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Peter KUKUCA
Departament of Measurement
Slovak Technical Univesity
Bratislava

MEASUREMENT OF REACTIVE POWER OF NONSINUSOIDAL VOLTAGE AND CURRENT

Summary. The paper describes the measurement method of reactive power of nonsinusoidal voltage and current. Possibilities of construction of varmeter phase shifting circuits with noninductive ladder networks are given. Erros smaler than 1 % in frequency range 50 Hz - 2 kHz have been achieved.

1. Introduction

Reactive power Q of nonsinusoidal voltage and current is most often defined as follows

$$Q = \sum_{k=1}^{\infty} U_k I_k \sin \varphi_k \qquad (1)$$

where U_k, I_k are the RMS values of the k-th harmonic of the voltage and the current and φ_k is the phase shift between the voltage and the current of the k-th harmonic.

Measurement of this reactive power is very difficult, because there is no instrument directly measuring this quantity. The methods used under sinusoidal conditions are not suitable because of their frequency dependance and other limitations.

The equation (1) can be rewritten

$$Q = \sum_{k=1}^{\infty} U_k I_k \cos (\varphi_k - 90^\circ) \qquad (2)$$

This equation shows that the measurement of reactive power can be completed with an active wattmeter with a phase shifting circuity in one input.

Realisation of this circuit called a Hibert transformer was reported e.g, in [1]. The solution with frequency dependent negative resistors (FDNR) is not very convenient, because many electronic parts have to be used only a small frequency range can be obtained. The Hilbert tranformer with 15 operational amplifiers, 23 precise resistors and 11 precise capacitors of different values and with some common passive components has an amplitude and phase shift error 1 - 5 % in the frequency range 50 Hz - - 450 Hz.

The equation (2) can be modified as follows

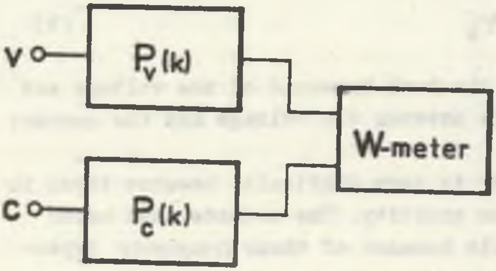
$$Q = \sum_{k=1}^{\infty} P_k\, U_k\, \frac{1}{P_k}\, I_k \cos\left[\varphi_k - \alpha_k - (90^o - \alpha_k)\right] \qquad (3)$$

what enables another approach to the measurement of reactive power (See Fig.1). Circuits influencing phase shift and amplitude of the signals are placed in both inputs of the wattmeter [2] . If the transfer functions of these circuits for the k-th harmonic are

$$P_v(k) = P_k\, e^{j(90^o - \alpha_k)} \qquad (4)$$

$$P_c(k) = \frac{1}{P_k}\, e^{-j\alpha_k} \qquad (5)$$

where P_k and $1/P_k$ are frequency dependent amplifications and $90^o - \alpha_k$ and $-\alpha_k$ are frequency dependent phase shifts, then the whole block diagram in the Fig.1 presents a varmeter. The discussed reactive power measurement methods based on the Budeanu's definition can be in many respects criticized - what is mare precisely described in paper [5]. Nevertheless these methods are analysed literature, for example, [6,7], where appriopriate circuits are indicated with their tested application. The present paper introduces other, checked by the author, solution, which is similar to of the method given in paper [6].



Rys.1 Schemat blokowy waromierza
Fig.1 Block diagram of the varmeter

2. Possibilities of phase shifting circuits construction

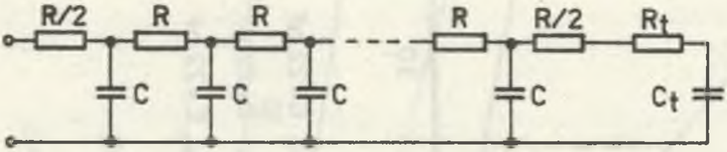
The input impedance $Z_1(k)$ of noninductive line, which is long enough, is equal to the wave impedance

