ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1999
Seria: BUDOWNICTWO z. 86 Nr kol. 1440

Jerzy SKRZYPCZYK

FUZZY METHODS IN THE ANALYSIS OF UNCERTAIN SYSTEMS

Summary. In the paper basic concepts of a new methodology of the fuzzy boundary element method and fuzzy finite element methods are presented. This article deals with fuzzy-set-valued mappings which are solutions of the fuzzy equations. Computational fuzzy problems and applications are considered in details for boundary potential problems with fuzzy Dirichlet and Neumann type boundary conditions and for fuzzy finite element problems. All structural and load uncertainties are assumed to be fuzzy values and the structure is discretized using FEM. This yields the elements of the stiffness matrix and the components of the force vector with uncertainties to be defined as fuzzy values.

METODY ROZMYTE W ANALIZIE SYSTEMÓW NIEPEWNYCH

Streszczenie. W pracy przedstawiono podstawowe założenia nowej metodologii metody rozmytych elementów brzegowych (MREB) i metody rozmytych elementów skończonych (MRES). W artykule wprowadzono pojęcia odwzorowań o wartościach rozmytych, które definiują rozwiązania równań rozmytych. Dyskutowane są szczegółowo problemy obliczeniowe i zastosowania dla brzegowych zadań teorii potencjału z rozmytymi warunkami brzegowymi typu Dirichleta i Neumana oraz dla problemów rozwiązywanych metodami rozmytych elementów skończonych. Założono, że wszystkie niepewności związane ze strukturą budowli oraz z obciążeniami są wielkościami rozmytymi, a struktura jest dyskretyzowana z wykorzystaniem MRES lub MREB. Założenia te powodują, że składowe macierzy sztywności i składowe wektora sił niepewnych są definiowane jako wielkości rozmyte.

1. Introduction

When a physical problem is transformed into the deterministic boundary problem, we usually cannot be sure that this modelling is perfect. The nature of this uncertainty can be discussed generally under three headings: human based uncertainty, system uncertainty and random uncertainty. The prediction of these three types of uncertainty is difficult and present methods, embodied in reliability theory, tend to concentrate on random uncertainty.

The boundary problem may not be known exactly and some functions i.e. the shape of a structure, material properties, boundary conditions, external or internal excitations, solutions etc. may contain unknown parameters. Especially, if they are known through some measurements they necessarily are subjected to errors. The analysis of the effect of these errors leads to the study of the qualitative and quantitative behaviour of the solution uncertainty. There is however, a fundamental difference between the nature of random uncertainty and that of human and system uncertainty. To analyse this type of uncertainty a mathematics which is directed at "vagueness" as distinct from randomness is required and this is the potential role of fuzzy sets. Many different interpretations are possible for terminology of uncertain aspects of the *Boundary Element Method* (BEM). We focus our attention on fuzzy-set-theoretic description of uncertain phenomena in BEM, and will refer to these approaches as *Fuzzy Boundary Element Method* (FBEM). These terms are used here to refer to the boundary element method which accounts for uncertainties in boundary conditions or material properties of a structure as well as the shape of a boundary. Such uncertainties are usually distributed on a boundary or within a domain of the structure and should be modelled as spatial or spatially-temporal fuzzy fields.

Applications of the FBEM appear to have been initiated in the 1995. The earliest application used the fuzzy boundary integral equation to solve fuzzy boundary value potential problem with uncertain boundary conditions and internal sources (cf. Burczyński & Skrzypczyk [6-8]). Then FBEM has been used for elastostatic problems (cf. Pilch [19], Skrzypczyk & Burczyński [9-10]). Modelling uncertainties as fuzzy variables or fuzzy processes suggests the use of fuzzy-set-theoretical methods, which are closely related to convex modelling of uncertainties. Only linear static problems are studied and applications to non-linear or dynamic problems are left for a future study. Using BEM to solve boundary value problem in some domain with prescribed boundary conditions on the boundary, one can obtain the *Boundary Integral Equation.* From now we assume that values of some of boundary conditions, material properties, internal prescribed fields and the shape of a boundary are uncertain and we'll model this uncertainty using fuzzy variables. We obtain the *Fuzzy Boundary Integral Equation* where all operations are in the fuzzy sense.

Singular integrals are understood in the sense of fuzzy principal values (cf. Skrzypczyk [20-21], Skrzypczyk & Burczyński [24-25]).

Illustrative examples from the potential theory are given to comment different aspects of the presented theory. Interval and trapezoid - type fuzzy boundary conditions are considered. To complete the presentation the potential problem in a fuzzy domain is discussed.

Many different interpretations are possible for terminology of uncertain aspects of the FEM's. We focus our attention on fuzzy-set-theoretic and non-stochastic description of uncertain phenomena in FEM, and will refer to these approaches as *Fuzzy Finite Element Method* (FFEM). All structural and load uncertainties are assumed to be fuzzy values and the structure is discretized using *Fuzzy Finite Elements.* This yields the elements of the stiffness matrix and the components of the force vector with uncertainties to be defined as fuzzy values. Hence fuzzy algebraic operations are applied in connection with FEM's.

