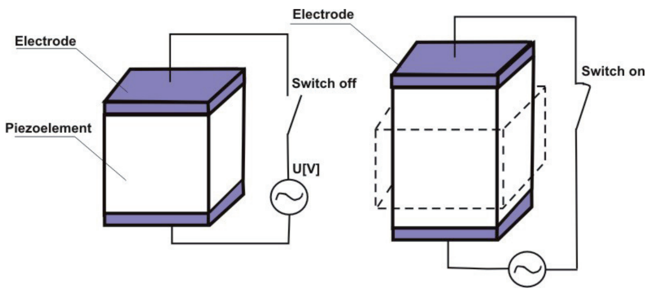

Fig. 1. Piezoelement with switch on and switch off power

The opportunity of applying graph method to non-classical determination of impedance of piezoelectrical plates will be analyzed.

2. System model under examination

In order to determine the characteristic of piezoelectric flexibility,the piezoelectric is modelled as an element described with four external parameters. Considering the mechanical system continuous in sections. Forces that affect movements at the element ends (Fig. 2).

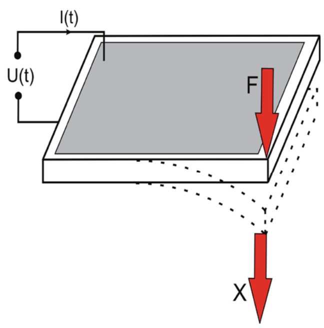

Fig. 2. Simple piezoelectric plate

Analyzing the phenomenon of piezoelectricity it is to consider one more electrical parameter, i.e. the voltage caused by movement. Supplementary parameter is also the current forced by generated voltage. Both, mechanical elements and piezoelectrical elements, as well, regarding the terminals can be analyzed as n-port.

It is an element determined with parameters necessary and sufficient to formulate relations between input-output parameters. Mathematic model of circuits flexibility shows the dependences between two-dimensional mechanical matrix presenting the force and the movement affected by that force. In general two vectors can be recorded as in formula 1.

$$A = \begin{bmatrix} U \\ I \end{bmatrix} \qquad B = \begin{bmatrix} F \\ x \end{bmatrix} \tag{1}$$

In general the relation between the input signal A and the response B is showed by the B = YA dependence.

The analysis will consist of creation of three matrixes describing the dependences of piezoelectric conducting layer, mechanical element and a matrix of transition. The matrix of input-output dependences can be obtained with general formula presented in [1-3, 8,13-15]

3. Matrix analysis of piezoelectric plate

On Figure 3 the substitute electrical diagram complying with the formula proposed by Mason is presented. The impedances recorded symbolically depend on the parameters of piezoelectrical plate such as overall dimensions, mechanical load, etc.[11-14, 4]

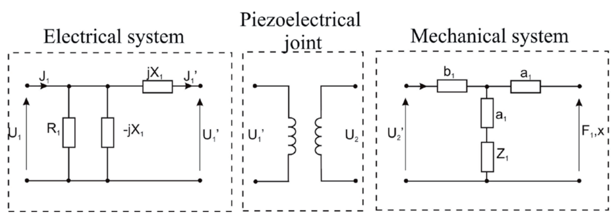

Fig. 3. Substitute disgram of piezoelectric

In order to construct a matrix describing electrical system the symbolic record presented on figure 3 has been applied.

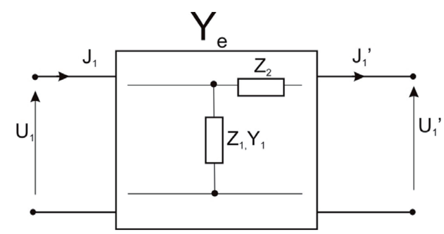

Fig. 4. Substitute diagram of electrical system

Matrix 2 describes the dependences of A,B,C,D parameters of figure 4 on enforcements, as well as it provides the explanations.

$$\begin{bmatrix} U_1 \\ J_1 \end{bmatrix} = \begin{bmatrix} 1 & jX_1 \\ \dfrac{R_1 - jX_1}{-jX_1 R_1} & -jX_1 \end{bmatrix} \begin{bmatrix} U_1^{'} \\ J_1^{'} \end{bmatrix} \tag{2}$$

Successively, the piezoelectrical joint was subject to analyzing as per analogy in electromechanical aspect is presented through transformer of ratio value equal n.

