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ADMISSIBLE STRESSES AS CRITERION FOR SLS

1. Introduction

Concrete structures which are repeatedly loaded, for example, bridges, must retain their elastic features in whole of their service life. According to CEB/FIP model code (MC78) [1] as well as to many national standards, e.g., [2,3,4] that condition is defined by the serviceability limit states (SLS). At SLS load stresses in steel and in concrete are compared to corresponding limit values of stress. Also crack widths and deflection values are chacked. The elastic behaviour of the structure is first of all guaranteed by established admissible stresses at SLS. The CEB/FIP model code (MC78) does not state admissible stresses for concrete, but while establishing them recommends taking into account specific features of the considered constructions. According to point 6.4.3 from MC 78 [1] and point 1.3 from FIP Recommendations [11] in SLS the stresses from external loads should fulfill the following relation

$$\sigma_d \leq \sigma_{adm} \quad (1)$$

where σ_d is the stress under SLS design loads and σ_{adm} is the admissible stress at SLS. Because the strength properties of concrete depends on the type of loads (sustained loads, repeated loads, fatigue loads) and on the stress state in considered elements, thus the value of σ_{adm} depends on this same influences. Having in mind this, the value of σ_{adm} may be expressed as a function of two factors γ_{mi} and $\gamma_{\sigma i}$ and characteristic

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strength of concrete in axial compression f'_{ck}

$$\sigma_{adm} = \frac{f'_{ck}}{\gamma_{mi}} \gamma_{\alpha i} \qquad (2)$$

In formula (2) γ_{mi} is a partial safety factor which depends on the type of loads action at SLS (index "i" define the type of loads action - see chapter 5) and $\gamma_{\alpha i}$ describe the properties of concrete in complex state of stress. For axial compression $\gamma_{\alpha i} = 1.0$. National standards, e.g.,[3,4] state different values of these stresses, but most of these values refer to axial state of stresses. The paper present a new method for defining the admissible stresses for concrete in the analyzied range of structure behaviour for axial and multiaxial state of stress. Bridge structures have been a given special attention. This method refers to basic microstructural properties of concrete which gave the basis for formulating criteria for elastic behaviour of concrete. This criteria take into account the nature of the load such as dead loads, live loads, fatigue loads sa well as the influence of environmental conditions. Taking these criteria into account, a probabilistic measure determining limit values of stress and next partial safety factor γ_{mi} and $\gamma_{\alpha i}$ which guarantee proper assumed probability of elastic behaviour of concrete has been defined.

2. Ψ^{I} and Ψ^{II} surfaces as limits of elastic behaviour of concrete

External loads cause changes in the structure of concrete such as the growth of existing microcracks and the formation of new ones. This phenomenon has been observed at any state of stress and one can distinguish three different phases at its course. Assuming that we describe properties of concrete, for example, in the space of principal stresses, there are three surfaces that set limits of these stages. They are:

- the surface setting the onset of stable fracture propagation (OSFP) Ψ^{I},
- the surface setting the onset of unstable fracture propagation (OUFP) Ψ^{II},

- failure surface (FS) Ψ^{0}.

Having in mind the aim of this paper the surface properties Ψ^{I} and Ψ^{II} are important.

2.1. Ψ^{I} surface properties

Ψ^{I} is a closed surface and symmetrical about the hydrostatic axis in the stress space as well as in strain space [5]. When the state of stress of concrete is defined by points $K(\sigma_1, \sigma_2, \sigma_3)$ which are situated inside the surface Ψ^{I} the most important proparties of concrete are the following:

- microcracks are stable and exist in the isolated parts of concrete and the direction of their increase is not explicitly stated,
- concrete may be treated as an isotropic body, and fulfills the requirements of Hoock's low for single and repeated loads,
- the stress path has a minor effect on the concrete properties,
- in the case of multiaxial compression state of stress the volume of the concrete decreases linearly.

Furthermore, it is observed that surface Ψ^{I} constitutes limit stress for sustained loads in which concrete may be treated as a linear viscoelastic body. For cyclic fatigue loads Ψ^{I} defines the fatigue strength of concrete which is independent of the number of cycles [6].

