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THEORETICAL MECHANICS

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ON FUZZY SINGULAR INTEGRATION

Summary. In this paper, a new concept of fuzzy number-valued singular integral of fuzzy number-valued function with respect to a non-fuzzy measure is introduced, and some elementary properties are considered. We deal with fuzzy-set-valued mappings of real variables whose values are normal, convex, upper semicontinuous and compactly supported fuzzy sets in $\mathbb{R}^n$.

O ROZMYTYCH CAŁKACH OSOBLIWYCH

Streszczenie. W pracy przedstawiono nową koncepcję rozmytej całki osobliwej, określonej na funkcji rozmytej, względem miary deterministycznej oraz jej elementarne własności. W pracy rozpatrywane są funkcje rozmyte zmiennych rzeczywistych, których wartościami są zbiory rozmyte normalne, wypukłe, półciągłe z góry, o nośniku zwartym.

ОБ РАЗМЫТЫХ СИНГУЛЯРНЫХ ИНТЕГРАЛАХ

Резюме. В работе представлена новая концепция размытого сингулярного интеграла, определённого на размытой фухнции, по детерминированной мере и его елементарные свойства. Рассматриваются также размытые функции вещественных переменных, значениями которых ябляются размытые, нормальные, выпуклые, полунепрерывные сверху и с компактным носителем множества.

1. INTRODUCTION

In applications boundary value problems play a very important role. One of the most preferable methods of solving boundary value problems is based on boundary integral equations theory [4,5].

Many practical applications are governed by the Poisson equation. Consider that we are seeking to find the solution of a Poisson equation in a Ω (two or three dimensional) domain,

$$\nabla^2 u(x) = \xi(x), \qquad x \in \Omega, \tag{1}$$

where ξ is a known source density function of position and with the following conditions on the Γ boundary of Ω:

(i) Dirichlet (essential) conditions of the type $u(x)=u_0(x)$, for $x\in\Gamma_1$,

(ii) Neumann (natural) conditions such as $q(x)=\partial u(x)/\partial n=q_0(x)$, for $x\in\Gamma_2$,

where n is the normal to the boundary, $\Gamma=\Gamma_1\cup\Gamma_2$, and functions u_0, q_0 are known.

After substituting the Laplace fundamental solution U and $Q=\partial U/\partial n$ and grouping all boundary terms together (i.e. in $\Gamma=\Gamma_1\cup\Gamma_2$), one obtains [4,5] a boundary integral equation of the form

$$c(x)u(x) + \int_\Gamma Q(x,y)u(y)d\Gamma(y) + \int_\Omega U(x,y)\xi(y)d\Omega(y) = \int_\Gamma U(x,y)q(y)d\Gamma(y), \quad (2)$$

for $x\in\Gamma$, where the integrals are in the sense of Cauchy Principal Value [4, 5,16,17].

Notice that when a physical problem is transformed into the deterministic boundary problem (1) or (2) we usually cannot be sure that this modelling is perfect. The boundary problem may not be known exactly and the functions u_0, q_0 and ξ may contain unknown parameters. Especially, if they are known through some measurements they necessarily are subjected to errors. The analysis of the efect of these errors leads to the study of the qualitative and quantitive behavior of the solution uncertainty.

If the nature of errors is random, then instead of a deterministic problem (1) or (2) we get a random boundary integral equation with stochastic functions and/or random coefficients, comp. [6]. But if the underlying structure is not probabilistic, e.g. because of subjective choices, then it may be appropriate to use fuzzy numbers instead of real random variables. A fuzzy number $\tilde{a}$ is a fuzzy set of real numbers, i.e. a function μ: $\mathbb{R}\longrightarrow[0,1]$ whose value $\mu(x|\tilde{a})$ is the grade of membership of x in $\tilde{a}$. This leads to a fuzzy boundary value problems and in consequence to fuzzy boundary integral equations (FBIE), comp. [7,8].

To make further considerations on FBIE's possible we have at first to generalize certain results of singular integration to fuzzy-valued mappings. Since a fuzzy-valued mapping is essentially a family of set-valued mappings we utilize results for integration of set-valued mappings [1,2,11,14,15,19].

