ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1986

Seria: GÓRNICTWO z. 143 Nr kol. 883

1st International Conference - Reliability and Durability
of Machines and Machinery Systems in Mining
1986 JUNE 16-18 SZCZYRK, POLAND

Walter BARTELMUS
Central Research and Design Institute of Opencast Mining
Wrocław, Poland

FUNDAMENTALS OF GEAR COMPUTATIONS FOR TOOTHED GEAR LIFE

Summary. The paper is an introduction to the problem of gear computations for the mesh life to be discussed. At present, it is accepted that, if mesh life should be in excess of a life corresponding to the limit number of cycles, a material is selected whose fatigue strength is equal to, or higher than, a load adequate to the unlimited life. The assumption of unlimited life reveals no reservation in such cases when a predominant external load is the steady load, thus producing a constant-amplitude load for the mesh. Materials subject to the random wide-band load do not show a feature of unlimited life. The paper presented assumptions for the toothed gear life computations by accepting a limited fatigue strength depending on a number of the cycles of changes in load. The above question is presented against a background of the existing procedure for unlimited life computations.

1. INTRODUCTION

Gears applied in opencast mining especially high-speed stages have considerably shorter life than complex ones. The reasons for such a state can be various, however, first of all they result from a way of operation but they may result from the selection of the constructional characteristics as well. The problem of selection of the constructional characteristisc with regard to the mesh life will be presented in this paper. The assumption on unlimited mesh life (when the number of cycles of changes in load exceeds the limit number of cycles) seems to be one of reasons for the shorter life.

In the carried out considerations on gear computations for toothed gear life the properties of material subjected to the random wide-band load show the limited strength independent of the number of cycles of changes in load. This paper and the work [1] are a attempt of representation of mutual relations between manner of selection of constructional mesh features, reliability and mesh life, diagnostic determination of the mesh state and forecasting of state.

2. STATE OF MESH LOAD

The starting point for consideration of the mesh life is the kind of internal or external load of the toothed gear.

The external load of gear can by classified as:

1 - constant (as predominat state of load)
2 - variable (random).

The load on teeth of gear is always variable even when constant external load occurs. The constant external load on cylindrical and bevel gear results in load with constant amplitude pulsing from zero for teeth. In the planetary gear the tooth of planet under constant external load are subjected to the alternating load. The mentioned above loads are narrow-band loads with constant amplitude. When gears operating the described loads impose one another. Considering internal load the influence of errors of mesh operation hausings, flexibility of the gear parts should be taken into account. The errors of mesh operation, flexibility of a mesh cause that load on tooth is of wide-band type see in |2| on function of stresses at the tooth base. To sum up the random load on mesh is assumed in the considerations.

The random character of load on mesh involves description of material properties, which are described by limited strength independent of the number of cycles of load on mesh.

A material does not show property of unlimited strength when is loaded relatively low.

3. NOTION OF MESH LIFF

The notion of mesh life is not fully defined. As the starting point for consideration of notion of mesh life it can be applied the notion of life and connected with it load capacity defined for rolling bearing.

Under individual life of bearing the number of load changes (number of rotations) or the number corresponding to the number of hours of effective operation, which bearing can be operated without first sign of the material fatigue on ball races or balls is meant.

The laboratory investigations and operational practice show the considerable scatter of bearing life under identical load conditions.

The appropriate definition of rolling bearing life has been found. It is defined as life which can be reached by 90% of rolling bearings belonging to the sufficiently large group of rolling bearings. Such a life is called a nominal life.

In order to increase reliability of some devices the appropriate corrections coefficients are introduced which enable to define increased reliability of 95% or 99%.

For rolling bearing the approximate rule of life is determined. It says that life 90% is five times less that the mean bearing life but the maximum life is four time greater than the mean life. It seems that mesh life should be defined similarly. In practice a designer has no data on material properties basing on them he can not define mesh life with determined reliability. The designer has most often the average value Z_0 or k_0. The only recommendations [5] and [6] present the other approach,which will be given in para.

