#### ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ

Seria: GÓRNICTWO z. 143

Nr kol. 883

1st International Conference - Reliability and Durability of Machines and Machinery Systems in Mining 1986 JUNE 16-18 SZCZYRK, POLAND

Balbir S. DHILLON Subramanyam N. RAYAPATI

University of Ottawa Ottawa, Canada

AVAILABILITY MODELING OF MAN-MACHINE SYSTEMS WITH CRITICAL HUMAN ERROR

> <u>Summary</u>: This paper presents two newly developed mathematical models representing maintainable parallel and standby redundant systems with critical human error. For both the models, Laplace transforms of state probability equations and steady state availability expressions are developed for cases when system repair rates are non-constant. Specific plots are shown for each model to demonstrate the impact of critical and non-critical human errors on system steady state availability.

## 1. INTRODUCTION

Humans interact with engineering systems in many ways. Examples of manmachine interactions may be seen in numerous real life situations, such as in nuclear power plants, computer operation rooms, cockpits of aeroplanes and so on. In the earlier reliability analysis, attention was given to equipment hardware reliability only and the human element reliability was neglected. This shortcoming was recognized by Williams [1] in the late fifties. According to some researchers [2] about 10-15 percent of the total equipment failures are directly due to human errors. Therefore, realistic system reliability analysis must include the reliability of the human element as well. Furthermore, not all human errors are critical. A critical human error is one that causes the breakdown of the entire system. For example, fire due to human error in a control room where a redundant system is located will cause the total system failure. Similar studies pertaining to the reliability analysis of systems with human error can be found in reference [3-6].

This paper presents availability analysis of two mathematical models, i.e., models I and II, representing maintainable redundant systems with

1986

critical human error. Model I deals with a two-unit active parallel system. Similarly, Model II is concerned with a two-unit cold standby system. The supplementary variable method [7] was used to develop equations for both the models.

# 2. ASSUMPTIONS

The following assimptions are associated with both the models: (i) Failures are statistically independent; (ii) both units are identical; (iii) critical human error rates are constant; (iv) unit failure rate due to hardware failures/non-critical human errors in constant; (v) rapair rate of a single unit is constant; (vi) a repaired unit is as good as 'new'; (vii) failed system repair rates are non-constant; (viii) switchover mechanism is perfect for the standby system (applicable to Model II only); (ix) at time t = 0, the system is put into operation (i.e., in the case of Model I both units start operating simultaneously, whereas in the case of Model II one unit starts operating and the other unit is kept as a cold standby).

#### 3. ANALYSIS

This section presents steady state availability analysis of Models I and II.

#### Model I

This model represents a two-unit active parallel system (i.e. both unite operating simultaneously). The system can fail only when both units are non-operative. In addition, it can fail either due to hardware failures/non-critical human errors of due to critical human errors. A single failed unit is repaired and put back to operation. The state epace diagram of the model is shown in Figure 1. The following symbols are associated with the model: i is the ith state of the system; i = 0 (both units operating normally), i = 1 (one unit failed due to hardware failures/non-critical human errors, other unit operating); i = 2 (both units failed due to hardware failures/non-critical human errors); i = 3 (systems failure due to critical human errors).

 $P_i(t)$  is the probability that the system is in state 1 at time t, for i = 0,1,2,3.  $P_j(x,t)$  is the probability that at time t the failed systems is in state j and has an elapsed repair time of x, for j = 2,3. cc(x) and g(x) denote repair rate and probability density function of repair times, respectively, when the system is in state 2 and has an elapsed repair time of x.  $\beta(x)$  and h(x) denote repair rate and pro-

## Availability modeling of man-machine systems....

bability density function of repair times, respectively, when the system is in state 3 and has an elapsed repair time of x. s is the Laplace transform variable. Is the constant failure rate of a unit due to hardware failures/non-critical human errors.  $\mathfrak{N}_1 (= 2\mathfrak{A})$  is the constant transition rate from state 0 to state 1.  $\mu$  is the constant repair rate from state 1 to state 0.  $\mathfrak{N}_{c_1}$  is the constant critical human error rate from state 1 to state 3, for 1 = 0,1.

