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THEORETICAL MECHANICS

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APPLICATION THE FRACTAL GEOMETRY FOR CREATING PICTURES OF TREES

Summary. Paper presents L-system as an example of application the fractal geometry for creating pictures of trees. For description L-system and receiving drawings program L-parser - the implementation of L-system was used.

WYKORZYSTANIE GEOMETRII FRAKTALNEJ DO GENEROWANIA OBRAZÓW DRZEW

Streszczenie. W artykule przedstawiono L-system, jako przykład wykorzystania geometrii fraktalnej do generowania obrazów drzew. Do opisu L-systemu i wykonania rysunków posłużono się programem L-parser, będącym implementacją L-systemu.

ИСПОЛЬЗОВАНИЕ ФРАКТАЛЬНОЙ ГЕОМЕТРИИ ДЛЯ ГЕНЕРИРОВАНИЯ КАРТИН ДЕРЕВЬЕВ

Резюме. В статье представлена L-система как пример использования геометрии фрактальной для генерирования картин деревьев. Для описания L-системы и выполнения рисунков была использована программа L-parser, которая является имплементацией L-системы.

1. INTRODUCTION

Classical geometry was good enough for description of the shape of products made by man, but it is not satisfactory tool for description of the forms of natural creation, for example trees, leaves, clouds and mountains. Now, we have a new tool - the fractal geometry, which can be used to make precise model of natural objects. Although the name " fractal geometry" was used for the first time by B. Mandelbrot, who wrote the book under the title "The fractal geometry of nature" (1980) we should remember that the beginning of fractal geometry was made much earlier, for example in XIX century the Hausdorff measure and dimension were defined. Now this dimension is used for definition fractal as a set which Hausdorff dimension is greater than topological dimension. Also in the papers others XIX century mathematicians, as Julia and Fatou for example, we can find many interesting not connected with fractals, although the sets which nowadays we know as fractals did not have such name in XIX century yet. The biggest development of fractal geometry has been dated since the 80-ties of our century. It is closely connected with the development of computer sciences. For generating images of fractal sets we usually use recursions algorithm. It means that faster running software give us an opportunity for analysing greater number of fractal sets and that is why the evolution of fractal geometry is depending on evolution of computer technology.

2. DESCRIPTION OF L-SYSTEM

One of the algorithm for generating fractal sets was written by biologist Aristed Lindenmayer in 1968 as the algorithm for modelling growing of plants. Because of the name of the author the algorithm is called generally L-system, but the name "String Rewriting System" seems to be more suitable to quote in that situation as it better describe the essence of the algorithm. In order to make the definition of the algorithm easier to understand the example will be given first. Suppose we have a word:

$$ab$$

and the rule of replacing the letter a and b:

$$a \to ab, \quad b \to aba$$

At the first step the word *ab* is replacing by a word *ababa*, at the next step letters of that new word are replacing again according to the same rules. It means that we receive a new word: *ababaababaab*.

Formal definition which was introduced by Lindenmayer [4] is given below:

Let V denote an alphabet, V^* the set of all words over V, and V^+ the set of all nonempty words over V. An L-system is an ordered triplet G = [V, ω, P], where V is the alphabet of the system, $w \in V^+$ is a nonempty word called the axiom and P is a finite set of productions $V \times V^*$.

3. PROGRAM L-PARSER AS AN IMPLEMENTATION OF L-SYSTEM

Graphical interpretation of a string which come into being by using the L-system is based on the turtle orientation commands - LOGO-like. For description of position of the turtle in the space and the direction in which this turtle will move we involve two coordinate systems: fixed - Cartesian coordinate system Oxyz and movable coordinate system connected with the turtle. Position of the turtle is described by the coordinates of the origin of the movable coordinate system in the fixed coordinate system $P_0(x_0, y_0, z_0)$. The direction in which the turtle will move is described by orthogonal and unit vectors $(\vec{H}, \vec{L}, \vec{U})$, which are base of movable coordinate system. Symbols (the letters of alphabet V) connected with the going the turtle ahead are collected in table 1:

Table 1. Symbols connected with the going the turtle ahead

Symbol	Effect
F	$P_0' = P_0 + \vec{H}$ Draw a segment P_0P_0'
F(a)	$P_0' = P_0 + a\vec{H}$ Draw a segment P_0P_0'
Z	$P_0' = P_0 + \frac{1}{2}\vec{H}$ Draw a segment P_0P_0'
Z(a)	$P_0' = P_0 + a\vec{H}$ Draw a segment P_0P_0'
f	$P_0' = P_0 + \vec{H}$
f(a)	$P_0' = P_0 + a\vec{H}$
z	$P_0' = P_0 + \frac{1}{2}\vec{H}$

z(a)	$P_0' = P_0 + a\vec{H}$
	$P_0' = P_0$

To draw the trunks of the fractal trees and their branches in proper, different thickness the line segments P_0P_0' is replaced by a prism. Bases of the prism are rectangles with the origins P_0 and P_0' and variou length of its side is stated in L-system.

Rotation of the turtle, it means rotation of the movable coordinate system connected with turtle can be expressed by the equation:

$$\left[\vec{H'}, \vec{L'}, \vec{U'}\right] = \left[\vec{H}, \vec{L}, \vec{U}\right] R \tag{1}$$

where R is rotation matrix.

As the rotation of the turtle about any axis can be represented by the sequence of the rotation about vectors $\vec{H}, \vec{L}, \vec{U}$ the certain importance have the rotation matrixes connected with rotation about those axis:

$$R_U(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_L(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$R_H(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

Symbols collected in table 2 cause rotation of the coordinate system connected with the turtle and as the result change the direction in which the turtle is going to move.

Table 2. Symbols connected with orientation of the turtle

Symbol	Effect
+	$[H',L',U'] = [H,L,U]R_U(\alpha)$ turn a around U vector
+(x)	$[H',L',U'] = [H,L,U]R_U(x)$ turn x around U vector

-	$[H',L',U'] = [H,L,U]R_U(-\alpha)$ turn -a around U vector
-(x)	$[H',L',U'] = [H,L,U]R_U(-x)$ turn -x around U vector
&	$[H',L',U'] = [H,L,U]R_L(\alpha)$ turn a around L vector
&(x)	$[H',L',U'] = [H,L,U]R_L(x)$ turn x around L vector
^	$[H',L',U'] = [H,L,U]R_L(-\alpha)$ turn -a around L vector
^(x)	$[H',L',U'] = [H,L,U]R_L(-x)$ turn -x around L vector
<	$[H',L',U'] = [H,L,U]R_H(\alpha)$ obrót o standardowy kąt a around H vector
<(x)	$[H',L',U'] = [H,L,U]R_H(x)$ turn x around H vector
>	$[H',L',U'] = [H,L,U]R_H(-\alpha)$ turn -a around H vector
>(x)	$[H',L',U'] = [H,L,U]R_H(-x)$ turn -x around H vector
	$[H',L',U'] = [H,L,U]R_U(\pi)$ turn π around U vector
%	$[H',L',U'] = [H,L,U]R_H(\pi)$ turn π around H vector

The standard angle α is stated for each L-system together with such parameters as number of recursion level and dimension of the first prism.

The set of symbols (letters of alphabet V) given above enable us to generate many interesting and very complicated sets, but in order to draw plants, and branching structure in generally it is necessery to involve two symbols yet. These symbols are in table 3:

Table 3. Symbols connected with drawing branching structure

Symbol	Effect
[	push current state
]	pop current state

At this moment we have enough symbols to draw the first fractal tree. It is two-dimensional tree in order to make the understanding of that example easier. The axiom is one-letter word: A, and the replacement rule is: A→F[&A]^A. Program L-parser needs apart from sating the axiom and the replacement rules following data: number of iteration (recursion level), standard angle and dimension of the first prism. L-system analysed by program L-parser is presented below and the result is presented at fig.1.

2	*/ number of recursion level - 2 at fig.1a and 4 at fig. 1b/*
30	*/ standard angle α /*
10	*/ dimension of the first prism /*
A	*/ an axiom /*
A=F[&A]^A	*/ rule of replacing the letter A /*
@	*/ koniec L-systemu /*

L-system describing three-dimensional tree differs from the L-system given above only in the replacement rule because in that situation the prism rotate in two planes:

A→F[&A][>(120)&A]<(120)&A].

