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NUMERICAL ANALYSIS OF MECHANISMS WITH STRUCTURAL DATA

Summary. Defined in the paper are the structural data for numerical generation and solving of equations of constraints and kinematic dependences occurring in spatial mechanisms with geometrical constraints and constraints independent of time. The procedures obtained have been used for computer generation of the equations of motion of a system with Lagrange structure, and for their numerical solution.

NUMERYCZNA ANALIZA MECHANIZMÓW Z DANymi STRUKTURALNYMI

Streszczenie. W pracy zdefiniowano dane strukturalne do numerycznego generowania i rozwiązywania równań więzów oraz określania zależności kinematycznych charakteryzujących ruch w mechanizmach przestrzennych z więzami geometrycznymi i skleronomicznymi. Uzyskane procedury zastosowano do komputerowego generowania równań ruchu układu o budowie Lagrange'a oraz do ich numerycznego rozwiązywania.

ЦИФРОВОЙ АНАЛИЗ МЕХАНИЗМОВ СО СТРУКТУРНЫМИ ДАННЫМИ

Резюме. В работе определены структурные данные, которые применено для цифрового строения и решения уравнений связей и для определения кинематических зависимостей, характеризующих движение в пространственных механизмах с геометрическими и склерономными связями. Полученные процедуры приспособлено для цифрового строения системы уравнений Лагранжа и их численного решения.

1. INTRODUCTION

Presented in the paper are the test results obtained during realization of the program "Design computerization of machines and installations technology and production" with in the framework of the task "Methods of computer designing of mechanisms. Analysis of

senitivity, synthesis, optimization and testings of chaotic motions" [4,6,7]. The aim of the task was the preparation of computer calculations procedures and programs which can be used in the process of designing spatial mechanisms with many degrees of freedom, with geometrical and independent of time constraints.

To carry out an analysis in a complex mechanism simpler elements are isolated as suggested by Artobolewski [1], this process not being connected with their classification, but the need for the determination of the equations of constraints. Suggested is also another, more general than the most frequently used, method of reciprocal location of the systems of coordinates of the members connected by means of rotational pairs and sliding pairs, known as Hartenberg and Denavit recording [2,8].

Thanks to the elaboration of an analytical description of a complex mechanism structure an analytical determination of the method of the system reduction, more fully described in paper [5], its digital kinematic and dynamic analysis is possible also in the range of the generation of the equations of motions and equations of constraints.

2. FORMULATION OF THE PROBLEM

It is accepted that a mechanism is a system composed of a finite number of links in the form of rigid bodies cooperating with each other. It is also assumed that the links directly cooperating with each other in reciprocal motion have one degree of freedom. In the mechanism in which there occur direct joints of links with a greater number of reciprocal motions, also a substitute system with indirect links of zero inertia should be developed, equal in the kinematic aspect to the output system.

In each link, three so-called characteristic points are distinguished which constitute a system of three noncolinear points rigidly connected with the particular body. Making use of these points it is possible to define a system of Cartesian coordinates rigidly connected with the selected link.

The point coordinates in the Cartesian system can be presented as elements of a column matrix of the structure

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad (1)$$

where x_i is a projection of the vector of the position of the point on to the i -th axis of the system for $i = 1, 2, 3$.

The relation between the coordinates of the link point in an inertial fixed system and the local system of coordinates connected with the particular link is as follows:

$$\mathbf{X} = \mathbf{A} \mathbf{X}' + \mathbf{U}, \quad (2)$$

where

\mathbf{X} , \mathbf{X}' - column matrices defined in accordance with dependence (1) with the elements determining the point coordinates in the fixed and local system, respectively,

- A** - square matrix of the link revolutions,
- U** - column matrix of the link translation.

If the coordinates of the characteristic points of a body in a fixed system are determined by the column matrices **R₁**, **R₂**, **R₃** written acc. to (1), then the matrix **U** in relation (2) can be defined as follows

$$U = R_1 \quad (3)$$

Matrix **A** of the structure

$$A = [E_1, E_2, E_3] \quad (4)$$

is formed by versor components of a nonfixed system of coordinates expressed in relation to a fixed system and presented as column matrices **E₁**, **E₂** and **E₃**, also defined in accordance with (1). For this purpose are defined the column matrix

$$D = R_2 - R_1, \quad F = R_3 - R_1, \quad (5)$$

whose rows determine, respectively, the coordinates of the vectors **d** and **f**. The columns in relationship (4) are, correspondingly, the coordinates of the versors determined from the dependence

$$e_1 = \frac{d}{|d|}, \quad e_2 = \frac{d \times f}{|d \times f|}, \quad e_3 = e_1 \times e_2, \quad (6)$$

where the sign "×" denotes the vector product, and the symbol |·| denotes the vector length.