$$Z_1(k) = \sqrt{\frac{R}{2\, j\pi k f_o C}} \qquad (6)$$

where k is the order of the harmonic and f_o is the frequency of the fundamental harmonic. The phase of this impedance is -45° independent of the frequency and its magnitude decreases with the slope of 10 dB/decade. This enables us to realize the circuits with the tranfer functions

$$P_v(k) = \pm \sqrt{k} e^{j45^\circ} \tag{7}$$

$$P_o(k) = \pm \frac{1}{\sqrt{k}} e^{-j45^\circ} \tag{8}$$



Rys.2 Przesuwnik fazowy z wykorzystaniem linii drabinkowej RC
Fig.2 Phase shifting circuits with RC - ladder network

For low-frequency applications the RC ladder networks are to be used (Fig.2). The values of R and C and the number of cells depend on the frequency range and the required accuracy.

Because it is not possible to use an infinite number of cells, the problem of the first order is the termination of the network. The length of the line is critical for the lower limit of the frequency range. Because of that the best termination of the ladder network is the wave impedance for the lowest frequency i.e. for k=1. For the terminal impedance

$$R_t = \sqrt{R/4\pi f_o C} \tag{9}$$

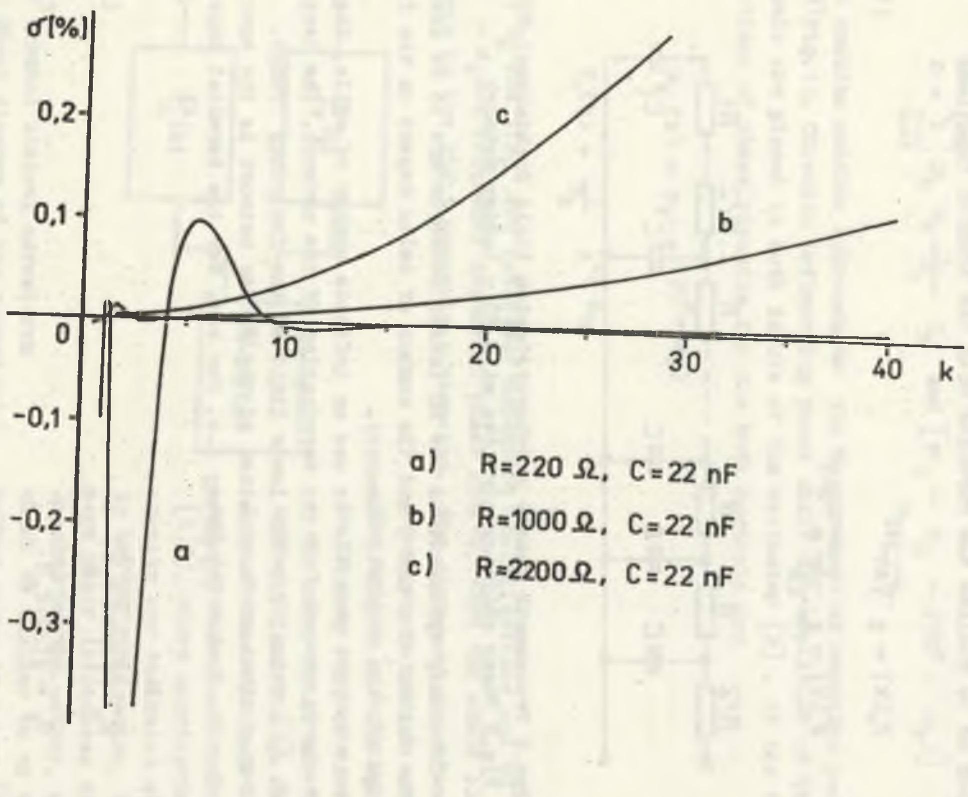
$$C_t = \sqrt{C/\pi f_o R} \tag{10}$$

Some calculated dependences of amplitude and phase shift errors of 50-cells ladder networks are shown in Figs.3 and 4.