2.2 Ψ^{II} surface properties

Ψ^{II} is an open surface as well as a failure surface Ψ^{0}, in the stress space and in the strain space. According to [5] Ψ^{II} is nonsymetrical about the hydrostatic axis. When the stresses of concrete are defined by points $K(\sigma_1, \sigma_2, \sigma_3)$ which are between Ψ^{I} and Ψ^{II} surfaces, the most important properties of concrete are the following:

- microcrack are stable but they begin to form branches whose directions is compatible with the direction of maximum principal stress,
- the concrete properties begin to exhibit (in the limited range)

characteristics of an anisotropic body,

- in the case of multiaxial compression the volume of concrete decreases nonlinearly, so that a minimum value is reached for stress defined by Ψ^{II} surface,
- stress path has a minor effect on concrete properties,
- the number of stress cycles carried by concrete is limited but it is quite big and for this kind of stresses concrete shows properties of a linear - elastic body [6],
- concrete shows properties of nonlinear viscoelastic body for sustained loads and the stress defined by Ψ^{II} surface determunes the strength of concrete for sustained loads.

3. Criteria of elastic behaviour of concrete

Considering the above properties of concrete, the following criteria defining elastic behaviour of concrete in multiaxial states of stress are posited.

1. For all sustained loads - surface Ψ^{I}.
2. For the sum of sustained loads and cyclic loads causing such a stress state that at least one of the principal stress stress components is negative (tension) - surface Ψ^{II}.
3. For the sum of sustained loads and cyclic loads causing such a stress state that all principal stress components are positive (multiaxial compressoion) - surface Ψ^{II}.

The stresses defined by surface Ψ^{II} must be reduced when loads have a fatigue chatacter.

The above criteria guarantee that:

- structure for considered live loads combinations behaves elasticly,
- permanent strain caused by repeated loads and sustained loads are stabilized, and moreover, the concrete behaves according to the model of linear viscoelastic body,
- concrete fracture of a fatigued nature is ruled out.

In no case of structure can stresses defined by surface Ψ^{II} be exceeded. Thus, no fast directed marge of microcracks into branch system will result. Concrete structure will then not change to such a degree that concrete tightness could decrease and that the permeation of agressive compounds into its inside could be enabled. Hence, the mentioned criteria of concrete elastic behaviour also guarantee that the durability and concrete resistance to unfavourable environmental condition will not decrease.

4. The definition of admissible stress measure

Let the stress tensor from loads in any structure point be defined in terms of principale stress components σ_j (j=1,2,3). Let us compute the value

$$\xi^{\alpha} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} \quad \text{and} \quad \alpha_j = \frac{\sigma_j}{\xi^{\alpha}} \quad \text{for } j = 1,2,3 \qquad (3)$$

Value ξ^{α} is the absolute value of a vector with components σ_j and α_j is direction cosine of that vector in the space of principal stresses. Cosines α_j explicitly determine the straigth line l, which crosses in point K^I with coordinates σ_1^I, σ_2^I, σ_3^I the surface Ψ^I, in point K^{II} with coordinates σ_1^{II}, σ_2^{II}, σ_3^{II} the surface Ψ^{II}, and in point K^o with coordinates σ_1^o, σ_2^o, σ_3^o, the surface Ψ^o. The vectors $\overline{OK}^I$, $\overline{OK}^{II}$, $\overline{OK}^o$, where points O is the origin of the coordinate system are collinear, and their absolute values are the following:

$$|\overline{OK}^I| = \left[\sum_{j=1}^{3} \left(\sigma_j^I \right)^2 \right]^{0.5} = \omega^I \qquad (4a)$$

$$|\overline{OK}^{II}| = \left[\sum_{j=1}^{3} \left(\sigma_j^{II} \right)^2 \right]^{0.5} = \omega^{II} \qquad (4b)$$

$$|\overline{OK}^o| = \left[\sum_{j=1}^{3} \left(\sigma_j^o \right)^2 \right]^{0.5} = \omega^o \qquad (4c)$$

Considering the fact that the strength properties of concrete are random variables, the location of surfaces Ψ^I, Ψ^{II} and Ψ^o in space of principal stresses may be defined only with a certain probability. It follows that coordinates of points K^I , K^{II} ,and K^o, and the measures ω^I , ω^{II} and ω^o assigned to them, are also random variables. The criteria defined in point 3 circumscribe the limit surface of the elastic behaviour of concrete, depending on the type of the acting load. The surface in question is Ψ^I or Ψ^{II}. It is suggested that the probabilistic measure of stress for this range of concrete activity be defined in the following way.