2. ELEMENTARY CONCEPTS AND RESULTS

In this paper, the following concepts and notations will be used. $\mathbb{R}^n$ was reserved for the set of n-dimensional reals, $(\mathbb{R}^n,|\cdot|)$-denotes the Euclidean space with metric $|\cdot|$, $\mathbb{R}_+$ - the set of nonnegative reals, Γ is an arbitrary fixed n-dimensional manifold in Euclidean space, $\mathcal{A}$ is a σ-algebra formed by the subsets of Γ, $(\Gamma,\mathcal{A},d\Gamma)$ is a classical complete and finite measure space (nonfuzzy). In the following cl(A) denotes the closure of a set $A\subset\mathbb{R}^n$, $\vee,\wedge$ will stand for 'supremum' and 'infimum' respectively.

Let the symbol $\mathcal{P}_c(\mathbb{R}^n)$ denotes the power set of $\mathbb{R}^n$. Define the addition and scalar multiplication in $\mathcal{P}_c(\mathbb{R}^n)$ as usual Kaleva [14,15].

Let $I(\mathbb{R})$ denote the set of all closed bounded intervals $\bar{z}=[z^-,z^+]$ on the real line $\mathbb{R}$, where z^- and z^+ denote the end points of $\bar{z}$. We call further elements of $I(\mathbb{R})$ interval numbers [3,10,18].

DEFINITION 2.1. For interval numbers $\bar{a},\bar{b}\in I(\mathbb{R})$, we define:

(i) $\bar{a}\le\bar{b}$ iff $a^-\le b^-$ and $a^+\le b^+$;

(ii) $\bar{a}\bullet\bar{b} := \{a\bullet b\colon a\in\bar{a},\ b\in\bar{b},\ "\bullet"="+,-,\cdot,/"\}$,

where (ii) gives a general method to determine obvious algebraic operations on interval numbers and which results in the following formulas:

(iii) $\bar{a}+\bar{b}=\bar{c}$ iff $c^-=a^-+b^-$ and $c^+=a^++b^+$;

(iv) $\bar{a}-\bar{b}=\bar{c}$ iff $c^-=a^--b^+$ and $c^+=a^+-b^-$;

(v) $\bar{a}\cdot\bar{b}=\bar{c}$ iff $c^-=\min(a^-b^-,a^-b^+,a^+b^-,a^+b^+)$ and $c^+=\max(a^-b^-,a^-b^+,a^+b^-,a^+b^+)$;

(vi) $1/\bar{a}=\bar{c}$ iff $c^-=1/a^+$ and $c^+=1/a^-$ whenever $0\notin[a^-,a^+]$;

(vii) $\bar{a}/\bar{b}=\bar{c}$ iff $\bar{c}=\bar{a}\cdot(1/\bar{b})$;

(viii) $d(\bar{a},\bar{b}):=|a^--b^-|\vee|a^+-b^+|$ is called the distance between interval numbers $\bar{a}$ and $\bar{b}$. ■

It is easy to see that, if a and b are real numbers, then $d(a,b)=|a-b|$.

For further information we refer to [3,18].

DEFINITION 2.2. Let $\mathcal{F}(\mathbb{R}^n)$ is the fuzzy power set of $\mathbb{R}^n$ i.e.

$$\mathcal{F}(\mathbb{R}^n) := \{\mu\colon\ \mu\colon\mathbb{R}^n\to[0,1]\}.$$

A fuzzy vector is a fuzzy set $\tilde{a}$ with membership function $\mu(x|\tilde{a})$ in $\mathcal{F}(\mathbb{R}^n)$ satisfying the following conditions:

(i) $\tilde{a}$ is normal i.e. there exists an $x\in\mathbb{R}^n$ such that $\mu(x|\tilde{a})=1$;

(ii) $\tilde{a}$ is fuzzy convex i.e. for every $\lambda\in]0,1]$, $\tilde{a}_\lambda := \{x: \tilde{a}(x)\geq\lambda\}$ is a closed interval, denoted by $[a_\lambda^-, a_\lambda^+]$;

(iii) $\tilde{r}$ is upper semicontinuous;

(iv) $\tilde{r}_0 := \mathrm{cl}\{r\in\mathbb{R}^n: \mu(r|\tilde{r})>0\}$ is compact.