Number of cells can be reduced by dividing the network, into two parts with the same wave impedances, but different time constants of the cells (Fig.5). Following equation applies here

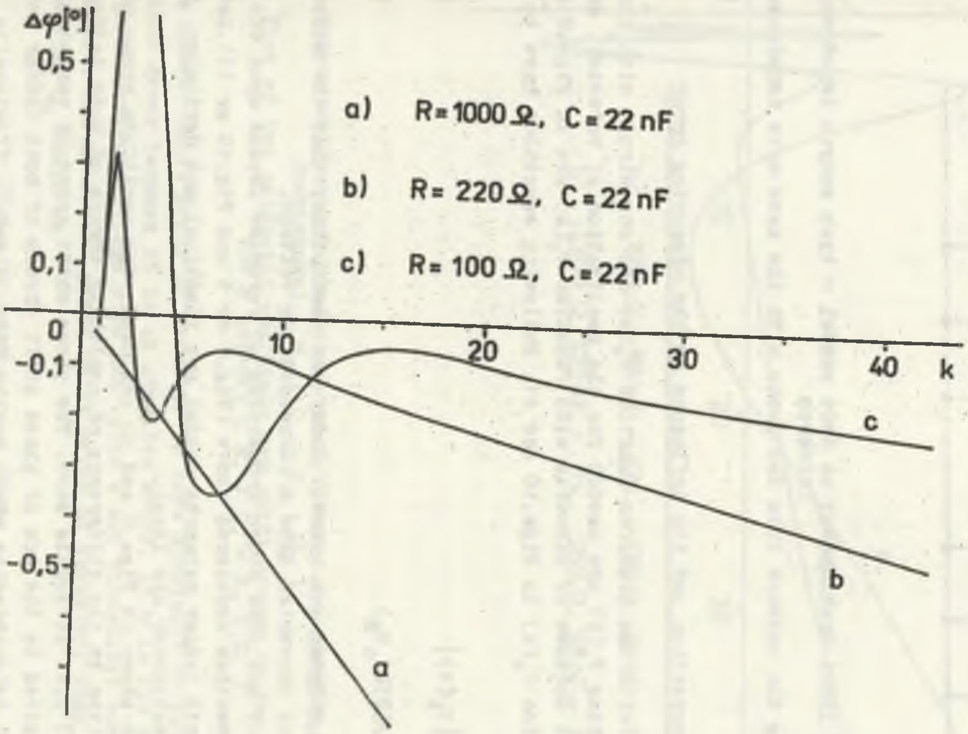
$$R_1/C_1 = R_2/C_2 \tag{12}$$

Calculated dependences of amplitude and phase shift errors of two such ladder networks are shown in Figs.6 and 7.



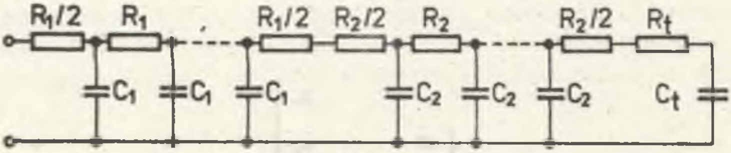
Rys.3. Wykres obliczonych błędów amplitudy i fazy dla 50-elementowej linii drabinkowej RC

Fig.3. Plot of the calculated amplitude and shift errors of 50-cells ladder network



Rys.4. Wykres obliczonych błędów amplitudy i fazy dla 50-elementowej linii drabinkowej RC

Fig.4. Plot of the calculated amplitude and shift errors of 50-cells ladder network



Rys.5 Podział linii drabinkowej na dwie części o tych samych impedancjach falowych

Fig.5 Dividing the network into two parts with the same wave impedaces

3. Varmeter connection and its influence on the measuring error

One circuit with the transfer function $P_v(k)$ and one ciruit with the transfer function $P_c(k)$ are needed for the realization of varmeter, according to Fig.1 Schemes of circuits with function $P_v(k)$ are in Figs.8 and 9, with function $P_c(k)$ in Figs.10 and 11. Following equations have to be valid

