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THEORETICAL MECHANICS

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AXIAL DEFORMATIONS OF AN ELASTIC BAR IMMERSED IN THE NONLINEAR ELASTIC MEDIUM
PART 1. THE BASIC EQUATIONS

Summary. The problem of axial deformations of a bar with the nonlinear elastic response of the medium in which the bar is immersed has been discussed . The case analysed models a lot of engineering problems in mechanics of elastic bodies as well as in other fields of technology (heat conduction through a rod, horizontal motion of material point with nonlinear resistance, etc.). The non-linear equations of the problem have been obtained which in the linearisation process lead to the linear boundary value problem.

ROZCIĄGANY- ŚCISKANY PRĘT SPRĘŻYSTY ZANURZONY W NIELINIOWYM OŚRODKU SPRĘŻYSTYM
CZĘŚĆ 1. PODSTAWOWE RÓWNANIA

Streszczenie. Rozpatrzono zagadnienie ściskania - rozciągania pręta z nieliniową reakcją sprężystą, wywołaną oporem ośrodka, w którym zanurzony jest pręt. Analizowany tutaj przypadek modeluje wiele problemów technicznych w mechanice ciał odkształcalnych, a także i w innych dziedzinach techniki (ustalona wymiana ciepła przez pręt, ruch punktu materialnego wzdłuż prostej pod wpływem siły zależnej od położenia punktu itp.). Uzyskano nieliniowe równanie problemu, które w procesie linearyzacji prowadzą do zadań brzegowych liniowych.

РАСТЯЖЕНИЕ И СЖАТИЕ УПРУГОГО БРУСА ПОГРУЖЕННОГО В НЕЛИНЕЙНОЙ УПРУГОЙ СРЕДЕ
ЧАСТЬ 1. ОСНОВНЫЕ УРАВЕНИЯ

Резюме. Рассматривается вопрос сжатия и растяжения бруса с нелинейной упругой реакцией, вызванной сопротивлением среды, в которой погружен

брус. Анализируемый пример моделирует много тэхнических проблем в механике нетвердых тел и в других отраслях техники (определенный теплообмен в брусе, движение материальной точки вдоль простой под действием силы зависимой от положения точки итп.) Были выведены нелинейные уравнения проблемы, которые в процессе линеаризации ведут к линейным краевым задачам.

1.INTRODUCTION

The paper focuses on the axial deformation problem for an elastic bar subjected to the response of the medium in which the bar is immersed. We apply the linear theory of bars of a variable cross-section, made of a Hooke's material. The problems discussed here model several technical processes both in mechanics of elastic media (one - dimension problems) and other fields (a heat conduction through a rod, a horizontal motions of a material point with nonlinear resistance). We will apply notations given in Refs. [3] and [4]. The original element in the paper is the inclusion of the nonlinear reaction of the groud. So formulated a problem leads to the nonlinear equations or to the nonlinear functionals. The algorithm of the finite elements method explified here will be used in the second part of the paper.

2. BASIC RELATIONS

From the local equlibrium conditions for an axial deformation state of a bar we obtain the differential equation

$$\frac{dN(x)}{dx}+n(x)-k\left[u(x)\right]u(x)=0,\ \forall x\in(0,L), \tag{2.1}$$

where

$$N(x)=F(x)\sigma(x), \tag{2.1'}$$

in which *N(x)* defines an axial force in the bar; *n(x)* - an external longitudinal loading; $k\left[u(x)\right]$ - a nonlinear coefficient of a medium interacting with the bar which can also be defined as the nonlinear function of elastic foundation. In (2.1) $u(x)$, $x \in (0, L)$ is a displacement function of a cross - section of the bar. We assume that the function

$$(0,L)\ni x\mapsto F(x), \tag{2.2}$$

is - in the general case - a weak monotone (bars of the weakly variable cross - section):

$$\{[\max F(x) - \min F(x)]/\min F(x)\} < \delta,\ x \in (0, L), \tag{2.2'}$$

where δ is sufficient small number.