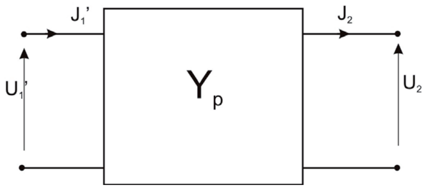

Fig. 5. Substitute diagram of piezoelectrical joint

The matrix describing the circuit presented on figure 5 with a formula 3 can be recorded by analogy to create matrix 2.

$$\begin{bmatrix} U_1^{'} \\ J_1^{'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N} & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} U_2 \\ J_2 \end{bmatrix} \qquad (3)$$

Mechanical system characterized by Z_a, Z_b, Z_c impedance parameters will be subject to analyzing (Fig.6).

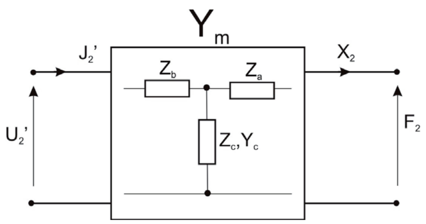

Fig. 6. Substitute diagram of mechanical part

$$\begin{bmatrix} U_2^{'} \\ J_2^{'} \end{bmatrix} = \begin{bmatrix} 1+\dfrac{Z_b}{Z_a+Z_1} & Z_a+Z_b+\dfrac{Z_aZ_b}{Z_a+Z_1} \\ \dfrac{1}{Z_a+Z_1} & 1+\dfrac{Z_a}{Z_a+Z_1} \end{bmatrix} \begin{bmatrix} F_2 \\ X_2 \end{bmatrix} \qquad (4)$$

Matrix 5 was obtained after chain joint of 2, 3, 4 matrixes, the matrix obtained in this manner describes dependences of voltage generated on piezoelectric plates depending on the force producing the effect on it.

$$Z = \begin{bmatrix} \frac{1}{n}(\frac{1+Z_b}{Z_a+Z_1})+\frac{jX_1n}{Z_a+Z_1} & \frac{1}{n}\frac{Z_a+Z_b+Z_aZ_b}{Z_a+Z_1}+jXn(\frac{1+Z_a}{Z_a+Z_1}) \\ \frac{-(R_1-jX_1)}{jX_1R_1n}(\frac{1+Z_b}{Z_a+Z_1})-\frac{jX_1n}{Z_a+Z_1} & \frac{-(R_1-jX_1)}{jX_1R_1n}(\frac{Z_a+Z_b+Z_aZ_b}{Z_a+Z_1})-jX_1n\frac{1+Z_a}{Z_a+Z_1} \end{bmatrix} \qquad (5)$$

4. A graph model of one plate

The model is presented on figure 7. The accepted model is characterised by the segment constant section and piezoelectric properties. Dislocations or current marked as ${}_1s_1$, ${}_1s_2$, caused by exciting forces or voltage ${}_2s_1$, ${}_2s_2$ [5, 6, 9, 16]. Linear dislocations are measured on the y axis starting from the beginning of the co-ordinate system. By mapping vertexes into dislocation due to equation 6 in a manner that ${}_1s_i \in {}_1S$ and ${}_1x_i \in {}_1X$, where i =0..2 hyper graph models the tested piezoelectric can be obtained by equation 6.

$$f: \quad {}_1S \rightarrow {}_1X, \qquad (6)$$

Relationships between the ${}_1s_1$, ${}_1s_2$ bar ends dislocations and forces that act on the bar ends are written down in equation 7.

$$\underset{f}{{}^kX} = \left[{}^kX, f\right] \qquad (7)$$

$${}_1S = Y\, {}_2S \qquad (8)$$

Equation 8 can be represented by the matrix form:

$$\begin{bmatrix} {}_1s_1 \\ {}_1s_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} {}_2s_1 \\ {}_2s_2 \end{bmatrix}, \qquad (9)$$