4. DESCRIPTION OF MATERIAL PROPERTIES-MEAN LIFE

The mean life of material subjected to the variable load can be described by the equation

$$LS^m = \text{const} \tag{1}$$

L - number of cycles of changes in load (number of maximums),

S - level of stress N/mm^2.

If maximum values of stresses correspond to the different levels then their cumulative reaction is taken into account f.ex. Polmgren-Miner hypothesis, which can be presented in the simplest way:

$$\sum_{i=1}^{n} \frac{N_i}{L_i} = 1 \tag{2}$$

The given above relation can be formulated in the following words.

If a material sample has life expressed by a number of maximums L_1 at a given level S_1 and is subjected to N_1 changes in load then it will be utilized its ability to transfer loads in the degree expressed by the fraction N_1/L_1.

If the sample is subjected to a load of the level S_1, S_2, S_3 which can be composed in a certain sequence then the relation (2) is satisfied.

To ilustrate further utilization of the equations (1) and (2) it is assumed that a sample is subjected to load of narrow band noise having mean value equal to 0.

For this case the distribution of maximums can be described by Raylsigh distribution of probability density function p (S). The number of maximums in a interval of load variation dS amounts to:

$$p(S)\,ds = \frac{S}{\sigma^2}\exp - \frac{S^2}{2\sigma^2}\,ds \tag{3}$$

σ – standard deviation of stresses.

The total number of maximums for a sample having life T is equal (4) if frequency of changes in loads amounts to f_o – number of stress transitions trough 0.

$$N_i = f_o\,Tp(S)\,ds \tag{4}$$

The level of damage caused by stress at the level S_o having band width dS is equal to

$$f_o T\,\frac{P(S)\,ds}{L_i} \tag{5}$$

Applying equation (1)

$$L_i = \frac{const}{S^m} \tag{6}$$

Substituting in (2) expression (5) and (6) we obtained:

$$1 = \int_0^\infty f_o T\,\frac{p(S)dS}{const/S^m} = \frac{f_o\,T}{const}\int_0^\infty S^m p(S)dS \tag{7}$$

Thus, we can compute life of an element T

$$T = \frac{const}{f_o(\sqrt{2}\,\sigma)^m\,\Gamma\,(1 + \frac{m}{2})} \tag{8}$$

The obtained result corresponds to a mean life.

The above relation was verified in practice in [3] the obtained scatter of life is ten times great. Deriving equation (8) the random load and determined relation (1) describing the average material properties have been assumed.

In practice material properties are random. This problem is under consideration in [4]. The basic parameters describing material properties are: exponent m, limit number of cycle L_o and in case of toothed gear corresponding to it bending strength $Z_o\ \frac{N}{mm^2}$ and fatigue strength ot surface stresses $k_o\ \frac{N}{mm^2}$.

The following values of m and L_o for hardened material are assumed the most frequently:

for bending of teeth $m = 8$ $L_o = 5 \cdot 10^6$

for contact strenght $m = 6$ $L_o = 5 \cdot 10^7$

In case of quenched and tempered materials

for bending of teeth $m = 8$ $L_o = 5 \cdot 10^6$

for contact strenght $m = 3.5$ $L_o = 5 \cdot 10^7$

According to AGMA Standard [5]

for bending of tooth $m = 5.68$ $L_o = 6 \cdot 10^6$ – for bavel gear

according to recommendations [6]

for contact strength $m = 6$; the limit number of cycles-lacks.

Two cases of material behaviour can be determined in the description of material properties. In case of load on mesh with constant apmlitude predominantly it shall be assumed that if assumed life of gear is greater then limit number of cycles of changes in load the theoretical life at certain load is unlimited. In case of random wide-band load the material properties are described by the curve (1) within whole range of cycles of changes in load.

5. FUNDAMENTALS OF DETERMINATION OF EQUIVALENT LOAD AND EQUIVALENT NUMBER OF CYCLES FOR UNLIMITED LIFE

The methods applied nowadays, for gear life computations are based on notion of unlimited mesh life (when number of cycles of changes in load exceeds limit number of cycles).