Stress $\sigma(x)$ is defined by Hooke`s constitutive equation

$$\sigma(x) = E\varepsilon(x),\ \sigma(x) \in (-\sigma_H, \sigma_H),\ \sigma_H \rangle 0, \tag{2.3}$$

where in the linear theory

$$\varepsilon(x) = \frac{du(x)}{dx},\ x \in (0, L). \tag{2.4}$$

Combining the equations (2.1) - (2.4) we obtain the differential equation of the problem

$$-\frac{d}{dx}\left[EF(x)\frac{du(x)}{dx}\right] + k[u(x)]u(x) = u(x),\ x \in (0, L). \tag{2.5}$$

We assume that the bar axial stiffness satisfies the condition

$$EF(x) \geq C_1 = const \rangle 0,\ x \in (0, L), \tag{2.6}$$

and the response of elastic foundation is restricted by

$$L^2 k(x) \geq C_2 = const \rangle 0,\ x \in (0, L). \tag{2.6'}$$

The inequalities (2.6), (2.6') are essential for the formal description of the problem.

3. BOUNDARY - VALUE PROBLEMS

The following kinds of boundary conditions will be considered:

a) displacement boundary conditions:

$$u(x)\big|_{x=0} = u_0,\ u(x)\big|_{x=L} = u_L; \tag{3.1}$$

b) traction boundary conditions:

$$N(x)\big|_{x=0+} = \left[EF\frac{du(x)}{dx}\right]_{x=0+} = -P_0,$$
$$N(x)\big|_{x=L-} = \left[EF\frac{du(x)}{dx}\right]_{x=L-} = -P_L, \tag{3.2}$$

where the right - hand sides of (3.2) are not independent, being related by the global equlibrium equation:

$$P_0 + P_L + \int_0^L n(x)dx - \int_0^L k\left[u(x)\right]u(x)dx = 0. \tag{3.2'}$$

To uniquely define the displacement field under conditions (3.2) we have to assume that the bar is fixed in the selected cross - section of a coordinate x^*:

$$u(x^*) = u^*, \tag{3.2''}$$

c) mixed boundary conditions

$$u(x)\big|_{x=0} = u_0,\ N(x)\big|_{x=L-} \equiv \left[EF(x)\frac{du(x)}{dx}\right]_{x=L-} = P_L, \tag{3.3}$$

or

$$N(x)\big|_{x=0} \equiv \left[EF(x)\frac{du(x)}{dx}\right]_{x=0+} = -P_0,\ u(x)\big|_{x=L} = u_L. \tag{3.3'}$$

The boundary value problems considered in particular problems are defined by the equation systems:

1) (2.5), (3.1);
2) (2.5), (3.2), (3.2'), (3.2");
3a) (2.5), (3.3);
3b) (2.5), (3.3').

4. AN APPROXIMATION OF THE DISPLACEMENT FIELD

The construction of approximate solutions to the problems formulated above, will be based on the finite element method, cf. eg. [1], [2]. Here we will apply the following kinds of approximation:

4.1. A Linear Aproximation

The interval (*0, L*), related to the axis of the bar is divided by the sequence of points:

$$x_1 \equiv 0,\ x_2, \dots, x_{e-1}, x_e, x_{e+1}, \dots, x_{N_e}, x_{N_e+1} \equiv L,\ \ x_e < x_{e+1}. \tag{4.1}$$

into the intervals (x_e, x_{e+1}), e=1, 2, ..., N_e, whose lengths are equal to

$$h_1 = x_2 - x_1, \dots, h_{e-1} = x_e - x_{e-1}, h_e = x_{e+1} - x_e, \dots, h_{N_e} = x_{N_{e+1}} - x_{N_e}, \tag{4.1'}$$

and then the interval considered (as the set of points on the bar axis) is described as

$$(0, L) = (x_1 = 0,\ x_{N_{e+1}} = L) = \left[\bigcup_{e=1}^{N_e} (x_e, x_{e+1})\right] \cup \left[\bigcup_{e=2}^{N_e} \{x_e\}\right] \tag{4.2}$$

