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SENSITIVITY OF SCHWEDLER SHELLS ON STATIC SINGULAR LOAD

Summary. Derivation of basic equations of the lattice shells by matrix metod. Static analysis of spherical Schwedler shells with the top ring loaded by vertical systems of nodal singular forces. Influence of boundary conditions on course of internal forces and displacements. It is assumed that nodes of analysed shells are hinged and rigid. On the top ring two limit boundary conditions are prescribed. Horizontal rings are divided stepwise on 6, 12, 18 and 24 regular angles. Obtained results and from those followed conclusions for practical structural design of Schwedler shells are postulated.

1.Introduction

The purpose of this paper is investigation of the one type of one-layered lattice spherical shells having a hole on their top. Analysed shells are loaded by vertical singular loads acting at each of the nodes of the top ring. Three systems of loads are studied (Fig. 1.1). Results of these types of loads represent the same value.

Two boundary conditions on the top ring of the shells are considered:

- free edge and
- perfectly rigid top ring in horizontal plane.

Individual shells are are solved as a *TRUSSS* and *FRAMED* system (*TRUSS* and *BEAM* elements are used).

Three types of rise - span ratious (f/d) are taken into account. Analysed shells consist 6, 12, 18 and 24 regular angle assembly of ring - members.

Obtained results are graphically presented and in the end of paper summarized into conclusions and recommendations for engineering practice.

Since methods of solution of the problem are well - known, we shall presente only for informations of the reader, the basic system of equations which are modified form of equations [1].

We assume that shells:

- consist nodes and prismatic members which are mutually connected,
- are loaded by the vertical nodal forces,
- arise connected space convex polyhedrons,
- are pin - jointed immovable supported on the bottom edge,
- are made from homogeneous, isotropic and elastic materials whose mechanical properties are time - independent,

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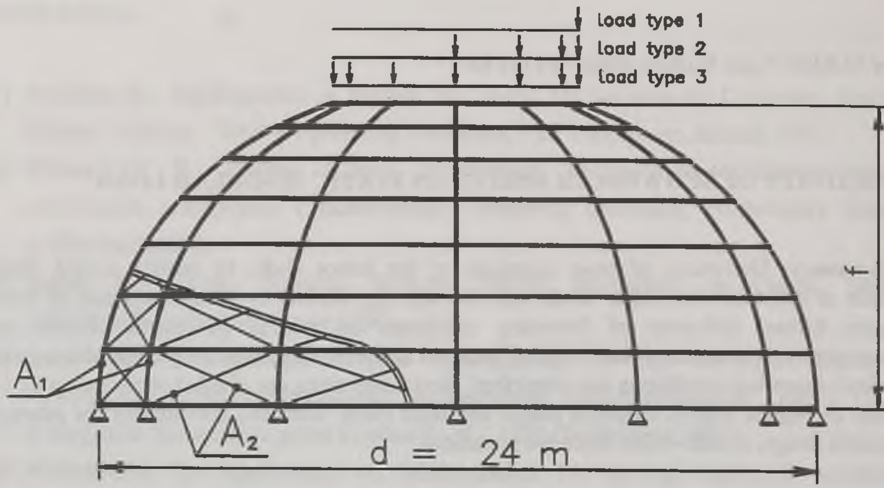

Fig.1.1 Geometrical scheme of the shells analysed. Cross-section area of the individual members, $A_1 = 10.1\text{cm}^2$, $A_2 = 2.5\ \text{cm}^2$. Young's modulus of elasticity of the material E = 210 GPa

- as a system behave according the linear theory of elasticity.

It is obviously that full analysis of the task have to contain the control of stability criteria of lattice shells [2]. This task in this paper is omitted.

Thoughout the paper, we shall use the right-handed local coordinate system (x,y,z) so called member coordinate system, where x - axis identical with centroid line of member an y and z axes coincide with the principle axes of the cross - section.

A set of coordinates used for defining the joint generalized displacement of a structure or the loading (in general sense) applied to will be called a set of global coordinates X, Y, Z.