Let random variable ω be ω^I when the surface Ψ^I is the limit stress (criterion 1), or let random variable ω be ω^{II} when surface Ψ^{II} is the limit stress (criterion 2).

Definition.

Quantile on the probability level p of the random variable ω ($\omega = \omega^I$, or $\omega = \omega^{II}$), on the condition that the value of random variable ω^o (measure of point K^o) equals ω^o_k, will be called admissible stress measure for elastic behaviour of concrete k^α_b.

This definition means that the probability of exceeding the elastic state, on the condition that the value of random variable $\omega^o = \omega^o_k$, equals p , and this can be formulated as:

$$P\,[(\omega / \omega^o = \omega^o_k) \le k^\alpha_b\,] = \int_{-\infty}^{k^\alpha_b} f_\omega(\,\omega / \omega^o = \omega^o_k)\,d\omega = p \qquad (5)$$

In Eq. (5) the function $f_\omega(\omega/\omega^o = \omega^o_k)$ is the conditional probability denisty function of the random variable ω, and ω^o_k is the quantile of the random variable ω^c at the probability level p_c, earlier defined. Assuming that the random variables ω^I, ω^{II} and ω^o are normal random variables (which is justified in the light of research), and taking into account Eq.(5), the formula for the admissible stress measure for concrete at SLS is the following:

$$k^\alpha_b = \bar{\omega}\,[\,1 - \nu_\omega\,(\,r_{\omega,\omega^c}\,k(p_c) + k(p)\sqrt{1 - r^2_{\omega,\omega^c}}\,)] \qquad (6)$$

The relationship (6) takes into account two states significant for concrete structutre :

- elastic behaviour of concrete dependant on the mean value $\bar{\omega}$

and on the coefficient of variation ν_{ω}, and

- failure state characterized by the random variable ω^{o} whose parameter is value $k(p_o)$. The value of the quantile ω_k^o is a function of that parameter. Besides, both states are related to each other by means of the correlation coefficient r_{ω,ω^o}. Depending on the accepted probability level, $k(p)$ is set; for instance for $p = 0.05$ and $p_o = 0.05$ the value $k(p) = k(p_o) = 1.645$. By computing Eq.(6) the formal assumption was made that the probability p and p_o are not greater than 0.50, which leads to (because of normal distribution of function ω and ω^o) $\omega_k^o \leq \bar{\omega}^o$ and $k_b^{\alpha} \leq \bar{\omega}$.

It follows from Eq.(6) that for certain value r_{ω,ω^o} the value k_b^{α} is minimal. This minimal value k_b^{α} is obtained when

$$r_{\omega,\omega^o} = \frac{k(p_o)}{\sqrt{k(p)^2 + k(p_o)^2}} \qquad (7)$$

Substituting Eq. (7) to Eq. (6), the minimum value of k_b^{α} with respect to r_{ω,ω^o} is obtained. This value is expressed as

$$k_b^{\alpha min} = \bar{\omega}\left(1 - \nu_{\omega}\sqrt{k(p)^2 + k(p_o)^2}\right) \qquad (8)$$

Equation (8) may be used when there are no reliable data on the correlation value between the states defined by the surfaces Ψ^{I} and Ψ^{o} or Ψ^{II} and Ψ^{o}.

Testing whether the elastic state has not been exceeded will consist in showing that the following inequality has been fulfilled:

$$k_b^{\alpha} \leq \xi^{\alpha} \qquad (9)$$

This comparsion should be made for all possible load combinations respecting the rule saying that the measure values ξ^{α} and k_b^{α} must correspond to the same direction in the space of principle stresses. The superscript α of these two measures points to this direction. Having in mind the relatinship (2) and Eq. (6) or (8) we may compute the values of partial safety factors γ_{mi} and γ_{ai} at SLS. This method will be presented in following chapter.