Given variable load can be reduced to the form of load presented in Fig. 1 where load given in form of moment is grouped from the greatest to the smallest with determination of number of load cycles.

Making use of equation (1) and (2) the equivalent number of cycle can be found:

$$L_e = L_w \left[k_1 \left(\frac{M_1}{M_{max}}\right)^m + k_2 \left(\frac{M_2}{M_{max}}\right)^m + \dots k_n \left(\frac{M_n}{M_{max}}\right)^m \right] \tag{9}$$

where

$$k_n = \frac{N_n}{L_w} \qquad L_w = \sum_{i=1}^{m} N_n$$

L_e - equivalent number of cycles,

L_w - required number of cycles,

M_n - load [Nm] corresponding to number of cycles N_n

$M_{max} = M_1$ - maximum load [Nm]

m - exponent of expression (1)

If equivalent number of cycles $L_e > L_o$, where L_o - limit number of cycles then as computional load is assumed $M_1 = M_{max}$ where $L_e < L_o$ the computional load can be determined by means of the formula:

$$M_{comp} = M_{max} \sqrt[m]{\frac{L_e}{L_o}} \tag{10}$$

Because additionaly to the load characteristic Fig. 1 the notion of nominal load M_{nom} is used the following equation is applied:

$$M_{comp} = K_p \cdot M_{nom} \tag{11}$$

where

K_p - overload factor.

When there is no load characteristic to determine factors K_p the appropriate tables, f.ex. acc. to [7] table 24 page 136, are used. To determine K_p it is necessary to know kind of driven machine and kind of driving motor. The given number K_p can be utilized to the computation of pinion of the single stage. For the wheel gear the numer of cycles is less respoctively and it results from gear ratio, furthermore if number of changes in load is less than L_o then K_p for gear will be different.

When load characteristic is not known and tables are used, $L_e = L_o$ can be assumed and gear ratio of the stage is equal u then number of wheel cycles amounts to L_o/u. When substituting to the formula (10) and making use of (11) we obtain

$$M_{comp} = K_p \cdot \sqrt[m]{\frac{1}{u}}\, M_n = K_{p2} \quad M_n \tag{12}$$

$$Kp_2 = K_p \cdot \sqrt[m]{\frac{1}{u}}$$

If load characteristic is known then $L_e/_u$ should be computed when $L_e/_u > L_o$ then $K_p = K_{p2}$ and when it is less the following relation is satisfied

$$M_{comp} = K_p \sqrt[m]{\frac{L_e}{L_o}\ \frac{1}{u}}\ M_n \tag{13}$$

Thus the relation (9) and (13) respectively should be applied for the gear of higher stages.

6. DETERMINATION OF EQUIVALENT LOAD AT THE LIMITED GEAR LIFE

The equivalent load will be determined applying assumption of limited life within the whole range of mesh life. Thus it is assumed that material properties are decribed by the curve (1).

The equivalent load can be determined as a result of transformation of the formula (9) assuming that $L_e = L_w$ but $M_{max} = M_e$.

$$M_e = \sqrt[m]{\sum_{i=1}^{n} k_n M_n^m} = M_{comp} = K_p' M_{nom} \tag{14}$$

The factor K_p' is different from K_p in expression (11) because the method of its computing is different.

When applying the principle of limited strenght the bending and contact material strength can be computed for the defined number of changes in load taking into account that for particular wheel in gear it is different:

The required number of cycles for each wheel L_{w_i} is derived from the formula

$$L_{w_i} = T_w n_i \cdot 60 \text{ (cycles)} \tag{15}$$

where

T_w - assumed gear life in hours,

n_i - rotational speed of i - pinion or wheel.

If assuming that contact strength is equal to k_o for the limit number of cycles then contact strength L_{w_i} amounts to

$$k_i = k_o \sqrt[m]{\frac{L_o}{L_{w_i}}} \tag{16}$$

k_i - contact strength for given wheel or pinion corresponding to number of cycles L_{w_i}.