Applications of the special kind of FFEM called *Interval Finite Element Methods* (IFEM's) appear to have been initiated in the 1994. The earliest applications used the interval arithmetic formulation and all uncertainties are assumed to be defined in bounded intervals. The first formulation of the *Fuzzy Finite Element Method* (FFEM) appeared in 1996, (cf. Skrzypczyk [21]). Modelling uncertainties as fuzzy variables or fuzzy processes suggests the use of fuzzy-set-theoretical methods, which are closely related to convex modelling of uncertainties. Only linear static problems are studied and applications to non-linear or dynamic problems are left for a future study.

The proposed technique leads to computation of fuzzy characteristics of response quantities. The procedure can be used to solve a wide variety of important structural engineering problems with different kinds of uncertainties including uncertain shape of structure's boundary.

Illustrative examples of frames and trusses are studied to comment how to obtain good approximations of fuzzy solutions.

Presented methods give the complete methodology how to obtain good approximations of solutions of uncertain boundary problems with use of fuzzy analysis.

2. Basic Definitions and Notion

In the paper we use the following notion. R^n denotes the set of n-dimensional reals, $\left(R^n, |\cdot|\right)$ - n-dimensional Euklidean space with the metric $|\cdot|$, $R(R_+)$ the set of reals (nonnegative reals respectively), Γ is reserved for k-dimensional (k<n) manifold in Euklidean space R^n.

Let further I(R) (similarly $I(R^n)$) denote the set of all closed, bounded intervals $\bar{z}=[z^-,z^+]$ on real line R (R^n respectively), where z^- i z^+ denote end points of the interval $\bar{z}$. We call further elements of sets I(R) ($I(R^n)$) interval numbers (interval vectors respectively) (cf. Alefeld & Herzberger [1], Bauch et al. [2], Moore [16], Neumaier [18]).

Let $F(R^n)$ be the class of fuzzy sets in R^n, i.e. the set of maps (cf. Dubois & Prade [12], Kacprzyk [13], Negoita & Ralescu [17])

$$F(R^n):=\{\mu: R^n \to [0,1]\}.$$

We call a fuzzy number the set $\tilde{a} \in F(R^n)$ defined by the so called membership function $\mu(x;\tilde{a}),\ x \in R^n$ and satisfying some additional conditions (cf. Dubois & Prade [11-12]). Let further

$$\tilde{a}_\lambda := \{x \in R^n : \mu(x;\tilde{a}) \ge \lambda\},\ 0 < \lambda \le 1.$$

By $F^*(R^n) \subset F(R^n)$ we denote the set of all fuzzy numbers. Interval numbers are naturally the particular examples of fuzzy numbers.

3. Fuzzy Boundary Integral Equations

Using BEM to solve potential boundary value problem in a domain Ω with prescribed boundary conditions on the Γ boundary of Ω: Dirichlet (essential) conditions of the type $u(x)=u_0(x)$, for $x \in \Gamma_1$ and Neumann (natural) conditions such as $q(x)=\partial u(x)/\partial n=q_0(x)$, for $x \in \Gamma_2$, $\Gamma=\Gamma_1 \cup \Gamma_2$, one obtains

$$c(x)u(x)+\int_\Gamma Q(x,y)u(y)d\Gamma(y)+\int_\Omega U(x,y)\xi(y)d\Omega(y)=\int_\Gamma U(x,y)q(y)d\Gamma(y), \qquad x \in \Gamma \quad (3.1)$$

where $\xi(x)$, $x \in \Omega$ is a known source density function and U is a fundamental solution of the Laplace equation ($Q=\partial U/\partial n$), (see Brebbia & Dominguez [3], Brebbia et al. [4], Burczyński [5]). We now assume that values of some of boundary quantities, a source density function and a contour Γ are uncertain and we shall model this uncertainty using fuzzy variables.

Let $\tilde{u}_0, \tilde{q}_0, \tilde{\xi}$ and $\tilde{\Gamma}$ be fuzzy functions. Define

$$U_\lambda(x|\Gamma) := \left| \begin{array}{l} u: c(x)u(x) + \int_\Gamma Q(x,y)u(y)d\Gamma(y) + \int_\Omega U(x,y)\xi(y)d\Omega(y) = \int_\Gamma U(x,y)q(y)d\Gamma(y), \\ u_0(z) \in \overline{u}_{0\lambda}(z)\big|_{z\in\Gamma_1}, q_0(z) \in \overline{q}_{0\lambda}(z)\big|_{z\in\Gamma_2}, \xi(z) \in \overline{\xi}_\lambda(z), z \in \Omega, \Gamma \in \mathcal{M} \end{array} \right| \tag{3.2}$$

The conditional exact fuzzy solution $\tilde{u}_1(x|\Gamma), x \in \Gamma \in \mathcal{M}$ is defined as follows

$$\mu_\Gamma(y;\tilde{u}_1(x|\Gamma)) := \sup\{\lambda: y \in U_\lambda(x|\Gamma)\}, \qquad x \in \Gamma \in \mathcal{M}, y \in R^1. \tag{3.3}$$

and the exact fuzzy solution $\tilde{u}_1(x), x \in \tilde{\Gamma}$ is defined according to min-max composition of fuzzy relations

$$\mu(y;\tilde{u}_1(x)) := \sup_{\Gamma\in\mathcal{M}}\left(\mu(\Gamma;\tilde{\Gamma}) \wedge \mu_\Gamma(y;\tilde{u}_1(x|\Gamma))\right), \qquad y \in R^1. \tag{3.4}$$

Formula (3.4) describes the membership function function of the first-type fuzzy solution of boundary potential problem defined over a fuzzy domain.