In the particular intervals (x_e, x_{e+1}), e=1, 2, ..., N_e, instead of $u(x)$, $x \in (x_e, x_{e+1})$ we introduce the linear approximation: $u(x) \to u^{(e)}(x)$

$$u^{(e)}(x) = \left[N^{(e)}(x)\right]\left\{U^{(e)}\right\}\ e = 1,\ 2,\ \dots,\ \mathrm{N_e}, \tag{4.3}$$

in which $N^{(e)}(x)$ is a matrix of shape functions

$$\left[N^{(e)}(x)\right] = \left[N_e(x) \quad N_{e+1}(x)\right], \tag{4.3'}$$

where

$$N_e(x) = \frac{1}{h_e}(x_{e+1} + x),\ N_{e+1}(x) = \frac{1}{h_e}(x - x_e). \tag{4.3''}$$

Then $\left\{U^{(e)}\right\}$ is the column matrix of displacement modes:

$$\left\{U^{(e)}\right\} = \left\{u_e \quad u_{e+1}\right\}^T. \tag{4.4}$$

If in each interval (x_e, x_{e+1}), e=1, 2, ..., N_e, we introduce local coordinates:

$$\xi = x - x_e,\ d\xi = dx,\ \xi \in (0, h_e),\ x \in (x_e, x_{e+1}), \tag{4.5}$$

then the shape function and the shape function matrix are described:

$$\begin{aligned}
&N_e(x) \equiv N_e(\xi + x_e) =: \tilde{N}_e(\xi),\\
&\tilde{N}_e(\xi) = \frac{1}{h_e}(h_e - \xi) = 1 - \frac{\xi}{h_e};\\
&N_{e+1}(x) \equiv N_{e+1}(\xi + x_{e+1}) =: \tilde{N}_{e+1}(\xi),\\
&\tilde{N}_e(\xi) = \frac{1}{h_e}\xi, \quad = \xi \in (0, h_e);\\
&\left[\tilde{N}_e(\xi)\right] \equiv \left[\tilde{N}_e(\xi) \quad \tilde{N}_{e+1}(\xi)\right] = \frac{1}{h_e}\left[h_e - \xi \quad \xi\right] \equiv \left[1 - \frac{\xi}{h_e} \quad \frac{\xi}{h_e}\right]
\end{aligned} \tag{4.6}$$

Then the displacement functions in the particular intervals of the bar are defined in the new local coordinates as follows:

$$\begin{aligned}
&u^{(e)}(x) \equiv u(x_e + \xi) =: \tilde{u}^{(e)}(\xi),\\
&\tilde{u}^{(e)}(\xi) = \left[\tilde{N}^{(e)}(\xi)\right]\left\{U^{(e)}\right\} = \frac{1}{h_e}\left[h_e - \xi \quad \xi\right]\left\{U^{(e)}\right\} \equiv \left[1 - \frac{\xi}{h_e} \quad \frac{\xi}{h_e}\right]\left\{U^{(e)}\right\},\\
&\left\{U^{(e)}\right\} = \left\{u_e \quad u_{e+1}\right\}^T.
\end{aligned} \tag{4.7}$$

The variations of displacement functions (virtual displacement) are pointed out on the basis of (2.3) or (2.7):

$$\begin{aligned}
&\delta u^{(e)}(x) = \left[N^{(e)}(x)\right]\left\{\delta U^{(e)}\right\} \equiv \left\{\delta U^{(e)}\right\}^T\left[N^{(e)}(x)\right]^T,\\
&\delta\tilde{u}^{(e)}(x) = \left[\tilde{N}^{(e)}(x)\right]\left\{\delta U^{(e)}\right\} \equiv \left\{\delta U^{(e)}\right\}^T\left[\tilde{N}^{(e)}(x)\right]^T,
\end{aligned} \tag{4.8}$$

where in (4.8) the Cauchy`s formula on the transpose of the product of two matrices are applied.