Let P, p(D,d)) are loads (displacements) vectors. Then following relations are valid

$$P = \mathbf{T}p \qquad D = \mathbf{T}d \qquad (x,y,z) \rightarrow (X,Y,Z) \tag{1}$$

$$p = \mathbf{T}^t P d = \mathbf{T}^t D \qquad (X,Y,Z) \rightarrow (x,y,z) \tag{2}$$

where

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_0 & \mathbf{O} \\ \mathbf{O} & \mathbf{T}_0 \end{bmatrix}, \qquad \mathbf{T}_0 = \begin{bmatrix} \cos(X,x) & \cos(X,y) & \cos(X,z) \\ \cos(Y,x) & \cos(Y,y) & \cos(Y,z) \\ \cos(Z,x) & \cos(Z,y) & \cos(Z,z) \end{bmatrix}$$

is transformation matrix.

To define the orientation of members in space it is necessary to specify the angles which it makes, in an unambiguous manner, with the coordinate axes. For this purpose the idea of the "direction" of a member will introduced. Directions may be indicated on line diagrams by

arrows placed on the members. For example symbol s $\prec$ j means orientation "from *s* to *j*" or (*s, j*) is a member *s j*. The graph theory application of the problem is discussed e.g. [3].

2. Basic equations

Equilibrium equation

$$P^{(j)} - \sum_{s=1}^{m(j)} P_{sj}^{j} - \sum_{s=1}^{n(j)} P_{js}^{j} = 0 \qquad (X,Y,Z),\ (j) \in \Omega^{\Delta} \quad j = 1,2 \ldots N \tag{3}$$

where $(j) \in \Omega^{\Delta}$ is an arbitary joint of the given structure separated from the system Ω^{Δ} .

Compatibility equations

$$D^{(j)} - C_s^t D_m = \sum_{s=1}^{m(j)} \Phi_s^j P_{js}^j \qquad (X,Y,Z),\ (j) \in \Omega^{\Delta} \tag{4}$$

where Φ_s^j is partial fixity matrix for $\forall s \to (j) \in \Omega^{\Delta}$, $C_s = [I, I_1, \ldots I_s]$, $s = m(j) + n(j)$ is a connection matrix, suffix s is used to indicate members have a common joint (j).

In our case we considered

$$D^{(J)} = D_m \qquad (X,Y,Z),\ (j) \in \Omega^{\Delta} \tag{5}$$

Force - displacement equation of member

$$\left[p^s\, p^j\right]_{sj}^T = \begin{bmatrix} \mathbf{k}^{11} & \mathbf{k}^{12} \\ \mathbf{k}^{21} & \mathbf{k}^{22} \end{bmatrix}_{sj} \left[d^s d^j\right]_{sj}^T \qquad (x,y,z),\ s,j \in \Omega^{\Delta},\ s \prec j \tag{6}$$

where the size of the matrices $\mathbf{k}^{11}$, etc. depends on the number components in loads and displacement vector.

Boundary conditions

In the general case the boundary conditions for the analysed system Ω^{Δ} can be expressed as follows:

$$\mathbf{Q}^{(j)} D^{(j)} + \mathbf{S} R^{(j)} = B^{(j)} \quad (X,Y,Z),\ (j) \in \Omega^{\Delta} \qquad (j) = \langle O_1, O_2, \ldots O_s \rangle \tag{7}$$

a/ If the structure supporting nodes are perfectly rigid then holds
$D^{(j)} = O$, $Q^{(j)} = [Q_1, Q_2, \ldots Q_6]^{(j)}$ are "large numbers", $S = [O]$, $B^{(j)} = O$. In this case reactions $R^{(j)}$ must be finded from equilibrium equations (8).

b/ If the structure supporting nodes are elastically yielding then holds
$\mathbf{Q}^{(j)} = [\text{constants of subsoil}]^{(j)}$, $\mathbf{S} = [\mathbf{I}]$, $\mathbf{B}^{(j)} = \mathbf{O}$.