It is assumed that all the links of a mechanism are numbered with the successive natural numerals whose number is *N*. Also defined and numbered are the generalized coordinates describing the configuration of the cooperating pairs of links in the quantity *M*. Moreover, it is assumed that at least one of the coordinates should determine the position of the link cooperating with the frame (with the link of the number one).

The method of the link cooperation is determined by assigning the functions expressing the position of the characteristic points of the cooperating nonfixed link in a system of coordinates connected with the matching link, treated as a reference system. The position of the particular points of the mechanism and the equations of constraints are assigned indirectly by the structural data determining which links are matched with each other in kinematic pairs.

To a pair of cooperating links of the particular numbers is assigned a generalized coordinate with a number proper to it. This assignement characterizes the structure of the system and its existence can be presented in the following way by means of the numbers:

$$c_{ij} = \begin{cases} 0 & \text{for } i = j \text{ and when the } i\text{-th link is not cooperating with the } j\text{-th one} \\ k & \text{when there exists direct cooperation} \end{cases}, \quad (7)$$

where *c_{ij}* are elements of the so-called coincidence matrix **C** determined for *i, j = 1, 2, ..., N*,

whereas k is the number of the generalized coordinate describing the reciprocal position of the i -th and $-i$ -th link for $k = 1, 2, \dots, M$.

The structural parameters defined above, though depending on the method of numeration of generalized links and coordinates represent the structure of a real physical object. In the problems of dynamic analysis, the equations describing the evolution of the system in the form of Lagrange equations of type II [9], are solved in a system of independent coordinates so the independent coordinates should be isolated from all the coordinates. We assume as many of them as there are degrees of freedom of the mechanism. The numbers of coordinates are also treated as structural parameters - they do not refer to the physical system but are integrally associated with its mathematical model.

Such parameters are determined by the following vector \mathbf{cq} whose coordinates are the numbers of generalized coordinates selected as independent ones:

$$cq_i = k, \quad (8)$$

where

- cq_i - i -th coordinate of a vector whose value is the number of the successive,
- $i = 1, 2, \dots, I$,
- I - amount of the degrees of freedom of a mechanism,
- $k \in (1, 2, \dots, M)$.

Naturally, all the coordinates of the vector \mathbf{cq} must be different from each other. The choice of generalized independent coordinates is not fully optional. This will be demonstrated in the example in the further part of the work.

Instead of the structural data given in the form of the matrix \mathbf{C} and the vector \mathbf{cq} , it is convenient to make use, in numerical calculations, of other structural data.

Such processed structural data are defined in the form of four vectors with at least N_I coordinates, while

$$N_I = N + 2L_1 + L_2 - 1, \quad (9)$$

where

- L_1 - amount of closed kinematic chains in the whole mechanism,
- L_2 - amount of open kinematic chains of mechanism.

The following denotations of the particular vectors have been assumed:

- nrg** - for the numbers of links,
- nws** - for the numbers of generalized coordinates,
- rdw** - for the kind of coordinates,
- nrw** - for the numbers of equations of constraints.

The value of the first elements of vectors written below are established as

$$\begin{aligned} nrg_I &= 1, \\ nws_I &= 0 \text{ - conventional definition,} \\ rdw_I &= 1, \\ nrw_I &= 0. \end{aligned}$$

The above assumptions do not constitute any new restrictions for the structural data or the class of the systems tested. The first coordinate means that the examination of a mechanism starts from the fixed link.

The successive coordinates of the vectors mentioned are defined as follows:

- $nrg_i =$ the number of the link directly cooperating with the link nrg_{i-1} when nrg_{i-1} is not the last link of a closed kinematic chain (the reciprocal configuration of these links is described by the coordinate nws_i), or the least of the preceding coordinates nrg_j for which there exists a cooperating link not specified among the preceding elements of the vector nrg if the preceding link closes the kinematic chain;
- $nws_i =$ the least number of the coordinate describing the configuration of any of the mechanism elements with relation to the link of the no nrg_{i-1} , when the link nrg_i is not the first in the kinematic chain,
0, otherwise;
- $rdw_i = 1$, if the coordinate with the number nws_i is independent, or when $nsw_i = 0$,
0, otherwise;
- $nrw_i = nrw_{i-1}$, when the link nrg_i is not the last link in the closed kinematic chain,
 $nrw_{i-1} + 1$, otherwise.