We further mark derivatives of the shape function $N_e(x)$, $N_{e+1}(x)$, and the matrix of these derivatives to obtain the matrix of gradients of shape function by:

$$\left[B^{(e)}\right] \equiv \left[\frac{dN^{(e)}(x)}{dx}\right] \equiv \frac{1}{h_e}\{-1 \quad 1\},$$
$$\left[B^{(e)}\right]^T = \frac{1}{h_e}\begin{Bmatrix}-1\\1\end{Bmatrix}. \tag{4.9}$$

Using the basic equation $\varepsilon(x) = \frac{du(x)}{dx}$, $\left(\rightarrow \varepsilon^{(e)}(x) = \frac{du^{(e)}(x)}{dx}\right)$ and introducing there the relations $u^{(e)}(x) = \left[N^{(e)}(x)\right]\left\{U^{(e)}\right\}$ we obtain:

$$\varepsilon^{(e)}(x) := \frac{du^{(e)}(x)}{dx} \equiv \left[\frac{dN^{(e)}(x)}{dx}\right]\left\{U^{(e)}\right\} = \left[B^{(e)}\right]\left\{U^{(e)}\right\}$$
$$\equiv \frac{1}{h_e}[-1 \quad 1]\begin{Bmatrix}u_e\\u_{e+1}\end{Bmatrix}. \tag{4.10}$$

The variation of function (2.10) is written as:

$$\delta\varepsilon^{(e)}(x) = \left[B^{(e)}\right]\left\{\delta U^{(e)}\right\} \equiv \left\{\delta U^{(e)}\right\}^T\left[B^{(e)}\right]^T. \tag{4.11}$$

Stresses in cross - sections, belonging to the interval (x_e, x_{e+1}) are equal to

$$\sigma^{(e)}(x) = E\varepsilon^{(e)}(x) = E\left[B^{(e)}\right]\left\{U^{(e)}\right\} \equiv \frac{E}{h_e}[-1 \quad 1]\begin{Bmatrix}u_e\\u_{e+1}\end{Bmatrix}, \tag{4.12}$$

Then the axial forces in these intervals are defined by:

$$N^{(e)}(x) = F(x)\sigma^{(e)}(x) = EF(x)\left[B^{(e)}\right]\left\{U^{(e)}\right\} \equiv \frac{EF(x)}{h_e}\{-1 \quad 1\}\begin{Bmatrix}u_e\\u_{e+1}\end{Bmatrix} \tag{4.13}$$

4.2. A quadratic polynomial approximation

The solution will be found in the local coordinate system; here we propose the quadratic polynomial in which the degrees of freedom are the displacement of the cross-section of the bar at the beginning $\tilde{u}_e$, middle $\tilde{u}_{e+\frac{1}{2}}$, and at the end $\tilde{u}_e$ of the interval (x_e, x_{e+1}).

The solution leads to the relation:

$$\tilde{u}^{(e)}(\xi)=\begin{bmatrix}\tilde{N}_e & \tilde{N}_{e+\frac{1}{2}} & \tilde{N}_{e+1}\end{bmatrix}\{U^{(e)}\}=[\tilde{N}_e]\{U^{(e)}\}, \tag{4.14}$$

in which the particular elements are defined by:

$$\begin{aligned}
&\tilde{N}_e(\xi)=1-\frac{3}{h_e}\xi+\frac{2}{h_e^2}\xi^2\equiv(1-\frac{2\xi}{h_e})(1-\frac{\xi}{h_e}),\\
&\tilde{N}_{e+\frac{1}{2}}(\xi)=\frac{4\xi}{h_e}-\frac{4\xi^2}{h_e}\equiv\frac{4\xi}{h_e}-(1-\frac{2\xi}{h_e}),\\
&\tilde{N}_{e+1}(\xi)=-\frac{\xi}{h_e}+\frac{2\xi^2}{h_e^2}\equiv-\frac{\xi}{h_e}(1-\frac{2\xi}{h_e});\\
&[\tilde{N}^{(e)}(\xi)]=\begin{bmatrix}\tilde{N}_e(\xi) & \tilde{N}_{e+\frac{1}{2}}(\xi) & \tilde{N}_{e+1}(\xi)\end{bmatrix},\\
&\{U^{(e)}\}=\begin{Bmatrix}u_e & u_{e+\frac{1}{2}} & u_{e+1}\end{Bmatrix}^T.
\end{aligned} \tag{4.15}$$

The variation of a displacement function is defined by:

$$\delta\tilde{u}^{(e)}(\xi)=[\tilde{N}^{(e)}(\xi)]\{\delta U^{(e)}\}\equiv\{\delta U^{(e)}\}^T[\tilde{N}^{(e)}(\xi)]^T. \tag{4.16}$$