Alternatively let substitute $\tilde{u}_0, \tilde{q}_0, \tilde{\xi}$ and $\tilde{\Gamma}$ for u_0, q_0, ξ and Γ respectively and let all operations be considered in the fuzzy sense. Thus we consider further the fuzzy analogue of eq. (3.1), as follows

$$\tilde{c}(x)\tilde{u}(x) + \int_{\tilde{\Gamma}} Q(x,y)\tilde{u}(y)d\Gamma(y) + \int_{\tilde{\Omega}} U(x,y)\tilde{\xi}(y)d\Omega(y) = \int_{\tilde{\Gamma}} U(x,y)\tilde{q}(y)d\Gamma(y), \qquad x \in \tilde{\Gamma} \tag{3.5}$$

Fuzzy integrals are understood in the sense of fuzzy principal value.

Assume, that we are looking for the interval - type solution

$$\tilde{u}_\lambda(x) = \left[u_\lambda^-(x), u_\lambda^+(x)\right], \quad x \in \Gamma \in \mathcal{M}, \tag{3.6}$$

where $0 \le \lambda \le 1$. Taking λ-cuts, $\forall 0 \le \lambda \le 1$ of the fuzzy eq. (3.5) we obtain formally the infinite set of interval boundary integral equations as follows

$$\begin{gathered} c(x)\left[u_\lambda^-(x), u_\lambda^+(x)\right] = \int_\Gamma Q(x,y)\left[u_\lambda^-(y), u_\lambda^+(y)\right]d\Gamma(y) + \\ + \int_\Omega U(x,y)\left[\xi_\lambda^-(y), \xi_\lambda^+(y)\right]d\Omega(y) = \int_\Gamma U(x,y)\left[q_\lambda^-(y), q_\lambda^+(y)\right]d\Gamma(y), \qquad x \in \Gamma \in \mathcal{M} \end{gathered} \tag{3.7}$$

4. Finite Elements With Fuzzy Parameters

A discrete model of a linear finite element with uncertain parameters is represented by the stiffness matrix with elements, which are fuzzy numbers. Respectively a discrete model of an uncertain field of loads is the vector with fuzzy values. Discretization of a continuum in "uncertainty" conditions using the technique based on finite element method will be presented on the example of a linear bar element. Assume the cross section area of the element is denoted A, Young's modulus of the material - E. About parameters we assume that they are fuzzy values (physical uncertainties). Similarly we can assume that boundary conditions as well as the length of the element (model uncertainty) are fuzzy values. They will be denoted in agreement with the notion introduced for fuzzy values, cf. Fig.1.

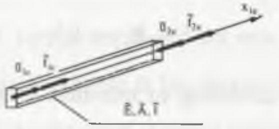


Fig.1. Fuzzy bar element

A field of element displacements, which is the fuzzy vector $\tilde{\mathbf{U}}_e(x)$ will be approximated by a linear combination of the fuzzy node displacements $\tilde{\mathbf{U}}_e$, multiplied by independent linear fuzzy interpolation functions, denoted as $\tilde{\mathbf{N}}(x)$, which are simply fuzzy polynomials.

Following, we have

$$\tilde{\mathbf{U}}_e(x) = \tilde{\mathbf{N}}(x)\tilde{\mathbf{U}}_e \quad (4.1)$$

$$\tilde{\mathbf{U}}_e^{\,T} = [\tilde{u}_1, \tilde{u}_2], \quad (4.2)$$

where

$$\tilde{\mathbf{N}}(x) = \left[1 - \frac{x}{\tilde{l}}, \frac{x}{\tilde{l}}\right]. \quad (4.3)$$

In an analogous way, as in the classical finite element method, cf. Ref. 8, we can obtain values of the local stiffness matrix, which are now fuzzy numbers

$$\tilde{K}_e = \frac{\tilde{A}\tilde{E}}{\tilde{l}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \quad (4.4)$$

Similarly we determine the vector of element forces.

More complicated elements can be considered similarly, i.e. details of analysis for beam elements can be found in Ref. [27].