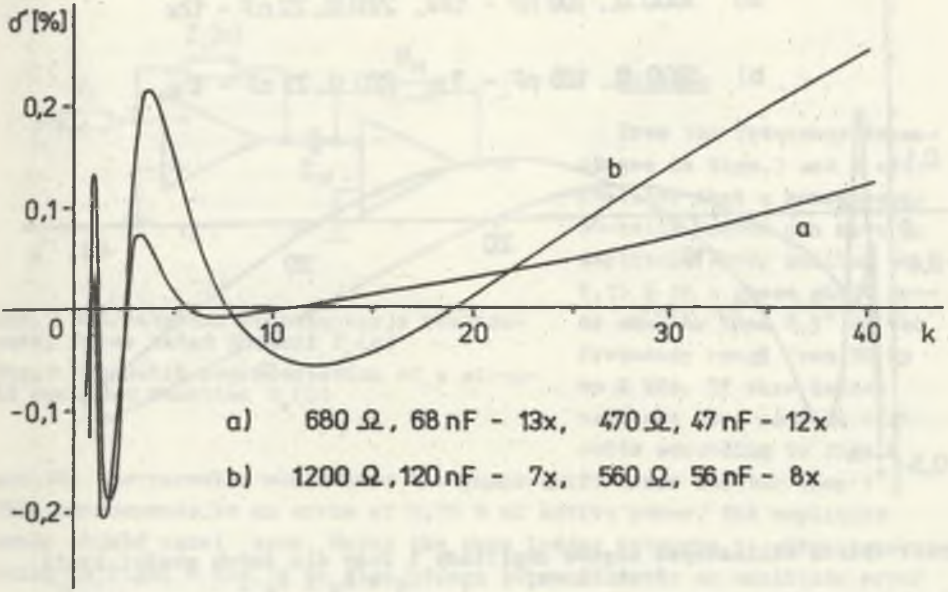
$$R_N = \left| Z_1(1) \right| \qquad (12)$$

$$R_M = 1/(2\pi f_0 C_M) \qquad (13)$$

Remark: These schemes are correct under the assumption, that the measured current was converted into a corresponding voltage.

The accuracy and some other properties of varmeter depend upon the choice of schematics mentioned above (Fig.8 or 9 and Fig.10 or 11).Let's assume that both ladder networks needed are identical and derivator, integrator and wattmeter are ideal.

If circuits shown in Figs.8 and 10 are used, the amplitude error of varmeter is given by the difference of amplitude errors of both ladder networks $Z_1(k)$ i.e. it equals zero. The phase shift error of varmeter in this case is given by the sum of phase shift errors of both ladder networks, i.r. it is doubled as shown in the Figs. 4 and 7. If circuits shown in Figs. 8 and 11 (in Figs. 9 and 10 respectively) are used, the phase shift error of varemeter is given by the difference of phase shift errors of both ladder networks $Z_1(k)$ i.e. it equals zero. The amplitude error of varmeter in this case is given by the sum of amplitude errors of both ladder networks, i.e. it is doubled as shown in the Figs. 3 and 6.



Rys.6 Wykres obliczonych błędów amplitudy i fazy dla dwóch części linii drabinkowej ($R_1 : C_1 = R_2 : C_2$)