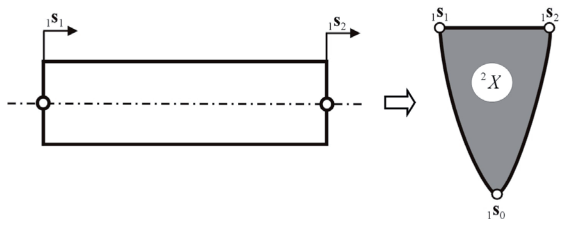

Fig. 7. The model and it is graph representation

On figure 7 graph modeled plate is presented. vertex ${}_1s_0$ is basic point. This electro-mechanical graph modelling is based on modeling mechanical system [1, 2, 7, 8-14].

5. A model of two connected plates

The model with two plates of the continuous section is presented in Fig. 8

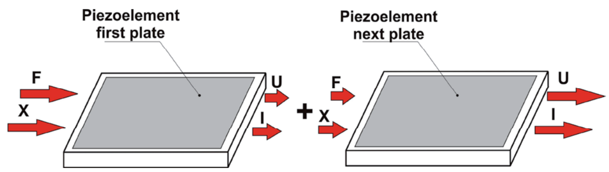

Fig. 8. An example of the connectd piezoelectric plates

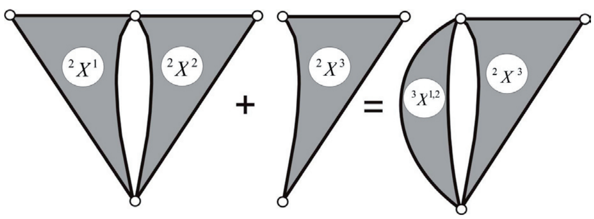

Fig. 9. Method of aggregation presented by graph

On figure 9 aggregation of two plates is presented. It is done by determination matrix impedance of two plates and later adding impedance of new plates. The main idea of aggregation is presented in author's article [1-7, 10,16,17]

Analogically to equation 5 relationships between dislocations of each section ends and forces causing dislocations were presented. It is showed by equation 5.

$$\begin{bmatrix} {}_2 s_1^{(n)} \\ {}_2 s_2^{(n)} \end{bmatrix} = \begin{bmatrix} Z_{11}^{(n)} & Z_{12}^{(n)} \\ Z_{21}^{(n)} & Z_{22}^{(n)} \end{bmatrix} \begin{bmatrix} {}_1 s_1^{(n)} \\ {}_1 s_1^{(n)} \end{bmatrix}, \tag{10}$$

The sum of the main flexibilities matrix and the additional flexibilities matrix was determined. After applying symbolical reductions the equation 11 was obtained.

$$Z = \begin{bmatrix} Z_a^{(n)} & Z_c^{(n)} & 0 \\ Z_d^{(n)} & Z_b^{(n)} + Z_a^{(n+1)} & Z_c^{(n+1)} \\ 0 & Z_d^{(n+1)} & Z_b^{(n+1)} \end{bmatrix} \tag{11}$$

Flexibilities of systems were determined in the equation 10 which is the recurrent specification of the following element of the complex system. After mathematical transformations the equation 11 was obtained.

For derivation of the equation 11 the correctness of the received solution by substitution of the same values describing each section and length of each section $L_1=(1/n)L$, $L_2=(1/n)L...L_n=(1/n)L$ was verified.

6. Conclusions

In this paper equivalent circuit model based on Mason's model for piezoelectric layer was proposed.

It was create the matrix describing input and output piezomaterial signals (equation 5).

The particular elements of the substitute system consist of three four-terminal networks that consider the system as mechatronic, i.e. consisting of electrical-mechanical elements. The particular study the analysis referred merely to input-output parameters that means a traditional n- port. On the base of analyse the phenomenon of piezoelectricity with graphs method the matrix is presented.After examination of one plate it was conected another piezoelectric plate fig. 8. This algebraic equation 11 is the author's idea for the flexibility calculation of systems compound of many elements. This is based on the matrix method and the application of the graph aggregation [15,13]. Futhure It was studed the opportunity to change characteristics to examine complex circuits will be analyzed.

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