It has been assumed that with a fixed link of the number one is connected a fixed system of coordinates. Then, dependences (2) for the particular links are established recurrently as below:

$$\begin{aligned} \mathbf{A}_{nrg_i} &= \mathbf{A}_{nrg_i}^{nrg_i} \left(q_{nws_i} \right), \\ \mathbf{U}_{nrg_i} &= \mathbf{A}_{nrg_i}^{nrg_i} \left(q_{nws_i} \right), \end{aligned} \tag{10}$$

$$\begin{aligned} \mathbf{A}_{nrg_{i+1}} &= \mathbf{A}_{nrg_{i+1}}^{nrg_{i+1}} \left(q_{nws_{i+1}} \right) \mathbf{A}_{nrg_i}, \\ \mathbf{U}_{nrg_{i+1}} &= \mathbf{A}_{nrg_{i+1}}^{nrg_{i+1}} \left(q_{nws_{i+1}} \right) \mathbf{U}_{nrg_i} + \mathbf{U}_{nrg_{i+1}}^{nrg_{i+1}}, \end{aligned}$$

where

- i - number of structural coordinate,
- q_i - generalized coordinate of the index i ,
- \mathbf{A}_j^k - matrix of relative rotation of the k -th link to the j -th link,
- \mathbf{A}_k - matrix of absolute revolution to the k -th link,
- \mathbf{U}_j^k - column matrix of relative translation of the k -th link to the j -th link,
- \mathbf{U}_k - column matrix of translation of the k -th link.

The quantities defined by relations (10) and their consequents are determined when fulfilled is the equality:

$$\text{nrw}_{j+1} - \text{nrw}_j = 0, \quad (11)$$

If for the successive element of the vector nrw the relation (11) is not met, then it means that the link of the number nrg_{j+1} closes the kinematic chain and equations of constraints determined by the equalities:

$$\begin{aligned} \mathbf{A}_{\text{nrg},i} - \mathbf{A}_{\text{nrg},i}^{\text{nrg},i} \mathbf{A}_{\text{nrg}} &= \mathbf{0}, \\ \mathbf{U}_{\text{nrg},i} - \mathbf{A}_{\text{nrg},i}^{\text{nrg},i} \mathbf{U}_{\text{nrg}} - \mathbf{U}_{\text{nrg}}^{\text{nrg},i} &= \mathbf{0} \end{aligned} \quad (12)$$

are obtained.

In many mechanisms containing only closed kinematic chains the so-called simple mechanisms can be isolated [4,6,7] each of which has no more degrees of freedom than the whole mechanism. In such a case, the equations of constraints, corresponding to the particular simple mechanisms, are solved successively, in a precisely defined order and independently of the equations of constraints assigned to the remaining parts of the mechanism.

The two methods of description of a mechanism structure presented in the paper have their disadvantages and advantages. The matrix of coincidence \mathbf{C} and the vector \mathbf{c}_q are a system of independent structural coordinates thanks to which it is easy to make changes in them. Thus, they are the parameters convenient for the user of the computer program. For numerical calculations necessary is the second set of data, or the vectors nrg , nws , rdw and nrw . These quantities are not independent since the change of one parameter requires, in general, a recomposing of the wholeset of data. In the algorithm of numerical calculations, the structural data of the first set are, in a standard way, processed into the second set of data, though a possibility of using as data only the second of structural parameters is foreseen.

Here attention should be called to a certain fact. Unfortunately, the transformation algorithm of the first structural quantities into the second ones gives results dependent on the method of numeration of links and generalized coordinates, as well as the those of independent coordinates. This refers not only to the difference in the sequence of coordinates, but also to the possibility of increasing the dimension of the second set of vectors of the structural parameters.

The program user can at once construct the second set of data of the structural parameters as foreseen in the algorithm of numerical calculations thanks to which it is possible to obtain the minimum dimension of these vectors determined by dependence (9). The numerical transformation of the first structural data into the second ones frequently gives a result of the dimension greater than the minimum one. The most unfavourable is the case when, in a closed kinematic chain, the independent coordinates determine the position of both its ends. In this case the maximum number of modified structural coordinates is:

$$N_1 = N + 4L_1 + L_2 - 1 \quad (13)$$

If in a mechanism of defined structure and assigned functions of independent coordinates, we express for the i -th body the coordinates of its certain point in a system rigidly joined with it through \mathbf{X}_i' in accordance with (1), then in a fixed system the coordinates of this point on the basis of (2) are expressed as follows:

$$\mathbf{X}_i = \mathbf{A}_i \mathbf{X}_i' + \mathbf{U}_i \quad , \quad (14)$$

where

$\mathbf{A}_i, \mathbf{U}_i$ - are determined in accordance with (10).