Let $\mathcal{F}^*(\mathbb{R}^n)\subset\mathcal{F}(\mathbb{R}^n)$ denote a set of all fuzzy vectors [9-15, 19, 20, 24].■

If $f:\mathbb{R}^n\times\mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a function then according to Zadeh's extension principle we can extend f to $\mathcal{F}^*(\mathbb{R}^n)\times\mathcal{F}^*(\mathbb{R}^n) \longrightarrow \mathcal{F}^*(\mathbb{R}^n)$ by the equation

$$\tilde{f}(\tilde{u},\tilde{v})(z) = \sup_{z=f(x,y)}\mu(x|\tilde{u})\wedge\mu(y|\tilde{v}), \tag{3}$$

It is well known that

$$\tilde{f}_\lambda(\tilde{u},\tilde{v}) = \tilde{f}(\tilde{u}_\lambda,\tilde{v}_\lambda), \qquad \forall\ \tilde{u},\tilde{v}\in\mathcal{F}^*(\mathbb{R}^n), 0\leq\lambda\leq1, \tag{4}$$

and f continuous [9, 10, 14, 15].

By $\mathcal{F}^*(\mathbb{R})$ we denote a set of all fuzzy numbers. Two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are called equal, $\tilde{a}=\tilde{b}$, if $\mu(x|\tilde{a})=\mu(x|\tilde{b})$ $\forall$ $x\in\mathbb{R}$. It follows that

$$\tilde{a} = \tilde{b} \Leftrightarrow \tilde{a}_\lambda = \tilde{b}_\lambda \quad \forall\ \lambda\in]0,1]. \tag{5}$$

For the arithmetic operations on fuzzy numbers we refer to [9,10]. The following results are well known and follows from the theory of interval operations, comp. Def.2.1.

LEMMA 2.1 [13,14]. If $\tilde{a},\tilde{b}\in\mathcal{F}^*(\mathbb{R})$, then for $\lambda\in]0,1]$,

$$(\tilde{a} + \tilde{b})_\lambda = [a_\lambda^- + b_\lambda^-, a_\lambda^+ + b_\lambda^+], \tag{6}$$

$$(\tilde{a} - \tilde{b})_\lambda = [a_\lambda^- - b_\lambda^+, a_\lambda^+ - b_\lambda^-], \tag{7}$$

$$(\tilde{a}\cdot\tilde{b})_\lambda = [\min(a_\lambda^-b_\lambda^-, a_\lambda^-b_\lambda^+, a_\lambda^+b_\lambda^-, a_\lambda^+b_\lambda^+),\ \max(a_\lambda^-b_\lambda^-, a_\lambda^-b_\lambda^+, a_\lambda^+b_\lambda^-, a_\lambda^+b_\lambda^+)], \tag{8}$$

$$(1/\tilde{b})_\lambda = [1/b_\lambda^+, 1/b_\lambda^-], \text{ whenever } \emptyset\notin[b_\lambda^-, b_\lambda^+], \tag{9}$$

$$(\tilde{a}/\tilde{b})_\lambda = (\tilde{a}\cdot(1/\tilde{b}))_\lambda. \blacksquare \tag{10}$$

Notice that $I(\mathbb{R})\subset\mathcal{F}^*(\mathbb{R})$.

DEFINITION 2.3. Define $\rho:\mathcal{F}^*(\mathbb{R}^n)\times\mathcal{F}^*(\mathbb{R}^n) \longrightarrow \mathbb{R}_+\cup\{\emptyset\}$ by the equation

$$\rho(\tilde{a},\tilde{b}) := \sup_{0\leq\lambda\leq1} d(\tilde{a}_\lambda, \tilde{b}_\lambda),$$

where d is the Hausdorff metric defined in $\mathcal{P}_c(\mathbb{R}^n)$. Then it is easy to show that ρ is a metric in $\mathcal{F}^*(\mathbb{R}^n)$.■

In addition, we see that $(\mathcal{F}^*(\mathbb{R}^n),\rho)$ is a complete metric space.