Governing equation of the system Ω^{Δ}

By substituting compatibility equation (4) and force - displacement equation (6) into equation (3) and after rearrangement the governing equation (equation of equilibrium expressed in the generalized displacements of nodes) has a form

$$\mathbf{K}D = P \qquad (X,Y,Z) \tag{8}$$

+ boundary conditions
where

$$D = \left[D^{(1)} D^{(2)} \ldots\ldots D^{(N)}\right]^{T} \qquad P = \left[P^{(1)} P^{(2)} \ldots\ldots P^{(N)}\right]^{T}$$

$$\mathbf{K} = \left[\mathbf{K}_1 \mathbf{K}_2 \ldots\ldots \mathbf{K}_N\right]^{T} \qquad \mathbf{K}_I = \left[\mathbf{K}_{I1} \mathbf{K}_{I2} \ldots\ldots \mathbf{K}_{IN}\right]^{T} \qquad (I = 1,2,\ldots N)$$

$$\mathbf{K}_{JJ} = \sum_{s=1}^{m(J)} \left(\mathbf{T}\mathbf{k}^{22}\mathbf{T}^{T}\right)_{sj} + \sum_{s=1}^{n(J)} \left(\mathbf{T}\mathbf{k}^{11}\mathbf{T}^{T}\right)_{js}$$

$$K_{ij} = \left(Tk^{22}T^{T}\right)_{ij} \qquad \text{if} \quad (i) \prec (j)$$

$$\mathbf{K}_{ij} = \left(\mathbf{T}\mathbf{k}^{11}\mathbf{T}^{T}\right)_{ji} \qquad \text{if} \quad (j) \prec (i)$$

$$\mathbf{K}_{ij} = [\mathbf{O}] \qquad \text{if} \quad (i) = (j)$$

It should now be clear that the assembly of the complete stiffness matrix of a structure from the individual member stiffness matrices depends only on the way the members which are conected together, and not on the geometry of the system. The same system of govering equations from the energy approach can be derived, e.g. [3].

3. Numerical analysis

In this contributions the spherical shells with radius of curvature r, diameter d = 24 m with three different rises f are solved.

Notations of shell

H - for the rise f = 9,6 m, or rise - span rations (f/d) = 0,4 (in Figures marked □)
M - for the rise f = 7,2 m, or rise - span rations (f/d) = 0,3 (in Figures marked O)
L - for the rise f = 4,8 m, or rise - span rations (f/d) = 0,2 (in Figures marked Δ)

Each structure over the circular plane as a regular 6, 12, 18 and 24 angle was modelled. The elements of the structures were assumed as a TRUSS (3 DOF) and BEAM (6 DOF) systems with the following combinations:

a) without top stiffener ring, b) with rigid top ring for TRUSS elements and c) ... and d) ... for BEAM elements.

Loadings were modelled by the singular vertical forces applied in the nodes of the top ring intensity 150 kN, where:

1) - is a symmetrically distributed load on lenght of top ring,
2) - is distributed on the half perimeter of the top ring,
3) - is concentrated in one node.

The grafical representations of the some results of solution along meridian bars configurations are given in Figs 3.1 -3.9. Dashed lines in figures represent the "boundary condition" b) or d).

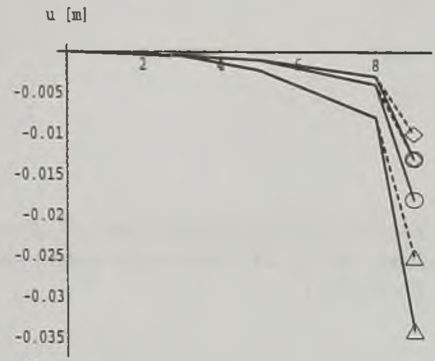

Fig.3.1. Vertical displacements of dome floors for the 6 regular angle system and load type 1)