Formula (14) determines the trajectories of the assigned point.

For the same point the coordinates of velocity can be written as a derivative of the dependence (14) in relation to time, or

$$\mathbf{V}_i = \frac{d\mathbf{X}_i}{dt} = \dot{\mathbf{X}}_i = \sum_{l=1}^I \left(\mathbf{X}_i' + \frac{\partial \mathbf{U}_i}{\partial q_l} \right) \dot{q}_l \quad , \quad (15)$$

where t - denotes time.

Analogically, on the basis of (15), the acceleration of a given point is determined by the relationship:

$$\mathbf{P}_i = \frac{d\mathbf{V}_i}{dt} = \ddot{\mathbf{X}}_i = \sum_{l=1}^I \left[\left(\frac{\partial \mathbf{A}_i}{\partial q_l} \mathbf{X}_i' + \frac{\partial \mathbf{U}_i}{\partial q_l} \right) \ddot{q}_l + \sum_{p=1}^I \left(\frac{\partial^2 \mathbf{A}_i}{\partial q_l \partial q_p} \mathbf{X}_i' + \frac{\partial^2 \mathbf{U}_i}{\partial q_l \partial q_p} \right) \dot{q}_p \dot{q}_l \right] \quad (16)$$

In the relations (15) and (16) the kinematic characteristics of a point are expressed by kinematic characteristics of independent generalized coordinates. It is assumed that a mechanism is a dynamic system with concentrated constants. Inertia of the i -th link is characterized by the mass m_i , the position of the mass centre expressed by vector \mathbf{U}_i^S and the matrix of the moments of inertia \mathbf{D}_i determined with relation to the mass centre and nonfixed system of coordinates rigidly bound with the body.

The kinetic energy of the system is presented as a square form of velocity of generalized coordinates of the structure

$$E = \frac{1}{2} \sum_{p,r=1}^I a_{pr} \dot{q}_p \dot{q}_r \quad (17)$$

where

E - is the kinetic energy of the system,

a_{pr} - are the coefficients being the functions of generalized coordinates.

In such a case the equations of motion in the form of Lagrange equations of the second type [9] have the following form:

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{q}_p} \right) - \frac{\partial E}{\partial q_p} = Q_p \quad (18)$$

where

Q_p - is the generalized force corresponding to the generalized coordinate q_p which is the function of the time, coordinates and their derivatives in relation to time,

$p = 1, \dots, I$

Carrying out the operation determined on the left side of equations (18) in relation to (17) we obtain

$$\sum_{r=1}^I \left[a_{pr} \ddot{q}_r + \sum_{l=1}^I \left(\frac{\partial a_{pr}}{\partial q_l} + \frac{\partial a_{lr}}{\partial q_l} \right) \dot{q}_l \dot{q}_r \right] = Q_p \quad (19)$$

The kinetic energy of a mechanism is determined from the relation:

$$E = \sum_{i=1}^N E_i = \frac{1}{2} \sum_{i=1}^N \int_{V_i} \mu_i \dot{\mathbf{x}}_i^T \dot{\mathbf{x}}_i dV_i \quad (20)$$

where

V_i, μ_i - determine, respectively, the space area taken by the i -th body and the function of its distribution of mass density,

$\dot{\mathbf{x}}_i^T$ - denotes the vector transposed in relation to the vector of velocity.

After substituting (15) into (20), carrying out of suitable operations and transformations (4), as well as comparing the dependence obtained from (17), we obtain the coefficients of motion (18) in the form:

$$a_{pr} = \sum_{i=1}^N \left[\sum_{j=1}^3 \left(m_i \frac{\partial u_{ij}^s}{\partial q_p} \frac{\partial u_{ij}^s}{\partial q_r} + \sum_{k,l=1}^3 \frac{\partial A_{ijk}}{\partial q_p} \frac{\partial A_{ijl}}{\partial q_r} J'_{ikl} \right) \right], \quad (21)$$

for

$$J'_{ikl} = \int_{V_i} \mu_i x'_{ik} x'_{il} dV_i, \quad (22)$$

where

u_{ij}^s , x'_{ij} , A_{ijk} - are respectively, the elements of the vectors \mathbf{U}_i^s , \mathbf{X}'_i and matrix \mathbf{A}_i .