Recall that the Hausdorff metric is defined as

$$d(A,B) := \inf\{\varepsilon: A\subset N(B,\varepsilon), B\subset N(A,\varepsilon)\},$$

where $A,B\in\mathcal{P}_C(\mathbb{R}^n)$ and $N(A,\varepsilon):=\{x\in\mathbb{R}^n: |x-y|<\varepsilon\ \forall\ y\in A\}$. This notation coincides with introduced in Def.2.1(viii) for n=1. Since $\alpha N(A,\varepsilon)=N(\alpha A,|\alpha|\varepsilon)$, $\forall\ \alpha\in\mathbb{R}$, we see that

$$\rho(\alpha\tilde{a},\alpha\tilde{b}) = |\alpha|\rho(\tilde{a},\tilde{b}), \qquad \forall\ \tilde{a},\tilde{b}\in\mathcal{F}^*(\mathbb{R}^n),\ \alpha\in\mathbb{R}. \tag{11}$$

From (6) it follows that

$$\rho(\tilde{a}+\tilde{c},\tilde{b}+\tilde{c}) = \rho(\tilde{a},\tilde{b}), \qquad \forall\ \tilde{a},\tilde{b},\tilde{c}\in\mathcal{F}^*(\mathbb{R}^n). \tag{12}$$

Comp. [9-14,19].

DEFINITION 2.4. For a sequence of fuzzy vectors $\{\tilde{r}_n\}\subset\mathcal{F}^*(\mathbb{R}^n)$, we say it is convergent iff $\{(\tilde{r}_n)_\lambda\}$ is a convergent sequence of interval vectors for all $\lambda\in]0,1]$, and the limit is defined as

$$\tilde{r}(x) := \sup\{\lambda: x\in\lim (\tilde{r}_n)_\lambda\}$$

simply write as $\tilde{r}_n \longrightarrow \tilde{r}$, or $\lim_{n\to\infty} \tilde{r}_n=\tilde{r}$.■

DEFINITION 2.5. For a sequence of fuzzy vectors $\{\tilde{r}_n\}\subset\mathcal{F}^*(\mathbb{R}^n)$, we say it is strongly convergent iff there exists an $\tilde{r}\in\mathcal{F}^*(\mathbb{R}^n)$ such that $\rho(\tilde{r}_n,\tilde{r}) \longrightarrow 0$, or simply said $\{\tilde{r}_n\}$ is strongly convergent to $\tilde{r}$, denoted with

$$\tilde{r}_n \xrightarrow{s} \tilde{r}, \text{ or } s\text{-}\lim_{n\to\infty} \tilde{r}_n=\tilde{r}.■$$

Obviously, $\tilde{r}_n \xrightarrow{s} \tilde{r}$ implies $\tilde{r}_n \longrightarrow \tilde{r}$ [10,13,14,19].

An interval-valued function is a special closed-valued set valued function $\bar{f}:\Gamma \longrightarrow I(\mathbb{R})$ [2,3,18,23]. It is usually written as $\bar{f}(x)=[f^-(x),f^+(x)]$, where

$$f^-(x)=\inf \bar{f}(x), \qquad f^+(x)=\sup \bar{f}(x).$$

A fuzzy-valued function is a mapping $\tilde{f}:\Gamma \longrightarrow \mathcal{F}^*(\mathbb{R})$ [9-15,19,20].

DEFINITION 2.6. Let $\tilde{f}:\Gamma \longrightarrow \mathcal{F}^*(\mathbb{R}^n)$ be a fuzzy function, $a\in\Gamma$, $\tilde{b}\in\mathcal{F}^*(\mathbb{R}^n)$. If

$$\forall\ \lambda\in]0,1],\ \varepsilon>0\ \exists\ \delta>0 :\ |x-a|<\delta \Rightarrow d(\tilde{f}_\lambda(x),\tilde{b}_\lambda)<\varepsilon$$

then $\tilde{f}(x)$ is said to converge to $\tilde{b}$ as $x \longrightarrow a$, written

$$\lim_{x\to a}\tilde{f}(x) = \tilde{b}.■$$

DEFINITION 2.7. Let $\tilde{f}:\Gamma \longrightarrow \mathcal{F}^*(\mathbb{R}^n)$ be a fuzzy function, $a\in\Gamma$, $\tilde{b}\in\mathcal{F}^*(\mathbb{R}^n)$. If

$$\forall\ \varepsilon>0\ \exists\ \delta>0 : \rho(x,a)<\delta \Rightarrow \rho(\tilde{f}(x),\tilde{b})<\varepsilon$$

then $\tilde{f}(x)$ is said to converge strongly to $\tilde{b}$ as $x \longrightarrow a$, written

$$\text{s-lim}_{x\to a}\tilde{f}(x) = \tilde{b}. \blacksquare$$

Comp. [9-15,19,20].