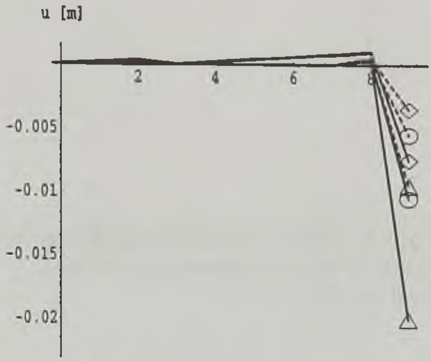

Fig.3.2. Vertical displacements of dome floors for the 24 regular angle system and load type 1)

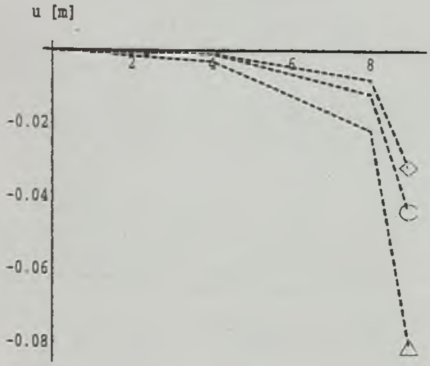

Fig.3.3. Vertical displacements of dome floors for the 6 regular angle system and load type 2)

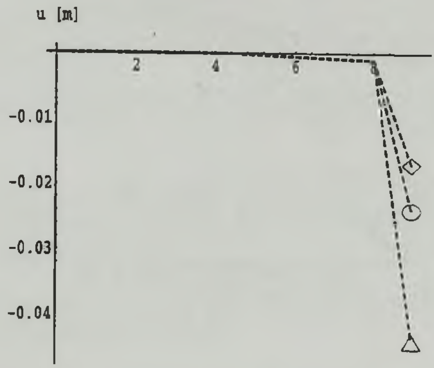

Fig.3.4. Vertical displacements of dome floors for the 24 regular angle system and load type 2)

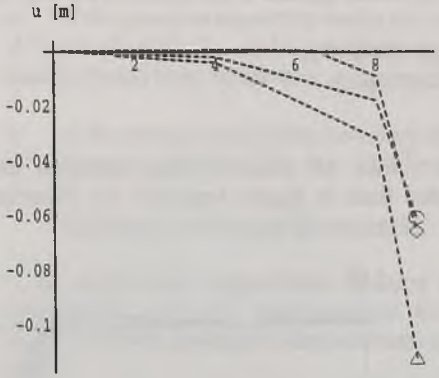

Fig.3.5. Vertical displacements of dome floors for the 6 regular angle system and load type 3)

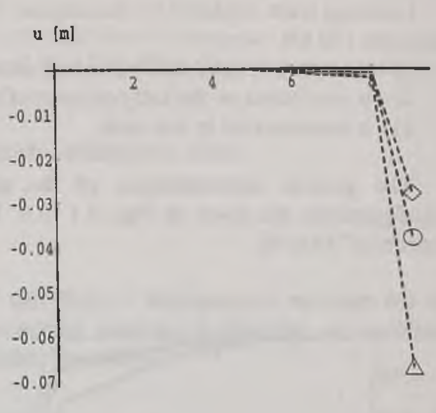

Fig.3.6. Vertical displacements of dome floors for the 24 regular angle system and load type 3)

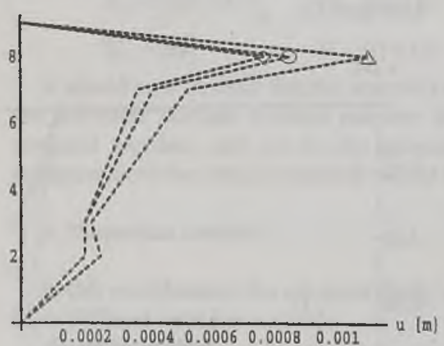

Fig.3.7. Horizontal displacements of dome floors for the 6 regular angle system and load type 1)

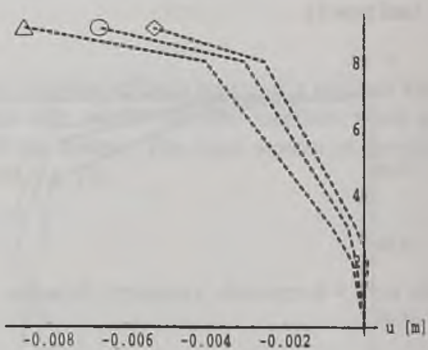

Fig.3.8. Horizontal displacements of dome floors for the 6 regular angle system and load type 2)