The tree from the fig. 2 will be modify now so that it will be more similar to those ones which we can see in reality. The first step in that direction will be differentiation branches as far as thickness and length are concerned. Symbols presented in table 4 are resposporsible for that:

Table 4. Symbols connected with increasing /decreasing size of drawing elements

Symbol	Effect
"	increment dimensions of the prism with 1.1
'	decrement dimensions of the prism with 0.9
"(x), '(x)	multiply dimensions of the prism with x
;	increment angle with 1.1
:	decrement angle with 0.9
;(x), :(x)	multiply angle with x
?	increment thickness of the prism with 1.4

!	decrement thickness of the prism with 0.7
?(x), !(x)	multiply thickness of the prism with x

The effect of symbol " ' " we are going to analyse using the L-system consisted of one-letter axiom: A and rules of replacement: A=FA, F='(2)F'(0.5). Results with the recursion level 2 and 4 are presented at fig.3. To clearly understand this L-system we can analyse string arising on the succeeding recursion level:

step I : FA

step II : '(2)F'(0.5)FA

step III: '(2)'(2)F'(0.5)'(0.5)'(2)F'(0.5)FA , which equal a string:

'(4)F'(0.5)F'(0.5)FA

As a result dimension each next prism is 0.5 time smaller than the last one.

Creating images of trees we should remember about Leonardo d'Vinci postulate according to which "all branches of a tree at every stage of its height put together are equal in thickness to the trunk below them". In the case of the tree from fig. 4 where the mother branch of diameter D gives rise to three daughter branches of equal diameter d this postulate yields the equation $D^2 = 3d^2$, which gives $d=1/\sqrt{3} \approx 0.58$. As we want to take into account the Leonardo d'Vinci postulate we have to add to the L-sysytem generating the tree from fig.3 a rule: $'(\frac{100}{58})F'(\frac{58}{100})$. The tree which we obtain then is presented at fig.4.

The symbol

\$ - roll until horizontal

roll the turtle around its own axis so that vector $\vec{L}$ is brought to a horizontal position. From the mathematical point of view, \$ modifies the turtle orientation in space according to the formula:

$$\vec{L} = \frac{\vec{V} \times \vec{F}}{\left|\vec{V} \times \vec{F}\right|}, \qquad \vec{U} = \vec{L} \times \vec{H}, \tag{2}$$

where $\vec{V}$ is a unit vector with direction opposite to the direction Oz axis of fixed coordinate system. To make clear how symbol \$ works in L-system there are two L-systems presented below. The results are presented at fig.5. Fig. 5a is graphical representation of L-system without symbol \$:

4 */recursion level/*

20 */standard angle/*

5 */dimension of the first prism/*

&(90)s */axiom/*

s=a>s /*rule of replacement*/

a=[ABC] /*rule of replacement*/

A=Z /*rule - draw $\vec{H}$ vector*/

B=+(90)Z /*rule - draw $\vec{L}$ vector*/

C=^(90)Z /*rule - draw $\vec{U}$ vector*/

@ /*end of L-system*/

Fig. 5b is graphical representation of the same L-system, but instead of the rule s=a>s the rule s=a>$s is applied.

The symbol $ usually is used together with the symbol t. These two symbols $ and t give an opportunity for taking into consideration influence of gravity on branches arragement. The bending of branches is simulated by slightly rotating the turtle in the direction of a vector $\vec{T}$ after drawing each segment, when $\vec{T}$ is an opposite vector to an axis Oz of fixed coordinate system. To introduce mathematical notation of the effect cause by symbol t in L-system we define:

T = [0,0,-1] in fixed Cartesian coordinate system

e - parameter, which can be connected with axis susceptibility to bending.