The measures of inertia (22) of the i -th link occurring in rotation about a fixed point are connected with the elements of the matrix of inertia \mathbf{J}_i of this link through relations:

$$J'_{ijk} = \begin{cases} -J_{ijk} & \text{for } k=l \\ 0.5(J_{i11} + J_{i22} + J_{i33}) - J_{ikl} & \text{for } k \neq l \end{cases}. \quad (23)$$

To obtain the complete form of equations (19) describing the motion of the system it is necessary to determine their right sides Q_p which are usually determined by calculating the variation of work - δL done by the external forces operating on the system:

$$\delta L = \sum_{i=1}^N \sum_{k=1}^{N_i} \sum_{j=1}^3 P_{ikj} \delta x_{ikj}, \quad (24)$$

where

A_{ikj} , δx_{ikj} - are respectively the j -th component of force and j -th component of the permissible by constraints, displacement of the point of application of force which operates at the k -th point of the i -th link of mechanism,

N_i - is the quantity of the forces operating on the i -th link.

It is further assumed that the points of application of forces in a nonfixed system are constant, whereas the concentrated moments can be replaced by pairs of forces. Taking into consideration that the coordinates of application of forces are the functions of independent generalized coordinates, and substituting relation (14) into (24), after transformation we obtain:

$$Q_p = \sum_{i=1}^N \sum_{j=1}^3 \left(P_{ij} \frac{u_{ij}}{q_p} + \sum_{l=1}^3 \frac{\partial A_{ijl}}{\partial q_p} S_{ijl} \right) \quad (25)$$

for

$$P_{ij} = \sum_{k=1}^{N_j} P_{ikj} \quad , \quad S_{ijl} = \sum_{k=1}^{N_j} P_{ikj} x_{ikl}$$

The equations of motion (19) can be solved, if the initial conditions are determined:

$$q_p(0) \quad , \quad \dot{q}_p(0) \quad \text{for } p = 1, \dots, I. \quad (26)$$

3. DESCRIPTION OF THE PROGRAM OF NUMERICAL CALCULATIONS

The computer analysis of mechanisms, with due consideration to structural data described in the previous chapter, has been divided into two programs:

MECH9K - for kinematic analysis,

MECH9D - for dynamic analysis.

Division into two programs has been assumed on account of the fact that the calculations in the range of kinematics are solved more quickly than the dynamic calculations, and moreover, interference of the program operator must start already at the level of constructing the program. These programs make possible an analysis of a wide class of spatial mechanisms. The programs are written in FORTRAN 77 and adapted for realization by means of widely available microcomputers of the type IBM AT. They are programs which are used for the same class of mechanisms as the previously developed programs MECH8K and MECH8D. Both versions of programs described in [4] have many common features - they operate in a similar way, have many procedures in common and the same form of results. The majority of input data are determined in the same way. The basic difference consists in a different description of the system structure and different determination of the geometric dependences occurring in the mechanism. Thanks to the assumption of the system structure description presented in the previous chapter there is no need for the user of the programs MECH9K and MECH9D to develop the analytical form of the equations of constraints since this is realized numerically during calculations.

The user must add to the program the procedure determining the trajectories of the three points characterizing the motion of each link in relation to the "neighbouring" link. In a kinematics problem required is the defining of the procedure which determines the functions of generalized independent coordinates depending on time, and in a dynamic problem it is necessary to determine the procedure expressing the particular external forces operating on a mechanism. The data are loaded to the program conversationally from the keyboard or from a set read by a disk station. These are the quantities determining the structure, geometry, kinematic or dynamic parameters of the system and the quantities controlling the calculations.

In the course of realization of the programs equations of constraints are numerical generated, and next solved by Newton's method [10]. In the subsequent stage of calculations in kinematics problems on the basis of dependence (14), (15) and (16) the kinematic characteristics (position, velocities, acceleration) of the assigned points are determined numerically. In the dynamic analysis, after solving the equations of constraints, the coefficients of the equations of motion (19) are determined numerically, from which after numerical integrating by means of the procedure MERSOL [3], the generalized coordinates are determined and their derivatives in relation to the time, and next - the remaining kinematic characteristics of the system.

The calculations results constitute the time dependent functions of generalized coordinates and coordinates of the assigned points and their velocities and accelerations which are registered in disk sets in tabulated form.