5. Methodology of Fuzzy Arithmetical Computations

If we now assume that the position of i-th point can vary from 1 to N one obtains a system of N fuzzy algebraic equations. This set of fuzzy equations can be expressed in matrix form as

$$\mathbf{H}_\lambda \tilde{\mathbf{U}}_\lambda = \mathbf{G}_\lambda \tilde{\mathbf{Q}}_\lambda + \tilde{\mathbf{V}}_\lambda, \tag{5.1}$$

where $\mathbf{H}_\lambda$ and $\mathbf{G}_\lambda$ are two NxN non-fuzzy matrices and $\tilde{\mathbf{U}}_\lambda, \tilde{\mathbf{Q}}_\lambda, \tilde{\mathbf{V}}_\lambda$ are fuzzy vectors of length N, $\forall\lambda\in]0,1[$. Notice that N_1 fuzzy values of $\tilde{u}_\lambda$ and N_2 fuzzy values of $\tilde{q}_\lambda$ are known on Γ_1 and Γ_2 respectively, hence there are only N fuzzy unknowns in the system of equations (3.7). One has to rearrange the system to obtain a standard system of fuzzy algebraic equations

$$\tilde{\mathbf{A}}_\lambda \tilde{\mathbf{X}}_\lambda = \tilde{\mathbf{F}}_\lambda, \qquad \forall\, 0 \le \lambda \le 1, \tag{5.2}$$

where $\tilde{\mathbf{X}}_\lambda$ is a fuzzy (interval) vector of unknown λ-cuts $\tilde{u}_\lambda$'s and $\tilde{q}_\lambda$'s fuzzy boundary values. Eq. (5.2) can now be solved and all the boundary values are then known. Let

$$X_\lambda := \left\{ \mathbf{X}: \mathbf{A}_\lambda \mathbf{X} = \mathbf{F}_\lambda,\ \mathbf{A}_\lambda = \left[a_{\lambda ij}\right],\ \mathbf{F}_\lambda = \left[f_{\lambda i}\right],\ a_{\lambda ij} \in \tilde{a}_{\lambda ij},\ f_{\lambda i} \in \tilde{f}_{\lambda i},\ i,j = 1,2,...,N \right\} \qquad \forall\, 0 \le \lambda \le 1. \tag{5.3}$$

Define $\tilde{\mathbf{X}}_1(\Gamma), \Gamma \in \mathcal{M}$, a fuzzy subset of R^N, by its membership function

$$\mu\left(\mathbf{x}; \tilde{\mathbf{X}}_1(\Gamma)\right) := \sup\left\{\lambda : \mathbf{x} \in X_\lambda\right\}, \qquad \mathbf{x} \in R^N, \Gamma \in \mathcal{M}. \tag{5.4}$$

We call $\tilde{\mathbf{X}}_1(\Gamma)$ an exact fuzzy conditional solution of FBEM for arbitrary $\Gamma \in \mathcal{M}$. Further we omit the parameter Γ, since it is fixed for further considerations.

Assume from now that no $\mathbf{A} \in \tilde{\mathbf{A}}_\lambda$ is singular $\forall\, 0 \le \lambda \le 1$. We wish to know the set of solutions $\tilde{\mathbf{X}}_1$ and its relation to the eq. (5.2), where the interval multiplication and addition are used to evaluate its left-hand side. We now try to solve eq. (5.2) for the $x^-_{i\lambda}$ and $x^+_{i\lambda}$, i=1,2,...,N, $0 \le \lambda \le 1$, and hope they are the λ-cuts of fuzzy numbers $\tilde{x}_i$, i=1,2,...,N. In any case, assume that this method does produce fuzzy numbers $\tilde{x}_i$, i=1,2,...,N.

Define $\tilde{\mathbf{X}}_2(\Gamma), \Gamma \in \mathcal{M}$, a fuzzy subset of R^N, by its membership function

$$\mu\left(\mathbf{x}; \tilde{\mathbf{X}}_2(\Gamma)\right) := \min_{1 \le i \le N}\left\{\mu\left(x_i; \tilde{x}_i\right)\right\}, \quad \mathbf{x} = \left[x_i\right] \in R^N, \Gamma \in \mathcal{M}. \tag{5.5}$$

We can prove that $\tilde{\mathbf{X}}_{1\lambda} \subseteq \left[\mathbf{X}^-_{2\lambda}, \mathbf{X}^+_{2\lambda}\right],\ \forall\, 0 \le \lambda \le 1, \Gamma \in \mathcal{M}$.

Many authors, Moore [16], Neumaier [18], Skrzypczyk & Pownuk [26] have discussed methods for computing an interval vector $\tilde{\mathbf{X}}_{2\lambda}$ containing $\tilde{\mathbf{X}}_{1\lambda}$. The exact calculation of $\tilde{\mathbf{X}}_{1\lambda}$ is for multidimensional problems very difficult. An interval vector $\tilde{\mathbf{X}}_{1\lambda}=\left[\mathbf{X}_{1\lambda}^{-},\mathbf{X}_{1\lambda}^{+}\right], \forall\ 0\leq\lambda\leq 1$ defines a region in an N-dimensional space bounded by the planes $x_i=x_{i\lambda}^{-}$ and $x_i=x_{i\lambda}^{+}$, i=1,2,...,N. Since $\tilde{\mathbf{X}}_{1\lambda}$ will usually not be a rectangle in R^N, we would expect $\tilde{\mathbf{X}}_{1\lambda}$ to be a proper subset of $\tilde{\mathbf{X}}_{2\lambda}$. Naturally, the smallest $\tilde{\mathbf{X}}_{2\lambda}$ is of interest. We shall use $\tilde{\mathbf{X}}_{2}$ as the approximate fuzzy solution of eq. (5.2).