Then, as a result of using symbol t in L-system vector $\vec{H}$ is rotated by angle α in the direction of vector $\vec{T}$, where:

$$\alpha = e\left|\vec{H} \times \vec{T}\right| \quad (3)$$

The following L-system and fig.6 which is its graphical representation show how symbol (letter) *t* can impact on L-system:

30 / *recursion level* /

10 / *standard angle* /

10 / *dimension of the first prism*/

AD / *axiom*/

/*rules of replacement:*/

A=B>A D=E>D

B=[&(45)C] E=[&(45)G]

C=FC G=tFG

@ /*end of L-system*/

Letter A in the axiom and rules of replacement which are at the left hand side are connected with the upper part of the set shown at fig. 6. The bottom part of this set is connected with letter D and the rules of replacement which are at the right hand side.

Fig.7 is included to show the effect of use of symbols $ and t when trees are generated. Each of the branches of trees, which are presented at fig. 7 consists of two segments. It was obtained by replacing the formula:

A=F[&A][>(120)&A]<(120)&A

by two formulas:

A=F[&A][>(120)&A]<(120)&A

B=FA

The tree presented at fig. 7a was generated without the symbols $,t , and the tree at the fig. 7b was obtained by processing L-system including symbols $ and t :

A=$t(0.2)F[&A][>(120)&A]<(120)&A

B=$t(0.2)FA

Characteristic feature of natural creatures is their uniqueness. As in L-parser program we can define the rotation angle in random way we can obtain trees which also possess that feature. We used symbols collected in table 5 to do that:

Table 5. Symbols connected with randomness

Symbol	Effect
~	turn in a random direction
~(x)	turn in a random direction with a maximum of x degrees

Randomness we can use also when we draw leaves. First, let us define leaves in such way that they are equilateral triangles. Apart from the symbols we have got to know by now we will also use new symbols connected with drawing polygons, which are in table 6:

Table 6. Symbols connected with drawing polygon shape

Symbol	Effect
{	start polygon shape
}	end polygon shape

The definition of mentioned triangle is following

L = {+(30)f(2)-(120)f(2)-120f(2)}

Required change of colour at this stage of drawing trees can be realised due to symbols collected in table 7:

Table 7. Symbols connected with changin colour of drawing elemnts

Symbol	Effect
c	Increment color index
c(x)	Set color index to x

Now we can complete previous definition of leaves in such way that we obtain a tree with green leaves attached to branches in a random way.

L =~ cc {+(30)f(2)-(120)f(2)-120f(2)}

Such tree is presented at fig.8.

All features of L-parser program presented by now were used to generate a tree which is put at fig.9. This tree was obtained as a graphical representation of the following L-system:

```
13
30
30
CCA
A=$t(.2)C[&BL[|zL]][>(120)&BL[|zL]]<(120)&BL[|zL]
B=$t(.2)CCA
C=!(0.95)~(5)FL[|zL]
F=c(9)'(1.15)F'(0.87)
L=[~c(3){+(30)f(2)-120f(2)-120f(2)}]
@
```

Trees generated by L-parser program can be written in DXF format and used in drawing made with AutoCAD.

References

[1] Barnsley, M. - "Fractals Everywhere", Academic Press, San Diego 1988

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[3] Peitgen, H.-O., Saupe D. "The Science of Fractal Images", Springer-Verlag, New York 1988

[4] Prusinkiewicz, P., Lindenmayer, A., "The Algorithmic Beauty of Plants", Springer-Verlag, New York 1990

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Streszczenie

W artykule został zaprezentowany jeden z algorytmów generowania zbiorów fraktalnych. Jest to algorytm opracowany przez biologa Aristida Lindenmayera w celu modelowania rozwoju rośliny. Zwykle algorytm ten nazywa się "L-system", ale jego istotę znacznie lepiej oddaje nazwa "String Rewriting System" ("System Przepisywania Łańcuchów"). Jest to algorytm rekurencyjny, który polega na zastępowaniu ciągów symboli otrzymywanych w kolejnych krokach algorytmu nowymi ciągami symboli zgodnie z określonym wcześniej regułami zastępowania. Otrzymany w ten sposób ciąg symboli jest reprezentowany graficznie na zasadzie podobnej do grafiki żółwia - LOGO. W niniejszym artykule pokazano w jaki sposób przy wykorzystywaniu L-systemu można otrzymywać obrazy drzew. W celu wykonania rysunków posłużono się programem L-parser, który jest implementacją L-systemu. Wszystkie rysunki zostały zapisane w formacie DXF i wydrukowane za pomocą AutoCad-a.