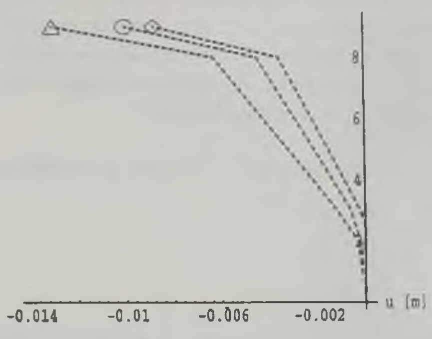

Fig.3.9. Horizontal displacements of dome floors for the 6 regular angle system and load type 3)

4. Conclusions

a) for symetrically loaded case (load system 1)) are not greater differences in the results (forces in the elements and displacements of nodes in the general sence) between variants a) - c) or b) - d) (TRUSS or BEAM system)

b) for non symetrical load it is valid for 6 and 12 regular angle system (in ground plane). For ring circle graduation on 18 or 24 parts the solution is non - stabil for TRUSS element system.

c) the highest (absolute) values of node displacements and internal forces in bars are focussed into the domain in which the load is acting.

d) depending on the parameters f, and the type and size of external load, there exists for a given type of dome, a constant number of meridian bar assembly such that an increase in their number does not affect the vertical and radial node displacements.

e) the effect of increasing the number of meridian bar assemblies (increasing the ring circle graduations) manifests itself markedly on the radial node dispalcements.

f) in the both cases (TRUSS and BEAM systems) for "boundary conditions" b) or d), the vertical displacements of nodes in the top ring domain diminish in compare with cases a) or c).

g) variation of the top ring stiffness in the radial direction locally affects the dome deformation.

h) for a given type of load, the stiffness of Schwedler domes is decisivety safequarded by the meridian and ring bar assemblies. The effect of diagonal bars manifests itself only where the load is acting.

i) in the case of cyclically symmetric domes that are symmetrically loaded, the variation of internal stresses and node displacements is aproximately the same, irrespective of whether the joints are taken as being perfectly rigid or as perfectly hinged. In given case it is thus enough to solve such domes for the "membrane state" of stress, and to complement the solution with "bending effect" for the boundary zones and the areas of singular load. This conclusion does not apply, however, in the case of an asymmetric load and

j) in the case of dome with top ring, this must be realised as a sufficiently stiffener structural element in the radial direction.

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Streszczenie

W pracy analizuje się wpływ pionowego obciążenia w postaci pojedynczych sił przyłożonych do górnego pierścienia kopuły Schwedlera. W rozwiązaniu wykorzystano metodę macierzową w ujęciu przemieszczeniowym. Przedstawiono równania równowagi, równania geometryczne, warunki zgodności i równania fizyczne, z których wyprowadzono równania konstytutywne problemu uzupełnione o warunki brzegowe. Materiał analizowanych kopuł jest homogeniczny, izotropowy i idealnie sprężysty opisany prawem Hooke'a. Rozpatruje się następujące typy warunków brzegowych: dolny brzeg nieprzesuwny oraz brzeg górny swobodnie podparty lub sztywno zamocowany w płaszczyźnie poziomej. Kopuły mają stałą liczbę pierścieni oraz zmienną liczbę południkowych żeber. Rozważane są trzy typy obciążeń: obciążenie górnego pierścienia na całym obwodzie, obciążenie działające tylko na połowę pierścienia oraz obciążenie pojedynczą siłą. Rozpatruje się dwa rodzaje więzów węzłowych: idealnie przegubowe połączenie prętów (3DOF) oraz węzły idealnie sprężyste (6DOF). Na wykresach przedstawiono wpływ obciążenia siłami oraz przyjętych więzów. W zakończeniu podano zalecenia odnośnie do zastosowań w praktyce oraz wykaz literatury, z której korzystano.