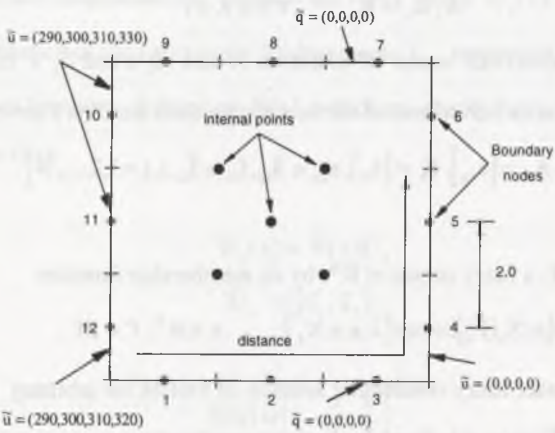


Fig.2. Fuzzy potential problem with fuzzy trapezoidal boundary conditions

6. Numerical Results

6.1. Trapezoidal boundary conditions

Consider now the simple potential problem as in [3] but with fuzzy boundary conditions of the trapezoidal membership functions. Such membership functions can be characterised as the ordered quadruple equivalent to the points of the trapezoid. Numerical values of the considered boundary values are given in details at fig. 2.

Since only boundary conditions are of the fuzzy character, the exact fuzzy solution has membership functions also of the trapezoidal shape. Results are presented at fig. 3. Values of the membership functions of the solution in the internal points have trapezoidal character too. The detailed values are omitted for simplicity (cf. Skrzypczyk & Burczyński [22-23]).

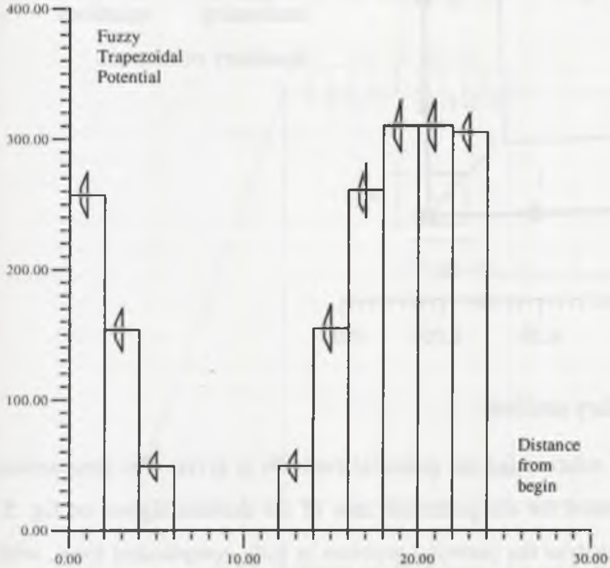


Fig.3. Trapezoidal membership functions of boundary solutions

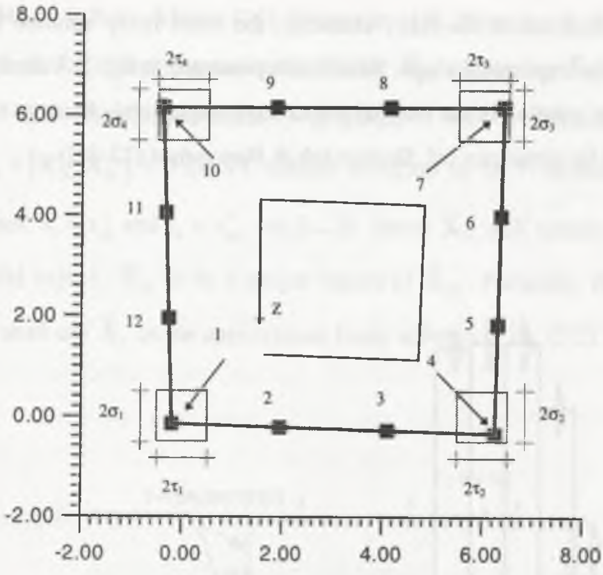
6.2. Potential Problem in the Fuzzy Domain

Now consider the same potential problem. Assume, that boundary functions are of interval character. This assumption is made for simplicity of calculations only. The methodology of calculations allow to consider any shapes of membership functions. Boundary conditions u_0 i q_0 are independent with respect to boundary fluctuations and $\tilde{\xi}(x) \equiv 0, \forall x \in \tilde{\Omega}$.

Additionally assume, that the considered domain is fuzzy - its boundary is a tetragon with apexes which can take uncertain positions. Let coordinates $(x_i, y_i, i = 1,2,3,4)$ be known with accuracy $(\pm\tau_i, \pm\sigma_i, i = 1,2,3,4)$ respectively see fig.4. In such a way, the considered domain is the fuzzy function of 8 fuzzy parameters.

At first we analyse the appropriate conditional solutions with respect to fuzzy boundary. We know, that each conditional solution is the interval function. Denote this solution

$$\tilde{u}(x|\Gamma) = \left[u^-(x), u^+(x)\right], \quad x \in \Gamma \in \mathcal{M}.$$



If all conditional solutions are known, we use max-min formula to obtain the global membership functions for interesting solutions - boundary or internal.