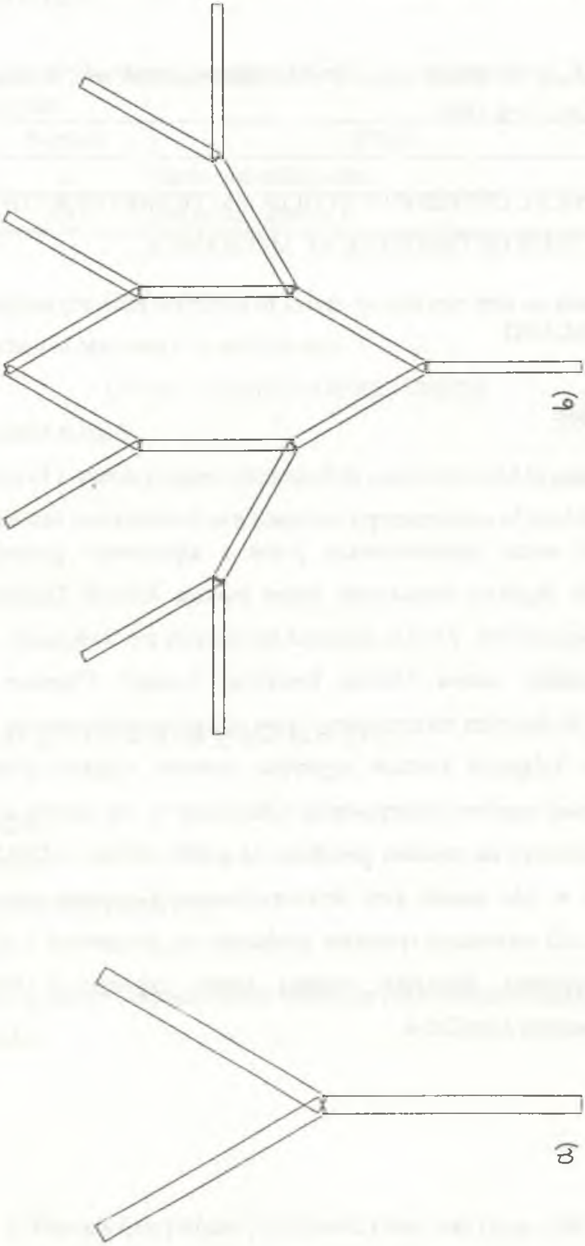

Fig. 1.

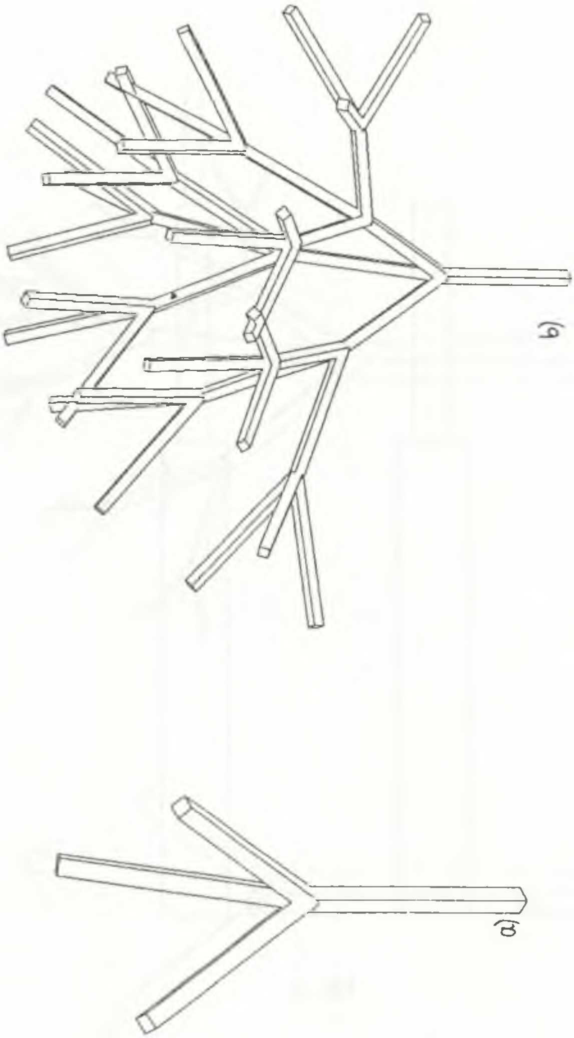

Fig. 2.