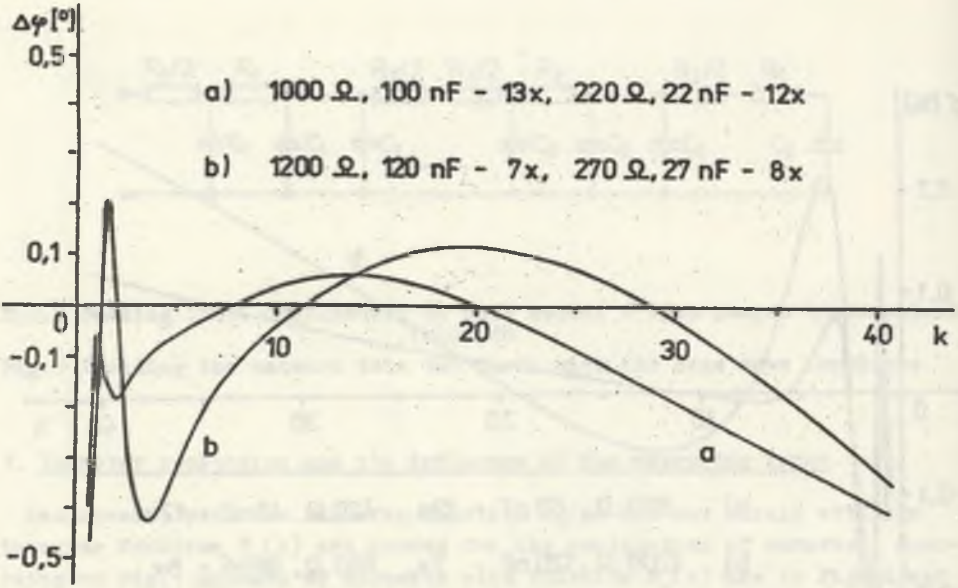
Fig.6 Plot of the calucated amplitude and shift errors of two parts of the a network ($R_1 : C_1 = R_2 : C_2$)

The phase shift error of varmeter is more difficult to calculate the amplitude error because it is an additive error. If φ is the phase shift between sinusoidal voltage with RMS value U and sinusoidal current with RMS value I and $\Delta\varphi$ is the phase shift error of varmeter, then the varmeter reading Q_m would be

$$Q_m = UI\sin(\varphi + \Delta\varphi) = Q\cos(\Delta\varphi) + P\sin(\Delta\varphi) \qquad (14)$$

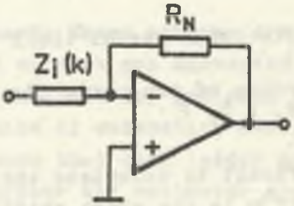
If the reactive power Q is small, the influence of the term $P\sin(\Delta\varphi)$ is significant and if Q is zero then the varmeter reading is not.

The amplitude error of varmeter is a multiplication error i.e. it causes the same relative error of reading independently from the reactive power value.



Rys.7 Wykres obliczonych błędów amplitudy i fazy dla dwóch części linii drabinkowej ($R_1 : C_1 = R_2 : C_2$)

Fig.7 Plot of the calculated amplitude and shift errors of two parts of network ($R_1 : C_1 = R_2 : C_2$)



Rys.8 Schematyczna reprezentacja realizowanej przez układ funkcji $P_v(k)$

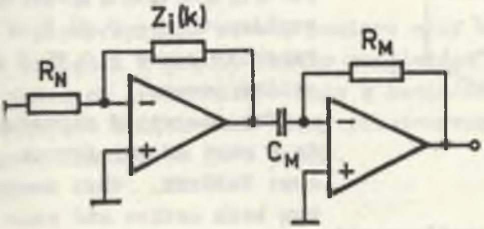
Fig.8 Schematic representation of a circuit realized function $P_v(k)$

Therefore it is more convenient to use such phase shifting circuit that phase shift error of the used ladder networks.

Complete scheme of electronic varmeter is more complicated because the output voltages should have no DC component. A proper method of operational amplifier offset voltage compensation was reported e.g. in [4].

The described method of reactive power measurement has a disadvantage that should be kept in mind. The phase shifting circuit with the transfer function $P_v(k)$ has the gain proportinal to the square root of the order of harmonic k and the phase shifting circuit with the transfer function $P_c(k)$ has the gain inversely proportional to the square root of the order of harmonics k. Signals with sharp edges cause in circuits with the transfer function $P_v(k)$ high voltage peaks. The output voltage of circuits with the transfer function $P_c(k)$ is small for higher frequencies.

The phase shifting circuits and the wattmeter used should therefore have a wide dynamical range.