5. Admissible stress measure and partial safety coefficents forreinforced and prestressed concrete structures

Equation (6) and (8) and the criteria of the elastic behaviour of concrete, formulated in point 3, are the basis for deriving detailed equations which lead to computing admissible stress measures for individual types of loads. Equations for k_b^{α} have been derived for:

- dead loads k_{b1}^{α} (partial safety factor γ_{m1})
- sum of dead loads and cyclic (live) loads in the case when one of the principal stress components σ_j is the less than 0 (tension) - k_{b2}^{α}, (partial safety factor $\gamma_{m2'}$) and in the case when all the principal tensor stress components σ_j are greater than 0 (multiaxial compression) - k_{b2}^{α} (partial safety factor γ_{m2}).

When deriving the equations the following assumptions are made:

- the probability of the fact that the random variable $\omega^{o} \leq \omega_k^{o} = 0.05$ and then $k(p_o) = 1.645$,
- for dead loads, the condition defined by equation (5) is met with $p = 0.50$, and then $k(p) = 0.0$,
- for dead and live loads condition (5) is met with $p = 0.05$, and then $k(p) = 1.645$.

The probability level p = 0.05 of fulfilled condition (5) for the sum of dead and live loads, is usualy assumed for materials properties at SLS. For dead loads, the meeting of condition (5) with p = 0.50, guarantees that criterion 1 of point 3 will be fulfilled on the avarage for all cross sections of the considered construction element. This is the sufficent requirements in order for deformations caused by the sustained loads not to be in the range of nonlinear creep of concrete. With these assumptions the equations for k_b^{α} for individual types of loads are the following:

$$k_{b1}^{\sigma} = \bar{\omega}^{-1}(1 - 1.645\, r_{\omega^{I}\omega^{o}}\, \nu_{\omega^{I}}) \qquad (10)$$

$$k^{\alpha}_{b2'} = \bar{\sigma}^{I}\left[1 - 1.645\,\nu_{\sigma^{I}}\left(r_{\sigma^{I},\sigma^{o}} + \sqrt{1 - r_{\sigma^{I},\sigma^{o}}}\right)\right] \quad (11)$$

$$k^{\alpha}_{b2} = \bar{\sigma}^{II}\left[1 - 1.645\,\nu_{\sigma^{II}}\left(r_{\sigma^{II},\sigma^{o}} + \sqrt{1 - r_{\sigma^{II},\sigma^{o}}}\right)\right] \quad (12)$$

It follows from the properties of the limit surfaces Ψ^{I} and Ψ^{II} that $k^{\alpha}_{b2'}$ and k^{α}_{b2} determine the minimum and the maximum values of limit stresses for the sum of dead and live loads. The manner of considering fatigue loads was described in [7].

5.1. Admissible stresses and partial safety factor for axial compression

In axial compression initiation stress σ_i $(\sigma^{I} = \sigma_i)$ correspond to the surface Ψ^{I}, critical stress $(\sigma^{II} = \sigma_{CR})$ to surface Ψ^{II}. and the strength of concrete f'_c $(\sigma^{o} = f'_c)$ corresponds to the sutface Ψ^{o}. Table 1 gives the admissible stresses for this state of stress. These values have been computed using the random symulation method described in [7] for estimating the unknow parametrs from Eqs. (10) - (12). In the

Table 1. Admissible stresses and partial safety factor γ_{mi} at SLS for uniaxial compression.