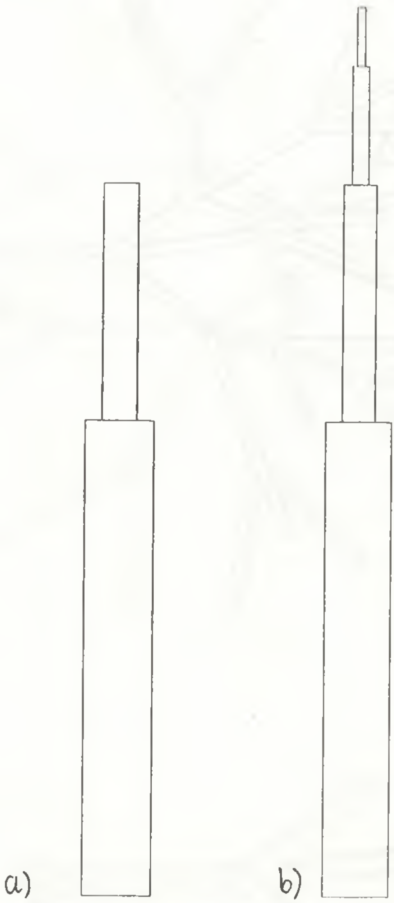

Fig. 3.

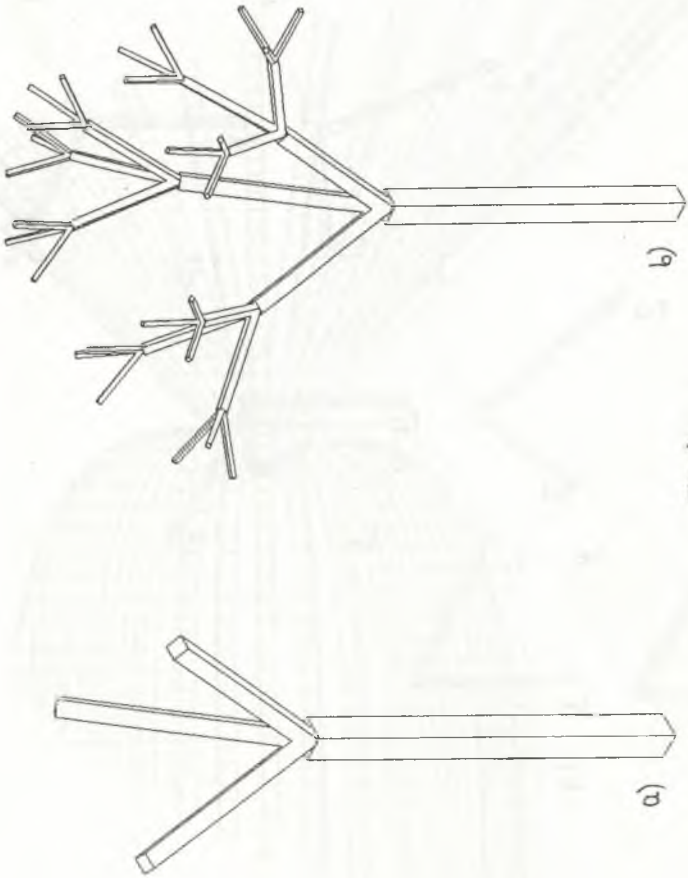

Fig. 4.