Fig. 4. Fuzzy domain of boundary problem

At fig. 5 the global boundary solution for the potential function is given. For comparison one conditional solution is presented for the particular case of the domain signed on fig. 5. There is no special problems to analyse the potential problem in fully complicated form, with all fuzzy elements - internal sources, boundary conditions and the shape of boundary. It only the problem of computational complexity.

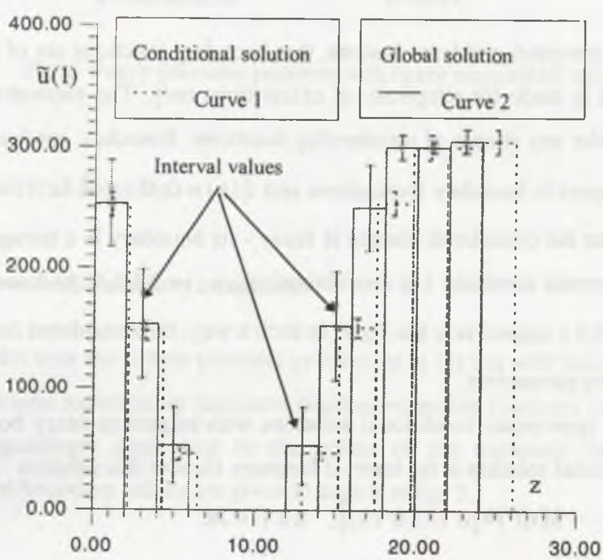


Fig. 5. Global fuzzy solution and the conditional one

6.3. Frame with Uncertain Loads

Consider two-dimensional frame structure with fuzzy node forces and fuzzy continuous load as presented in Fig. 6. Parameters of the frame are the following: E= 210 [GPa], A=2.0e-2 [m^2], I=2.0e-4 [m^4] .

Results for interval bending moments and shear forces are presented in Fig. 7.

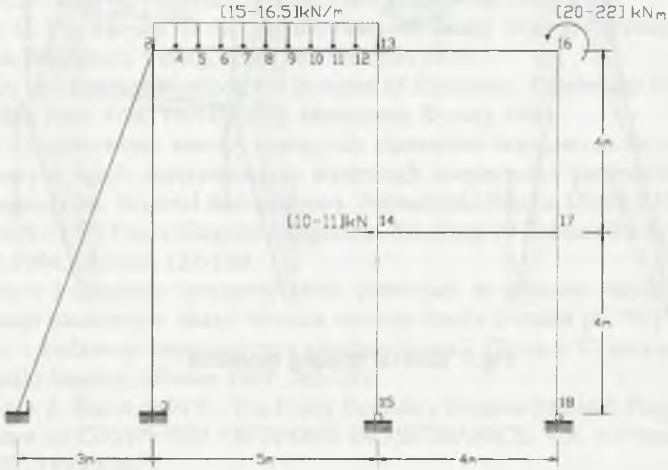


Fig.6. Scheme of the frame with uncertain parameters

7. Conclusions

This paper is a continuation of earlier works and summarise our knowledge about analysis with use of bou`ndary element method in fuzzy formulation. It was concerned with the new theoretical and computational methodology of the fuzzy analysis to boundary element method in potential theory and to fuzzy finite element analysis of uncertain structures. Applications were presented to potential problems with boundary conditions which are not sharply given and are characterised by fuzzy functions of interval type and trapezoidal-type membership functions.

A major conclusion is that fuzzy sets can be effectively used to estimate system uncertainty in boundary problems and random uncertainty can be calculated with a new techniques called FBEM and FFEM.