Concrete grades* f_{cu} [MPa]		B 20 (C 16)	B 25 (C 20)	B 30 (C 25)	B 40 (C 30)	B 50 (C 40)	B 60 (C 50)
Admissible stresses	k_{b1}	8.0	9.6	11.3	14.3	17.2	20.0
	γ_{m1}	2.50	2.60	2.65	2.80	2.90	3.00
	k_{b2}	10.3	12.8	15.4	20.5	25.6	30.8
	γ_{m2}	1.95	1.95	1.95	1.95	1.95	1.95
	$k_{b2'}$	6.5	7.9	9.4	12.5	15.2	17.6
	$\gamma_{m2'}$	3.10	3.15	3.20	3.20	3.30	3.40

* Concrete grades is defined in cubes in accordance with Polish code.

computations it was assumed that the mean value $\bar{\sigma}_i = 0.45$ and $\bar{\sigma}_{CR} = 0.80$. Table 1 gives also the values of partial safety factor γ_{mi}. These values were computed using formula (2). The value of factor $\gamma_{\alpha i}$ for axial compression is equal 1.0.

5.2 Partial safety factors $\gamma_{\alpha i}$ for biaxial state of stress

In the biaxial state of stress ($\sigma_1 \geq \sigma_2 \geq 0$; $\sigma_3 = 0$) the surfaces Ψ^{I}, Ψ^{II}, and Ψ^{o} get reduced to limit curves - Figure 1. Using experimental

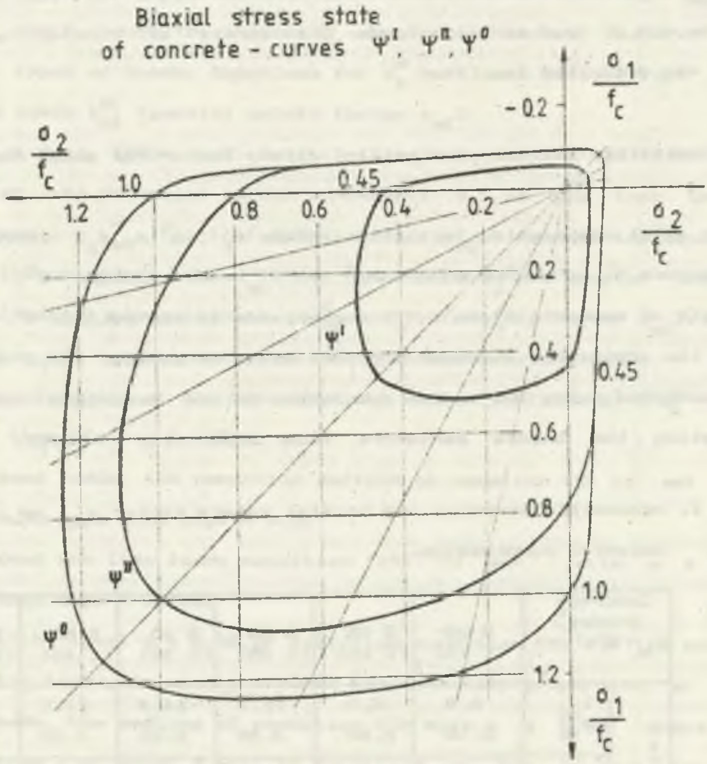

Fig. 1

results as well as expression (10) to (12), and Eq.2 partial safety factor $\gamma_{\alpha i}$ for biaxial state of stress were computed. Results of the computations for concrete grades B20 to B40 (concrete grade is defined on cubes in accordance with Polish code) are given in table 2. As there no reliable statistical data concerning the correlations between curves Ψ^{I} and Ψ^{o} or between Ψ^{II} and Ψ^{o} so for computing k_b^{α} the values of correlation coefficient defined by equation (7) were accepted. The manner of evaluating the coefficients of variation ν_{σ}^{I} i ν_{σ}^{II} was given in paper [8] using the information included in papers [5,9,10]. Table 2 gives values of partial safety factor $\gamma_{\alpha i}$ valid only for the direction of principal stress σ_1. Partial safety factor $\gamma_{\alpha i}$ in the second direction of principal stresses are obtained by multiplaying the values included in the table 1 by the ratio $\beta = \sigma_1/\sigma_2$. The values of partial safety factor γ_{mi} for biaxial state of stress are equal to values of γ_{mi} for axial compression, and are given in table 1. Admissible stresses at SLS for biaxial compression one can obtain using formula (2) and values of γ_{mi} from table 1 and $\gamma_{\alpha i}$ from table 2.