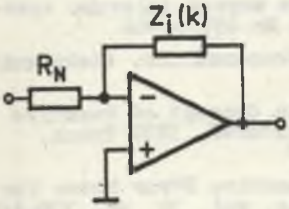


Rys.9 Schematyczna reprezentacja realizowanej przez układ funkcji $P_v(k)$

Fig.9 Schematic representation of a circuit realized function $P_v(k)$

Results

From the frequency dependences in Figs.3 and 4 one realizes that a homogeneous 50-cells ladder can have an amplitude error smaller than 0,15 % or a phase shift error smaller than 0,5° in the frequency range from 50 Hz to 2 kHz. If thes ladder networks were used in circuits according to Figs.8 and 10, the varmeter would have the phase shift error smaller than 1°. This corresponds to an error of 1,75 % of active power. The amplitude error should equal zero. Using the same ladder networks in circuits according to Figs. 8 and 11 or Figs. 9 and 10 would cause an amplitude error smmaller than 0,3 % of measured reactive power and a zero phase shift error. This is significantly better than in the previous case.



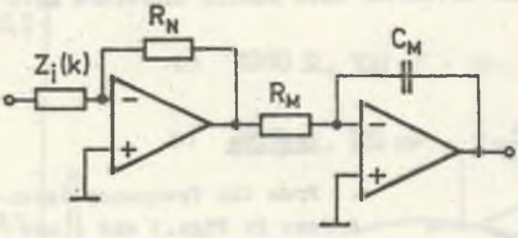
Rys.10 Schematyczna reprezentacja realizowanej przez układ funkcji $P_c(k)$

Fig.10 Schematic representation of a circuit realized function $P_c(k)$

The error of varmeter that could theoretically be reached with heterogeneous ladder networks (see Figs.6 and 7) are

- for 25-cells ladder networks: amplitude error < 0,4 % of rdg. phase shift error < 0,8°
- for 15-cells ladder networks: amplitude error < 0,5 % of rdg. phase shift error < 1,0°

A varmeter with input circuits shown in Figs. 8 and 11 with 50-cells homogeneous ladder networks has been built. Resistors and capacitors have not been chosen. The terminal resistor R_t was set for the best low frequency behaviour. The varmeter had the amplitude error smaller than ± 0,75 of reading and the phase shift error smaller than 16'. This corresponds to the total error smaller than ± 0,75 % of reactive power ± 0,5 % of active power.



Rys.11 Schematyczna reprezentacja realizowanej przez układ funkcji $P_c(k)$

Fig.11 Schematic representation of a circuit realized function $P_c(k)$

Later the orginal ladder networks were replaced by 25-cells heterogeneonus ones with chosen parts. The total error was smaller than ± 0,25 % of reactive power ± 0,25 % of active power,

The described varmeter is a part of the instrument VARWATT, that measures both active and reactive power of nonsinusoidal voltage and current. VARWATT has been using the modules of the system UNIWATT developed at the Department of Measurement of the Electrical Engineering Faculty of the Slovak Technical University.

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Recenzent: doc. dr hab. inż. Brunon Szadkowski

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POMIAR MOCY BIERNEJ NIESINUSOIDALNYCH PRZEBIEGOW NAPIĘCIA I PRĄDU

Sterszczenie

W pracy opisano metodę pomiaru mocy biernej niesinusoidalnych przebiegów napięcia i prądu. Podano możliwości konstrukcji waromierza bazującego na układach przesuwników fazy z bezindukcyjnymi liniami drabinkowymi. Osiągnięto dzięki temu błędy przetwarzania mniejsze niż 1% dla zakresu częstotliwości 50 Hz - 2 kHz.

ИЗМЕРЕНИЕ РЕАКТИВНОЙ МОЩНОСТИ НЕСИНУСОИДАЛЬНОГО НАПРЯЖЕНИЯ И ТОКА

Резюме

В работе описан метод измерения реактивной мощности несинусоидального напряжения и тока. Представлены возможности конструкции варметра основанного на системе фазовращателей с безиндуктивными цепными линиями для которого ошибки погрешности преобразования составляют не больше, чем 1% для диапазона частот 50гц - 2кгц.