We then set up the derivatives of the shape function (4.14) in the shape of the matrix (matrix of gradients of shape function) which is defined as $[\tilde{B}^{(e)}(\xi)]$:

$$\left[\tilde{B}^{(e)}(\xi)\right]=\frac{d}{d\xi}\left[\tilde{N}_e(\xi)\quad \tilde{N}_{e+\frac{1}{2}}(\xi)\quad \tilde{N}_{e+1}(\xi)\right]\equiv\left[\frac{d\tilde{N}^{(e)}(\xi)}{d\xi}\right]$$

$$=\left[-\frac{3}{h_e}+\frac{4\xi}{h_e^2}\quad \frac{4}{h_e}-\frac{8\xi}{h_e^2}\quad -\frac{1}{h_e}+\frac{4\xi}{h_e^2}\right] \tag{4.17}$$

$$=\frac{1}{h_e}\left[3\left(\frac{4}{3}\frac{\xi}{h_e}-1\right)\quad 4\left(-2\frac{\xi}{h_e}+1\right)\quad 4\frac{\xi}{h_e}-1\right]$$

Strain displacements (axis strain of bars) are equal to

$$\tilde{\varepsilon}^{(e)}(\xi)=\frac{du^{(e)}}{d\xi}\equiv\left[\frac{d\tilde{N}^{(e)}(\xi)}{d\xi}\right]\left\{U^{(e)}\right\}=\left[\tilde{B}^{(e)}(\xi)\right]\left\{U^{(e)}\right\},$$

$$\left\{U^{(e)}\right\}=\left\{u_e\quad u_{e+\frac{1}{2}}\quad u_{e+1}\right\}^T, \tag{4.18}$$

where $\left[\tilde{B}^{(e)}(\xi)\right]$ is defined as $(4.17)_3$.

Variation of strains, on the basis of (4.18), are equal to

$$\delta\tilde{\varepsilon}^{(e)}(\xi)=\left[\tilde{B}^{(e)}(\xi)\right]\left\{\delta U^{(e)}\right\}\equiv\left\{\delta U^{(e)}\right\}^T\left[\tilde{B}^{(e)}(\xi)\right]^T. \tag{4.19}$$

Stresses and axial forces at an arbitrary cross- section of the bar are defined:

$$\tilde{\sigma}^{(e)}(\xi)=E\tilde{\varepsilon}^{(e)}(\xi)\equiv E\left[\tilde{B}^{(e)}(\xi)\right]\left\{U^{(e)}\right\}$$

$$=\frac{E}{h_e}\left[3\left(\frac{4}{3}\frac{\xi}{h_e}-1\right)\quad 4\left(-2\frac{\xi}{h_e}+1\right)\quad 4\frac{\xi}{h_e}-1\right]\begin{Bmatrix}u_e\\ u_{e+\frac{1}{2}}\\ u_{e+1}\end{Bmatrix}, \tag{4.20}$$

$$\tilde{N}^{(e)}(\xi)=\tilde{F}(\xi)\tilde{\sigma}^{(e)}(\xi)\equiv E\tilde{F}(\xi)\left[\tilde{B}^{(e)}(\xi)\right]\left\{U^{(e)}\right\}.$$

4.3. A cubic polynomial approximation

As before we formulate the problem in the local coordinate system applying here the cubic polynomial approximation. The four parameters that define it are determined by the four degrees of freedom, that is the displacements and rotation angles in nodes:

$$\left\{U^{(e)}\right\}=\left\{u_e \quad \Phi_e \quad u_{e+1} \quad \Phi_{e+1}\right\}^T. \tag{4.21}$$

The solution of approximation problem leads to the equation:

$$\tilde{U}^{(e)}(\xi)=\left\{1 \quad \xi \quad \xi^2 \quad \xi^3\right\}[A]\left\{U^{(e)}\right\}, \tag{4.22}$$

where

$$[A]:=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\dfrac{3}{h_e^{\,2}} & -\dfrac{2}{h_e} & \dfrac{3}{h_e^{\,2}} & -\dfrac{1}{h_e} \\ \dfrac{3}{3h_e^{\,2}} & \dfrac{2}{3h_e^{\,2}} & \dfrac{2}{3h_e^{\,2}} & \dfrac{1}{3h_e^{\,2}} \end{bmatrix}. \tag{4.22'}$$