Table 2. Partial safety factor $\gamma_{\alpha i}$ at SLS for biaxial state of stress

	$\beta = \frac{\sigma_1}{\sigma_2}$	Concrete grades [MPa]								
		B 20 (C 16)			B 30 (C 25)			B 40 (C 30)		
		$\gamma_{\alpha 1}$	$\gamma_{\alpha 2'}$	$\gamma_{\alpha 2}$	$\gamma_{\alpha 1}$	$\gamma_{\alpha 2'}$	$\gamma_{\alpha 2}$	$\gamma_{\alpha 1}$	$\gamma_{\alpha 2'}$	$\gamma_{\alpha 2}$
c-c	0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.2	1.15	1.15	1.15	1.15	1.15	1.20	1.15	1.15	1.20
	0.4	1.25	1.25	1.20	1.25	1.25	1.15	1.25	1.25	1.20
	0.6	1.25	1.25	1.20	1.25	1.25	1.15	1.25	1.25	1.20
	0.8	1.25	1.25	1.15	1.25	1.25	1.10	1.25	1.25	1.15
	1.0	1.15	1.15	1.00	1.15	1.15	1.00	1.15	1.15	1.05
c-t	-0.02	-	0.90	-	-	0.90	-	-	0.90	-
	-0.04	-	0.85	-	-	0.85	-	-	0.85	-
	-0.10	-	0.55	-	-	0.55	-	-	0.55	-
	-0.20	-	0.30	-	-	0.30	-	-	0.30	-
	-0.40	-	0.20	-	-	0.20	-	-	0.20	-
t-t	-	-	-0.10	-	-	-0.10	-	-	-0.10	-

c-c - compression - compression, c-t - compression - tension,
t-t - tension - tension

6. Conclusion

The method of determining the admissible stresses σ_{adm} and partial safety material factors γ_{mi} and $\gamma_{\alpha i}$ at SLS described above consider simultaneous two significant working stages for concrete structure; elastic state and failure state. This method refers to the microstructural properties of concrete. The admissible stresses at SLS defined on the basis of this method, guarantee that for any case of loads a structure with a set probability will behave elasticly. The stress defined by k_b^{α} defines the admissible stresses at SLS. The values of admissible stresses and partial safety material coefficents included in tables 1 and 2 may be used in designing concrete structures subjected to cyclic and sustained loads.

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Summary

The analysis of microstructural properties of concrete both for dead and live loads in multiaxial state of stress has provided the basis for criteria which define the ultimate stresses of the elastic behaviour of concrete. Next the probabilistic measure of those stresses was defined. That measure was named admissible stresses at SLS. The value of the admissible stresses and partial safety material factors at SLS both for biaxial state of stress and uniaxial compression was calculated.

DOPUSZCZALNE NAPRĘŻENIA JAKO KRYTERIUM

W STANACH GRANICZNYCH UŻYTKOWANIA

Streszczenie

Graniczne naprężenia gwarantujące sprężyste zachowanie się betonu w złożonych stanach naprężeń określono w oparciu o mikrostrukturalne własności betonu. Rozważono własności betonu pod obciążeniami stałymi i zmiennymi. Nastepnie określono probabilistyczną miarę wyznaczającą te graniczne naprężenia. Miarę tą nazwano naprężeniami dopuszczalnymi w stanach granicznych użytkowania (SGU). Wartości naprężeń dopuszczalnych oraz częściowe współczynniki materiałowe w SGU podano dla osiowego ściskania oraz płaskiego stanu naprężeń.

ДОПУСКАЕМЫЕ НАПРЯЖЕНИЯ КАК КРИТЕРИЙ ВО ВТОРОМ ПРЕДЕЛЬНОМ СОСТОЯНИИ

Содержание

Предельные напряжения гарантирующие упругую работу бетона в сложном напряженном состоянии автор назначил на основании микроструктурных свойств бетона. Рассуждено свойства бетона при постоянных и переменных нагружениях. Автор определил вероятностную меру этих предельных напряжении и назвал допускаемыми напряжениями во втором предельном состоянии. В статии предложено величины допускаемых напряжении и частимых факторов для осего сжатия и плоского напряженного состояния.