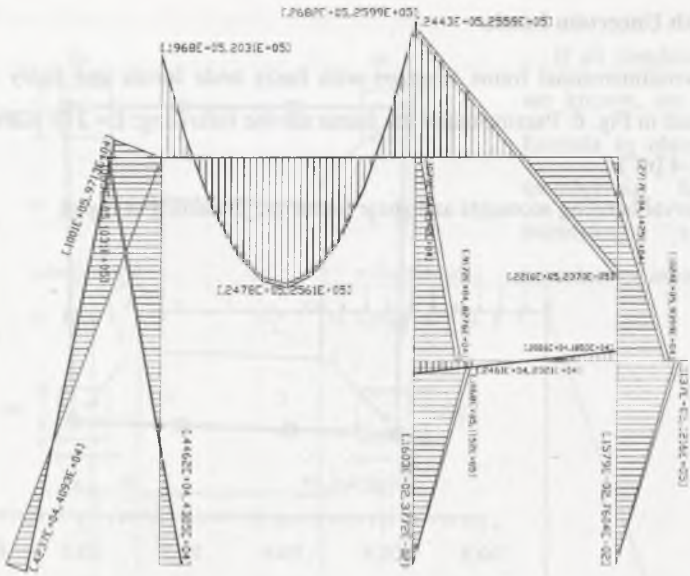


Fig.7. Interval bending moments

REFERENCES

1. Alefeld G., Herzberger J.: Introduction to Interval Computations, Academic Press, New York 1983.
2. Bauch H., Jahn K.,U., Oelschlagel, D., Susse, H., Wiebigke V.: Interval-mathematik, BSG B.G. Teubner Verlagsgeselschaft, Berlin 1987.
3. Brebbia C.,A., Dominguez J.: Boundary Elements - An Introductory Course, Comp. Mechanics Publ., Southampton, Boston 1989.
4. Brebbia C., Telles J.,C.,F., Wrobel L.,C.: Boundary Element Techniques - Theory and Applications in Engineering, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo 1984.
5. Burczyński T.: Metoda elementów brzegowych, WN-T, Warszawa 1995.
6. Burczyński T., Skrzypczyk J.: The Fuzzy Boundary Element Method: A New Solution Concept. Proc. of XII Polish Conference on COMPUTER METHODS IN MECHANICS, Warsaw - Zegrze, Poland, 9-13 May 1995, 65-66.
7. Burczyński T., Skrzypczyk J.: Stochastic And Fuzzy Aspects Of The Boundary Element Method, International Conference on Uncertain Structures: Analytical, Numerical and Experimental Methods, Cruise Ship in the Western Caribbean, March 3-10 1996.
8. Burczyński T., Skrzypczyk J.: The Fuzzy Boundary Element Method: A New Methodology. Sci. Fasc. of Silesian Tech. Univ., ser. Civil Eng. 83, Gliwice 1996, 25-42.
9. Burczyński T., Skrzypczyk J.: Fuzzy Aspects of The Boundary Element Method. Engineering Analysis with Boundary Elements, Special Issue: Stochastic Boundary Element Methods, 19, 1997, 209-216.
10. Burczyński T., Skrzypczyk J.: The Boundary Element Method For Fuzzy Systems. Proc. of the IASTED International Conference on Modelling, Simulation and Optimization, Singapore, August 11-14 1997, 24-27.

11. Dubois D., Prade H.: Fuzzy Real Algebra: Some Results. Fuzzy Sets and Systems, 2, 1979, 327-348.
12. Dubois D., Prade H.: An Approach to Computational Processing of Uncertainty, Plenum Press, New York, London 1988.
13. Kacprzyk J.: Zbiory rozmyte w analizie systemowej, PWN, Warszawa 1986.
14. Kaleva O.: Fuzzy Differential Equations, Fuzzy Sets and Systems, 24, 1987, 301-317.
15. Kaleva O.: The Cauchy Problem For Fuzzy Differential Equations, Fuzzy Sets and Systems, 35, 1990, 389-396.
16. Moore R.,E.: Interval Analysis, Prentice Hall, Englewood Cliffs 1966.
17. Negoita C.,V., Ralescu D.,A.: Applications of Fuzzy Sets to System Analysis, Ed. Technica, Birkhauser Verlag, Basel und Stuttgart 1975.
18. Neumaier A.: Interval Methods for Systems of Equations, Cambridge University Press, Cambridge, New York, Port Chester, Melbourne, Sydney 1990.
19. Pilch B.: Zastosowanie metody rozmytych elementów brzegowych do zagadnień dwuwymiarowych teorii sprężystości w warunkach niepewności parametrów konstrukcji, Praca magisterska, Wydział Budownictwa, Politechnika Śląska, Gliwice 1996 (niepubl.).
20. Skrzypczyk J.: On Fuzzy Singular Integration. Sci. Fasc. of Silesian Tech. Univ. ser. Civil Eng. 83, 1996, Gliwice, 121-130.
21. Skrzypczyk J.: Zadanie brzegowe teorii potencjału w obszarze rozmytym. Materiały Konferencji naukowej z okazji 70-lecia urodzin Józefa Głomba pt. "Wybrane problemy naukowo - badawcze mostownictwa i budownictwa", Gliwice 17 czerwca 1997, Wyd. Politechniki Śląskiej, Gliwice 1997, 243-251.
22. Skrzypczyk J., Burczyński T.: The Fuzzy Boundary Element Method. Proc. of XIII Polish Conference on COMPUTER METHODS IN MECHANICS, Vol. 4, Poznań, Poland, 5-8 May 1997, 1195-1202.
23. Skrzypczyk J., Burczyński T.: Fuzzy Aspects of The Boundary Element Method in Elastostatics. Proc. of XXXVI Sympozjon "Modelowanie w mechanice", Wisła 16-20 luty 1997, Zeszyty Naukowe Katedry Mechaniki Stosowanej 3, 1997, 25-30.
24. Skrzypczyk J., Burczyński T.: Metoda elementów brzegowych dla obszarów o kształcie rozmytym. Materiały XXVII Sympozjonu "Modelowanie w mechanice", Wisła 9-13 lutego 1998, Zeszyty Naukowe Katedry Mechaniki Stosowanej 6, 1998, 317-322.
25. Skrzypczyk J., Burczyński T.: Boundary Element Methods For Fuzzy Domain, IABEM International Symposium on Boundary Element Methods, Ecole Polytechnique, Palaiseau (Paris), France, May 26-29 1998.
26. Skrzypczyk J., Pownuk A.: Metody rozwiązywania układów interwałowych równań liniowych mechaniki rozmytej. Sympozjon "Modelowanie w mechanice", Wisła 16-20 luty 1997, Zeszyty Naukowe Katedry Mechaniki Stosowanej 4, 1997, 323-328.
27. Skrzypczyk J.: Metoda rozmytych elementów skończonych - Nowa metodologia, Proc. of XXXVI Sympozjon "Modelowanie w mechanice", Wisła 16-20 luty 1997, Zeszyty Naukowe Katedry Mechaniki Stosowanej 4, 1997, 317-322.