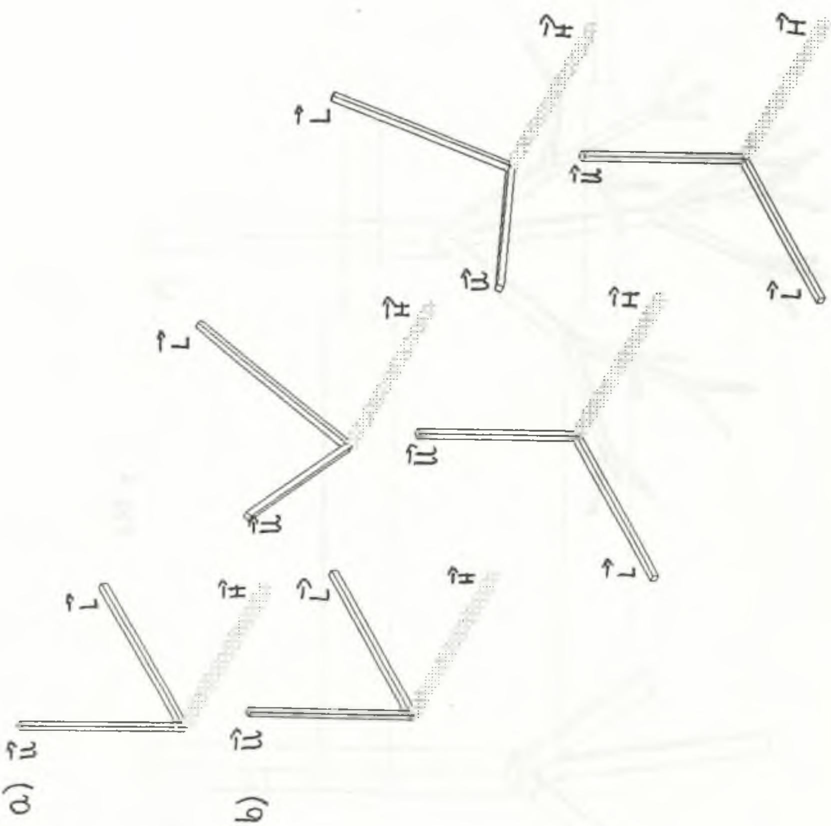

Fig.5.