DEFINITION 2.8. We say that a fuzzy-valued mapping $\tilde{f}$ is (strongly) measurable if for all $\lambda\in]0,1]$ the set-valued mapping $\tilde{f}_\lambda:\Gamma \longrightarrow \mathcal{P}_c(\mathbb{R})$ defined by

$$\tilde{f}_\lambda(x) := \left(\tilde{f}(x)\right)_\lambda$$

is (Lebesgue) measurable, when $\mathcal{P}_c(\mathbb{R})$ is endowed with the topology generated by the Hausdorff metric d [11,14,19]. ■

In the ordinary way (pointwise), we can define the operations, orders, convergences of interval-valued functions and fuzzy-valued functions.

LEMMA 2.2 [11,14,15,19]. Let $\bar{f}(x)=[f^-(x),f^+(x)]$. Then $\bar{f}$ is measurable iff f^- and f^+ are measurable. ■

3. FUZZY SINGULAR INTEGRALS

In this section, we first define the fuzzy integral of fuzzy-valued functions in a similar way to Aumann, and then we discuss the fuzzy singular integral of fuzzy-valued functions with respect to a non-fuzzy measure.

DEFINITION 3.1. Let $\tilde{f}:\Gamma \longrightarrow \mathcal{F}^*(\mathbb{R}_+)$ be a non-negative measurable fuzzy-valued function. The integral of $\tilde{f}$ over Γ, denoted $\int_\Gamma \tilde{f}(x)d\Gamma(x)$ is defined levelwise by the equation

$$\int_\Gamma \tilde{f}(x)d\Gamma(x) := \bigcup_{\lambda\in[0,1]} \lambda\left(\int_\Gamma \tilde{f}_\lambda(x)d\Gamma(x)\right) =$$

$$= \bigcup_{\lambda\in[0,1]} \lambda\left\{\int_\Gamma f_\lambda(x)d\Gamma(x):\ f_\lambda:\Gamma \longrightarrow \mathbb{R}_+ \text{ is a measurable selection for } \tilde{f}_\lambda\right\}.$$

where

$$\int_\Gamma \tilde{f}_\lambda(x)d\Gamma(x) = \left[\ \int_\Gamma f^-_\lambda(x)d\Gamma(x),\ \int_\Gamma f^+_\lambda(x)d\Gamma(x)\ \right]$$

is a usual integral of interval-valued function and is called the fuzzy integral of $\tilde{f}$ over Γ. ■

A strongly measurable mapping $\tilde{f}:\Gamma \longrightarrow \mathcal{F}^{*}(\mathbb{R}_{+})$ is said to be integrable over Γ if $\int_{\Gamma}\tilde{f}(x)d\Gamma(x) \in \mathcal{F}^{*}(\mathbb{R}_{+})$.

When the integral is taken over a subset $S\subset\Gamma$ we will write $\int_{S}\tilde{f}$.

It is obvious for $\mathcal{F}^{*}(\mathbb{R})$ that an each fuzzy number of $\mathcal{F}^{*}(\mathbb{R})$ can be presented as a fuzzy difference of two positive fuzzy numbers from $\mathcal{F}^{*}(\mathbb{R}_{+})$, so the extension of Def. 4.1 to all measurable fuzzy functions $\tilde{f}:\Gamma \longrightarrow \mathcal{F}^{*}(\mathbb{R})$ is easy.

Now, after giving the definition of fuzzy integrals of fuzzy mappings, we establish a fuzzy singular integral theory with respect to fuzzy-valued functions. It is the extension of Def.4.1.

Let $\tilde{h}:\Gamma \longrightarrow \mathcal{F}^{*}(\mathbb{R})$ be a fuzzy function defined almost everywhere (a.e.) on a certain manifold Γ which is endowed with some fuzzy metric (comp. part 2). Let x be a certain point on Γ; draw a ball with radius ε around x and remove from Γ the intersection of that ball with Γ. Suppose that the function $\tilde{h}$ is fuzzy summable over the remaining part Γ_{ε} of the manifold Γ, and that it is true for any $\varepsilon>0$. If the limit

$$\lim_{\varepsilon\to 0} \int_{\Gamma_{\varepsilon}(x)} \tilde{h}(y)d\Gamma(y)$$

exists in some fuzzy sense, it is called fuzzy singular integral of the fuzzy function $\tilde{h}$ over the manifold Γ. In particular, the manifold Γ can be the space $\mathbb{R}^{n}$ or a domain in $\mathbb{R}^{n}$.