Analogically for bending strength it is obtained

$$Z_i = Z_o \sqrt[m]{\frac{L_o}{L_{w_i}}} \tag{17}$$

It should be emphasized that exponents in formulae (16) and (17) are different.

The sources concerning material properties give the mean values of properties Z_o and k_o. Thus the determined life will concern the mean life with 50% reliability. This relation is changed by safety factor δ. The recommendation [5, 6] are the exception to the presented material properties. The material properties corresponding to the 99% reliability are assumed here. It is given as well that at 33% reliability properties of material can be decreased to 0,8 but in order to obtain maximum safety such properties of material should be assumed which should be increased by 25% or greater in proportion to properties corresponding to 99% reliability.

The standard [5] recommends to apply safety factor $\delta = 2$ this means to assume material properties 100% greater than those resulting from 99% reliability. Correlations between those values are illustrated by Fig. 2. To draw Fig. 2 the exponent m = 5,68 and $L_o = 6 \cdot 10^6$ cycles were assumed. Further it was assumed that material subjected to the random load indicates limited strength that in double logarithm system log Z/Z_o and log L gives straight line for the curve (1).

The following interpretation is applied: the line 99% is a line for which gear in 99% obtains limit number of cycle $L_o = 6 \cdot 10^6$. The line 33% determines that mesh reaches in 33% the limit number of cycles. If $\delta = 2$ is assumed then the line determined in this manner reaches 99% life for $3 \cdot 10^8$ cycles where $\delta = 1 \cdot 25$, then gear life will amount to $2 \cdot 10^7$ cycles i.e 99% reliability. The interpretation of recommendations [5, 6] presented above was extended referring to limited life on break, the standards recommend unlimited life with regard to break of bevel gear. The standards [5, 6] respecting computing of contact strength recommend proceeding such as presented in this paper i.e limited life.

The problem given in this paper, however, concerns meshes generally but not only bevel gear.

7. SUMMARY

The presented rules on computation of mesh life can be treated as an introduction to the problem and discussion. In the new devoleped toothed gears for opencast mines the compumations on mesh life were assumed according to the given here principles as valid principles. The principles of computations of toothed gears life should be reflected in branch standards.

The recommendations contained in this paper can be applied provided that appropriate data on materials are available. They should be determined in material investigations of mesh, however, it requires to simulate random variable loads resulting in wide band load on a tooth.

REFERENCES

[1] Bartelmus W.: Reliability and Diagnostics of Toothed Gears for Belt Conveyor Drives. Materials for Conference I COREM 86.

[2] Thomson R.A., Weichbrodt B.: Gear Diagnostics and Wear Detection. ASME Publication 69-VIBR-10.

[3] Crandall S.H., Mark W.D.: Random Vibration in Mechanical Systems Academic Press, New York 1963.

[4] Wireching P.H., Haugen E.B.: A General Statistical Model for Engineering Materials and Technology January 1974, pp 34-40.

[5] AGMa Standard for Rattint the Strength of Spiral Bevel Gear Teeth 223.01 Jan. 1964.

[6] Surface Durability Pitting Formula for Bevel Gear Teeth Gleason Works Rochester, New York, USA 1966.

[7] Muller L.: Toothed Gears - Designing, WNT, Warsaw 1979.

Recenzent: Prof. dr inż. Antoni JAKUBOWICZ

Wpłynęło do redakcji: luty 1986 r.