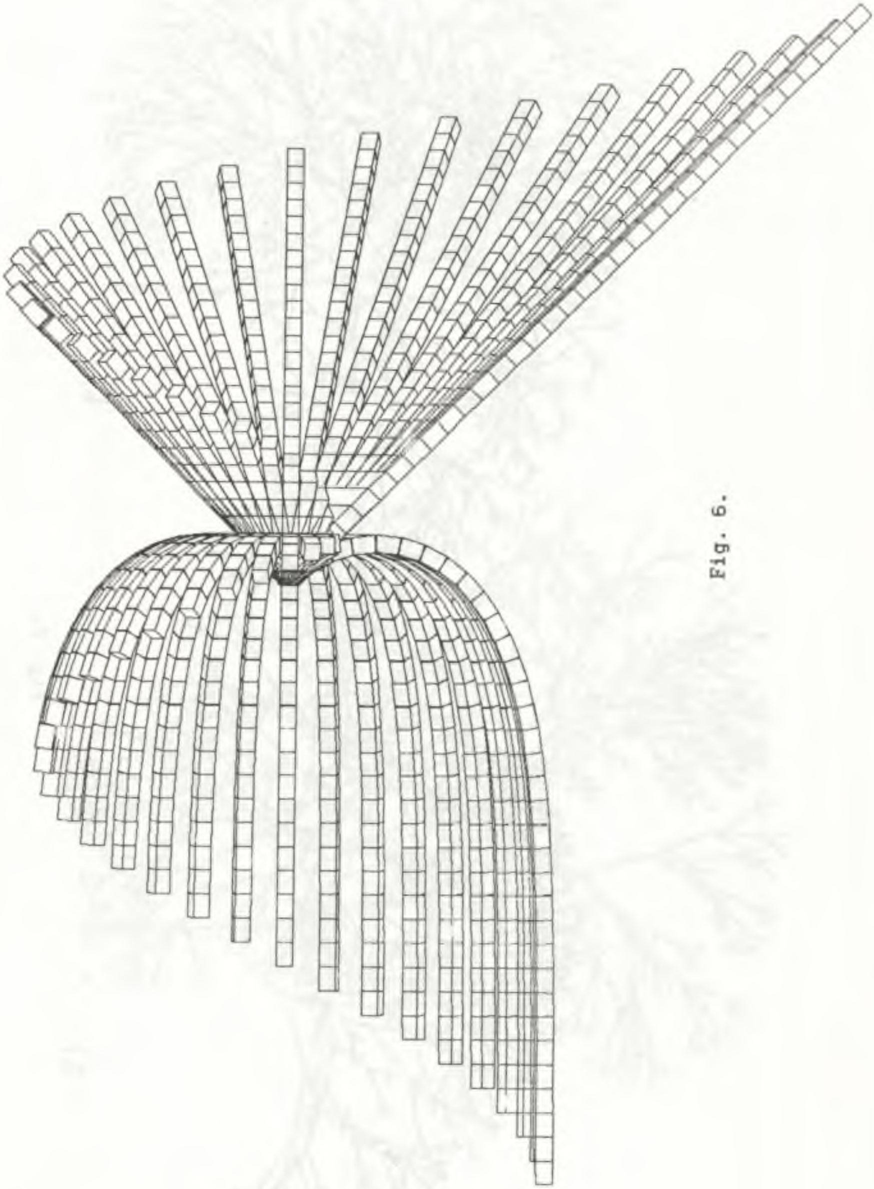

Fig. 6.

b)

a)

Fig. 7.

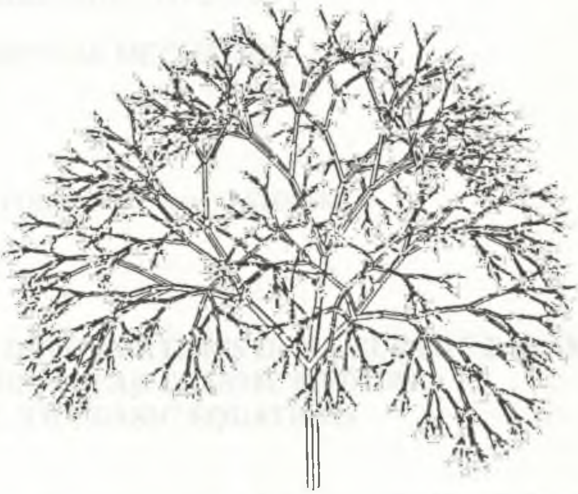

Fig. 8.

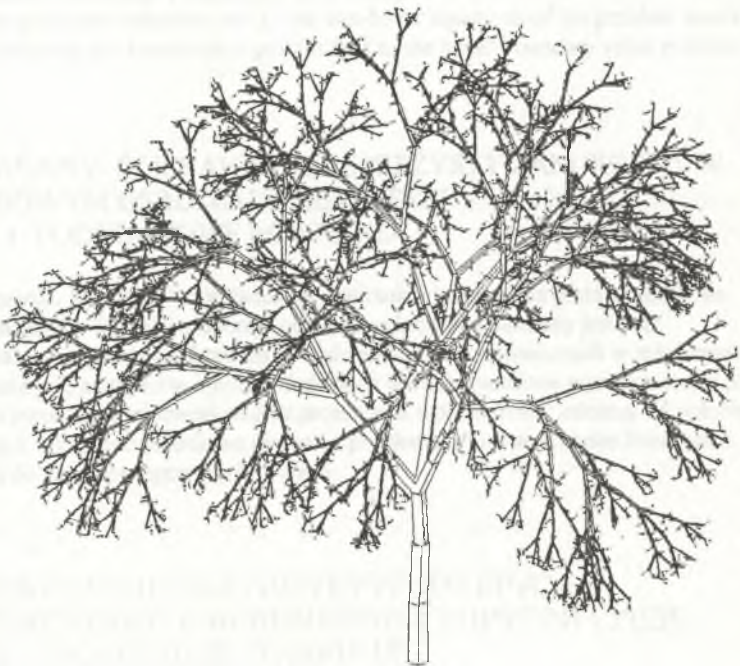

Fig. 9.