Streszczenie

Przy opisie problemu fizycznego jako deterministycznego problemu brzegowego, na ogół nie mamy pewności, że nasze modelowanie jest dokładne. Natura tej niedokładności może mieć różnorodny charakter: niepewności wynikające z niedoskonałości postępowania człowieka, niepewności systemu oraz niepewności losowe. Przewidywanie skutków tych niepewności jest trudne i obecne metody, które stanowią treść szeroko rozumianej teorii niezawodności, koncentrują się na niepewnościach przypadkowych.

Problem brzegowy może nie być znany dokładnie i niektóre funkcje, takie jak np.: kształt struktury, własności materiału, warunki brzegowe, zewnętrzne i wewnętrzne wymuszenia, rozwiązania itp. mogą zawierać parametry niepewne. Szczególnie jeśli są one znane poprzez pewne pomiary, są zawsze obarczone błędami. Analiza wpływu tych błędów prowadzi do badania jakościowego i ilościowego niepewności rozwiązań. Istnieje jednak fundamentalna różnica w naturze niepewności typu przypadkowego i tych niepewności, które mają źródła w działalności człowieka bądź działaniu systemu. Aby analizować niepewności innego pochodzenia niż przypadkowość, potrzebna jest „inna matematyka" od dotychczas stosowanej. I tutaj widać potencjalne możliwości wykorzystania zbiorów rozmytych.

Możliwe są różne interpretacje dla terminologii niepewnych aspektów metody elementów brzegowych (MEB). W pracy uwaga jest skoncentrowana na opisie wykorzystującym zbiory rozmyte i podejście to będzie dalej referowane jako *rozmyta metoda elementów brzegowych* (RMEB). Wprowadzone pojęcia są wykorzystane do opisu niepewności warunków brzegowych i własności materiałowych, jak również niepewności dotyczących brzegu obszaru. Takie niepewności są na ogół opisane na brzegu obszaru lub w jego wnętrzu i modelowane jako przestrzenne pola rozmyte.

Pierwsze zastosowania RMEB pojawiły się w 1995. Zastosowano wówczas rozmyte całkowe równania brzegowe do rozwiązania rozmytego zagadnienia potencjału z niepewnymi warunkami brzegowymi i niepewnymi źródłami wewnętrznymi (por. Burczyński & Skrzypczyk [6-8]). Później RMEB została zastosowana do zagadnień teorii sprężystości (por. Pilch [19], Skrzypczyk & Burczyński [9-10]). Stosując MEB do rozwiązania problemu brzegowego ze znanymi warunkami brzegowymi otrzymujemy całkowe równanie brzegowe. Jeżeli od tego momentu założymy, że wartości warunków brzegowych, własności materiału, wartości pól wewnętrznych oraz sam kształt brzegu mogą być niepewne, będą modelowane z wykorzystaniem teorii zbiorów rozmytych. Otrzymamy wówczas *rozmyte brzegowe równanie całkowe*, w którym wszystkie operacje maja charakter rozmyty. Całki osobliwe są rozumiane w sensie rozmytej wartości głównej (por. Skrzypczyk [20-21], Skrzypczyk & Burczyński [24-25]).

Przykładowe obliczenia z teorii potencjału ilustrują różnorodne aspekty przedstawionej teorii. W celu pełnego zilustrowania przedstawiono wyniki problemu potencjału w obszarze rozmytym.

Różne interpretacje są również możliwe dla terminologii niepewnych aspektów metody elementów skończonych. Skoncentrowano się, podobnie jak w przypadku MEB, na interpretacji rozmytej, różniącej się od podejścia probabilistycznego. Metoda jest dalej opisywana jako *metoda rozmytych elementów skończonych (MRES)*. Wszystkie niepewności strukturalne i dotyczące obciążeń konstrukcji są modelowane jako wielkości rozmyte.

Zastosowania specyficznej odmiany MES, zwanej *przedziałową metodą elementów skończonych*, pojawiły się po raz pierwszy w 1994. W metodzie tej zakładano, że niepewności są interwałami. Pierwsze sformułowanie *metody rozmytych elementów skończonych* pojawiły się w 1996 (por. Skrzypczyk [21]). Modelowanie wielkości jako zmiennych rozmytych wymaga stosowania specjalnych procedur postępowania.

Przykłady z teorii ram i kratownic w ujęciu rozmytym pozwalają na lepsze zilustrowanie tych stosunkowo nowych metod.