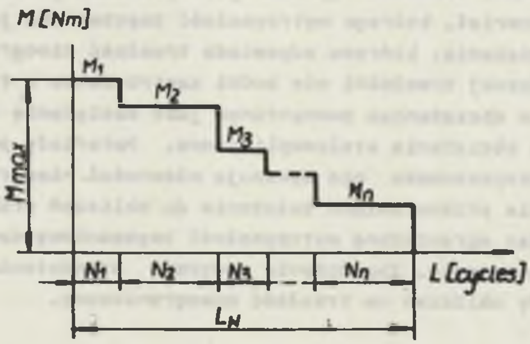

Fig. 1. Characteristic curve of mesh load cykl.-cycles
Rys. 1. Krzywa charakteryzująca obciążenie zazębienia

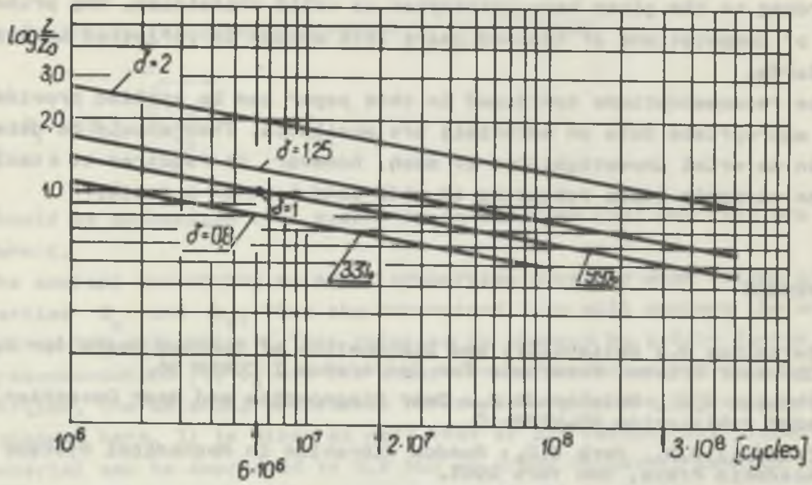

Fig. 2. Relation between realtive strength, reliability and life of the mesh

Rys. 2. Zależność między względną wytrzymałością, niezawodnością i trwałością zazębienia

PODSTAWY OBLICZEŃ PRZEKŁADNI ZĘBATYCH NA TRWAŁOŚĆ

S t r e s z c z e n i e

Referat jest wprowadzeniem do dyskusji nad zagadnieniem obliczeń zazębienia na trwałość. Obecnie przyjmuje się, że gdy trwałość zazębienia ma być większa niż trwałość odpowiadająca granicznej liczbie cykli, to dobiera się materiał, którego wytrzymałość zmęczeniowa jest równa lub większa od obciążenia, któremu odpowiada trwałość nieograniczona. Założenie nieograniczonej trwałości nie budzi zastrzeżenia w tych przypadkach, gdy dominującym obciążeniem zewnętrznym jest obciążenie stałe, co daje dla zazębienia obciążenie stałoamplitudowe. Materiały poddane obciążeniu losowemu szerokopasmowemu nie wykazują własności nieograniczonej trwałości. W referacie przedstawiono założenia do obliczeń przekładni na trwałość, przyjmując ograniczoną wytrzymałość zmęczeniową zależną od liczby cykli zmian obciążenia. Zagadnienie powyższe przedstawiono na tle istniejącej metody obliczeń na trwałość nieograniczoną.

ОСНОВЫ РАСЧЕТОВ ПРОЧНОСТИ ЗУБЧАТЫХ ПЕРЕДАЧ

Р е з ю м е

Доклад является приглашением к дискуссии над проблемой расчетов прочности зубчатых зацеплений. В настоящее время считается, что прочность зацепления зависит от усталостной вынооливости материала. Чтобы прочность зацепления была больше прочности, отвечающей предельному числу циклов, следует подобрать такой материал, усталостная выносливость которого равна или больше нагрузки, которой отвечает неограниченная прочность. Задание неограниченной прочности не вызывают возражений в тех случаях, когда доминирующая внешняя нагрузка является постоянной. Для зацепления такую нагрузку даёт постоянно-колебательная нагрузка. Материалы, подвергнутые случайной широкополосной нагрузке, не показывают овойств неограниченной прочности.

В докладе представлены данные для расчетов прочности передач, причем ограниченная усталостная выносливость зависит от числа циклов изменений нагрузки. Вышеперечисленные проблемы представлены на фоне существующего метода расчетов неограниченной прочности.