Introducing to (4.22) the matrix of shape function

$$\begin{aligned}\left[\tilde{N}^{(e)}(\xi)\right]&:=\left\{1 \quad \xi \quad \xi^2 \quad \xi^3\right\}[A] \\ &=\left\{\tilde{N}_e(\xi) \quad \tilde{N}_e'(\xi) \quad \tilde{N}_{e+1}(\xi) \quad \tilde{N}_{e+1}'(\xi)\right\},\end{aligned} \tag{4.23}$$

and the matrix of gradients of shape function

$$\left[B^{(e)}(\xi)\right]=\left[\frac{d\tilde{N}^{(e)}(\xi)}{d\xi}\right]=\left\{0 \quad 1 \quad 2\xi \quad 3\xi^2\right\}[A], \tag{4.24}$$

we write this equation in the traditional form:

$$\tilde{u}^{(e)}(\xi)=\left[\tilde{N}^{(e)}(\xi)\right]\left\{U^{(e)}\right\}. \tag{4.25}$$

The variation of the displacement function is as follows

$$\delta\tilde{u}^{(e)}(\xi) = \left[\tilde{N}^{(e)}(\xi)\right]\left\{\delta u^{(e)}\right\} \equiv \left\{\delta U^{(e)}\right\}^T\left[\tilde{N}^{(e)}(\xi)\right]^T. \quad (4.25')$$

Then strains and variation of strains are

$$\begin{aligned} \tilde{\varepsilon}^{(e)}(\xi) &= \frac{d\tilde{u}^{(e)}(\xi)}{d\xi} \equiv \left[\tilde{B}^{(e)}(\xi)\right]\left\{U^{(e)}\right\}^T, \\ \delta\tilde{\varepsilon}^{(e)}(\xi) &= \left[\tilde{B}^{(e)}(\xi)\right]\left\{\delta U^{(e)}\right\}^T \equiv \left\{\delta U^{(e)}\right\}^T\left[\tilde{B}^{(e)}(\xi)\right]^T, \end{aligned} \quad (4.26)$$

and stresses as rules axial forces are defined by the relations

$$\begin{aligned} \tilde{\sigma}^{(e)}(\xi) &= E\tilde{\varepsilon}^{(e)}(\xi) \equiv E\left[\tilde{B}^{(e)}(\xi)\right]\left\{U^{(e)}\right\}, \\ \tilde{N}^{(e)}(\xi) &= \tilde{F}(\xi)\tilde{\sigma}^{(e)}(\xi) \equiv E\tilde{F}(\xi)\left[\tilde{B}^{(e)}(\xi)\right]\left\{U^{(e)}\right\}. \end{aligned} \quad (4.27)$$

Conclusion

1. The new element in the papers concerned with this subiect is the inclusion of the nonlinearity of the ground,
2. Because of the nonlinear operator of the problem appearing in the description, the analysis of estimation of an approximate solution is a difficult issue for the "mathematical analysis"; this problem has not been presented here.
3. In the article the introductory material indispensable for the variational equations analysed in Refs. [5]; has been prepared; in [5] you will find the numerical analysis of the results obtained by the method of finite elements.

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Streszczenie

W pracy sformułowano podstawowe problemy brzegowe teorii prętów sprężystych, zanurzonych w ośrodku nieliniowym sprężystym, które są rozciągane - ściskane. Problem taki modeluje wiele technicznych zagadnień, np. kotwienie prętów stalowych w betonie, połączenia śrubowe, jeśli ograniczyć się do modelowania ciał sprężystych. Biorąc pod uwagę ogólność sformułowanych zadań przygotowano, - przy wykorzystaniu aproksymacji, liniowej, kwadratowej, kubicznej - opis pól przemieszczeń typowy dla metody elementów skończonych, którą będziemy wykorzystywać w dalszej pracy autorów do rozwiązania już szczególnych zadań, w zastosowaniu rozwiązywania równań wariacyjnych problemów technicznych. W celu rozwiązania zadań brzegowych sformułowanych tutaj problemów wymagane jest eksperymentalne wyznaczenie funkcji oporu ośrodka, w którym zanurzony jest pręt.