DEFINITION 3.2. Let $\tilde{h}:\Gamma \longrightarrow \mathcal{F}^{*}(\mathbb{R})$ be a measurable fuzzy-valued function integrable on every subset $\Gamma_{\varepsilon}(x)\subset\Gamma$, where $\Gamma_{\varepsilon}(x):=\{y\in\Gamma:\ |y-x|\geq\varepsilon>0\}$. If the integral $\int_{\Gamma_{\varepsilon}(x)}\tilde{h}(y)d\Gamma(y)$ converges to some fuzzy number as $\varepsilon \longrightarrow 0$ in the sense of Def.2.6 then this limit will be called the fuzzy singular integral of $\tilde{h}$ on Γ in the sense of Fuzzy Principal Value (F.P.V.). It is written as

$$(\text{F.P.V.})\int_{\Gamma}\tilde{h}(y)d\Gamma(y) = \lim_{\varepsilon\to 0} \int_{\Gamma_{\varepsilon}(x)} \tilde{h}(y)d\Gamma(y)$$

or simply $\int_{\Gamma}\tilde{h}d\Gamma(y)$. ■

DEFINITION 3.3. Assume the notation is the same as in Def.4.2. If the integral $\int_{\Gamma_\varepsilon(x)} \tilde{h}(y)d\Gamma(y)$ converges strongly to some fuzzy number as $\varepsilon \longrightarrow 0$ in the sense of Def.2.7 then this limit will be called the strong fuzzy singular integral of $\tilde{h}$ on Γ in the sense of Fuzzy Principal Value (F.P.V.). It is written as

$$(\text{s-F.P.V.})\int_{\Gamma} \tilde{h}(y)d\Gamma(y) = \text{s-lim}_{\varepsilon \to 0} \int_{\Gamma_\varepsilon(x)} \tilde{h}(y)d\Gamma(y)$$

or simply $\int \tilde{h}d\Gamma$.■

THEOREM 3.1. Let $\bar{h}:\Gamma \longrightarrow I(\mathbb{R})$ be a measurable interval-valued function and assume that there exists the fuzzy singular integral of $\tilde{h}$ on Γ. Then

$$(\text{F.P.V.})\int_{\Gamma} \bar{h}(y)d\Gamma(y) = \left[(\text{P.V.})\int_{\Gamma} h^{-}(y)d\Gamma(y),\ (\text{P.V.})\int_{\Gamma} h^{+}(y)d\Gamma(y)\right]$$

where $(\text{P.V.})\int hd\Gamma$ denotes the usual singular integral in the sense of Principal Value [16,17] i.e.

$$(\text{P.V.})\int_{\Gamma} h^{\mp}(y)d\Gamma(y) = \lim_{\varepsilon \to 0} \int_{\Gamma_\varepsilon(x)} h^{\mp}(y)d\Gamma(y).$$

Proof. It follows immediately from [14,15].■

Let us consider the fuzzy singular integral

$$\tilde{v}(x) := \int_{\mathbb{R}^n} r^{-n}\tilde{h}(x,\theta)\tilde{u}(y)dy, \tag{13}$$

where x and y are points in the space $\mathbb{R}^n$, and $r=|y-x|$, $\theta=(y-x)/r$. The point x is called pole, the function $\tilde{f}(x,\theta)$ fuzzy characteristic and the function $\tilde{u}(y)$ fuzzy density of the fuzzy singular integral (13).

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Streszczenie

W pracy przedstawiono nową koncepcję rozmytej całki osobliwej, określonej na funkcji rozmytej, względem miary deterministycznej zdefiniowanej na konturze nierozmytym oraz jej elementarne własności. Rozpatrywane są całki osobliwe w sensie silnym i słabym. Całki rozmyte w sensie Lebesgue'a rozumiane są w sposób wprowadzony do teorii przez Aumanna. W pracy rozpatrywane są funkcje rozmyte zmiennych rzeczywistych, których wartościami są zbiory rozmyte normalne, wypukłe, półciągłe z góry, o nośniku zwartym.