Subsequently, the calculation results can be presented graphically, or used for further numerical processing, e.g. investigating the sensitivity of a mechanism to the changes of its parameters.

4. EXAMPLE OF DETERMINING STRUCTURAL DATA

The scheme of mechanism presented in Fig. 1, with one degree of freedom, is formed of two closed kinematic chains. If we assume q_1 as the independent coordinate, then it is not possible to isolate from the system, a closed kinematic chain having one degree of freedom in which the coordinate q_1 would be an independent generalized coordinate.

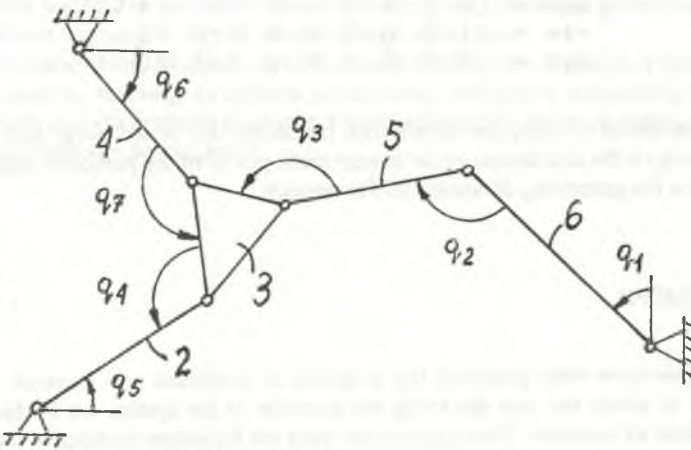


Fig. 1.

The algorithm of numerical calculations takes into consideration only the case in which the amount of the degrees of freedom of closed kinematic chains forming the mechanism is not greater than the number of the degrees of freedom of the whole system.

For the example under consideration, in the case of assigning the first set of structural data, we should assume as an independent coordinate, one of the coordinates describing the motion of nonfixed links in relation to the fixed link in the second closed kinematic chain i.e. q_5 or q_6 .

If we assume q_5 as the independent coordinate, then the first set of structural data defined by the dependences (7) and (8) takes the form:

$$C = \begin{bmatrix} 0 & 5 & 0 & 6 & 0 & 1 \\ 5 & 0 & 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 7 & 3 & 0 \\ 6 & 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$$cq = [5]$$

However, the second set of structural data, on the basis of the definition accepted in the second chapter, or as a result of the realization of the computer program, is represented by the following vectors:

$$\begin{aligned} nrg &= [1 \ 2 \ 3 \ 4 \ 1 \ 1 \ 6 \ 5 \ 3] , \\ nws &= [0 \ 5 \ 4 \ 7 \ 6 \ 0 \ 1 \ 2 \ 3] , \\ rdw &= [1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0] , \\ nrw &= [0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 2] . \end{aligned}$$

During realization of computer calculations, the second set of structural data is used for the determining of the coordinates of the characteristic points of the particular links in a fixed system and for the generating of equations of constraints.

5. CONCLUSIONS

In the paper have been presented the programs of kinematic and dynamic analysis of mechanisms, in which the data describing the structure of the system are loaded as input data in the form of numerals. The program user need not formulate analitically the equations of constraints.

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Streszczenie

W pracy zdefiniowano pewne dane liczbowe umożliwiające opis struktury szerokiej klasy mechanizmów przestrzennych o wielu stopniach swobody z geometrycznymi i skleronomicznymi więzami. Takie ujęcie zagadnienia pozwala uniknąć analitycznego formułowania zależności geometrycznych określających położenia poszczególnych punktów mechanizmu, równań więzów oraz analitycznego określenia sposobu redukcji współrzędnych uogólnionych opisujących ruch układu.

Na podstawie przyjętego opisu struktury mechanizmu, utworzono algorytm i procedury obliczeń numerycznych, które zastosowano do opracowania programu komputerowego wyznaczania charakterystyk kinematycznych zadanych punktów układu (składowe wektora położenia, prędkości i przyspieszenia). Następnie po modyfikacji i uzupełnieniu o odpowiednie dodatkowe procedury utworzono program cyfrowego generowania równań ruchu o budowie Lagrange'a oraz ich numerycznego całkowania.

Programy analizy kinematycznej i dynamicznej mechanizmów napisano w języku Fortran i przystosowano do realizacji za pomocą powszechnie dostępnych mikrokomputerów typu IBM AT. Praca zawiera przykład tworzenia danych opisujących strukturę mechanizmu, które stanowią dane liczbowe do programu.