The following are the system of differential equations and boundary conditions associated with Model I:

$$\frac{dP_{0}(t)}{dt} + (\mathfrak{H}_{1} + \mathfrak{H}_{0})P_{0}(t) = \mu P_{1}(t) + \int_{0}^{\infty} P_{2}(x,t) \alpha(x) dx +$$

$$\int_{0}^{0} P_{3}(x,t) \delta(x) dx$$
(1)

$$\frac{dr_1(t)}{dt} + (\lambda + \lambda_{c_1} + \mu)P_1(t) = \lambda_1 P_0(t)$$
(2)

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \sigma(x)\right] P_2(x,t) = 0$$
(3)

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta(x)\right] P_3(x,t) = 0$$
(4)

$$P_{0}(0,t) = \& P_{1}(t)$$
(5)

$$P_{3}(0,t) = \mathcal{P}_{0}(t) + \mathcal{P}_{0}(t)$$
 (6)

At time t = 0,  $P_0(0) = 1 c_1 P_1(0) = P_2(x,0) = P_3(x,0) = 0$ . By solving the above differential equations with the aid of Laplace transforms we get

$$P_{0}(s) = (s+\lambda+\lambda_{c_{1}}+\mu)/\left\{(s+\lambda+\lambda_{c_{1}}+\mu)\left[s+\lambda_{1}+\lambda_{c_{0}}-\lambda_{c_{0}}h(s)\right]-\lambda_{1}\left[\mu+\lambda g(s)+\lambda_{c_{1}}h(s)\right]\right\}$$
(7)

$$g(a) = \int_{0}^{\infty} e^{-\theta X} g(x) dx, \quad h(a) = \int_{0}^{0} e^{-\theta X} h(x) dx,$$

$$\begin{array}{c} -\int \sigma(x) dx & -\int \beta(x) dx \\ g(x) = \sigma(x) = 0 \quad \text{and} \quad h(x) = \beta(x) = 0 \end{array}$$

$$P_{1}(s) = \frac{\mathfrak{R}_{1}}{s + \mathfrak{R}_{1} + \mathfrak{R}_{0} + \mu} P_{0}(s)$$
(8)

$$P_{2}(s) = \frac{3 \, \hat{3}_{1}}{s + \hat{3}_{c_{1}} + \hat{3}_{c_{1}} + \mu} \left[ \frac{1 - q(s)}{s} \right] P_{0}(s)$$
(9)

$$P_{3}(s) = \left[\vartheta_{c_{0}} + \frac{\vartheta_{1}\vartheta_{c_{1}}}{s+\vartheta_{1}+\vartheta_{c_{1}}+\iota^{1}}\right] \left[\frac{1-h(s)}{s}\right] P_{0}(s)$$
(10)

The Laplace transform of the instantaneous availability of the two unit parallel system is given by

$$AV_{p}(s) = P_{0}(s) + P_{1}(s)$$
 (11)

The steady state availability of the two-unit parallel system is

$$AV_{ss} = \lim_{s \to 0} \left\{ s \ AV_{p}(s) \right\}$$
(12)

Special case Models: This section presents special case models of Model I when the system repair time distributions are described by

$$g(x) = \frac{\mu_1(\mu_1 x)^{n-1} e^{-\mu_1 x}}{(n-1)!} \text{ and } h(x) = \frac{\mu_2(\mu_2 x)^{n-1} e^{-\mu_2 x}}{(n-1)!},$$

where  $\mu_1, \mu_2$  and n are Erlangian parameters; n is a positive integer, <u>Special case Model 1</u>: For n = 1,  $g(x) = \mu_1 e^{-\mu_1 x}$  and  $h(x) = \mu_2 e^{-\mu_2 x}$ . The Laplace transform of the instantaneous availability of the two-unit parallel systems is given by

$$AV_{p}(s) = \sum_{i=1}^{4} E_{i}s^{4-i}/s \sum_{j=1}^{4} c_{j}s^{4-j},$$
 (13)

Availability modeling of man-machine systems...

$$A_{1} = \vartheta + \vartheta_{c_{1}} + \mu \cdot A_{2} = \mu_{1} + \mu_{2}, A_{3} = \mu_{1}\mu_{2}, A_{4} = \vartheta_{1} + \vartheta_{c_{0}} + \mu_{2}, A_{5} = \vartheta_{1}\mu_{2},$$

$$B_{1} = 1, B_{2} = A_{1} + A_{2}, B_{3} = A_{1}A_{2} + A_{3}, B_{4} = A_{1}A_{3}, C_{1} = 1, C_{2} = A_{1} + A_{4} + \mu_{1},$$

$$C_{3} = A_{1}\mu_{1} + A_{4}(A_{1} + \mu_{1}) + A_{5} + \mu \cdot C_{4} = A_{1}A_{4}\mu_{1} + A_{5}(A_{1} + \mu_{1}) - \vartheta_{1}(A_{2}\mu + \vartheta_{1}\mu_{1} + \vartheta_{c_{1}}\mu_{2}),$$

$$E_{1} = B_{1}, E_{2} = B_{2} + \vartheta_{1}, E_{3} = B_{3} + \vartheta_{1}A_{2}, E_{4} = B_{4} + \vartheta_{1}A_{3}.$$

Thus, the steady state availability of the two-unit parallel system is:

$$AV_{SB} = \lim_{B \to 0} \left\{ s \ AV_{p}(s) \right\} = \frac{\mu_{1}\mu_{2}(3+3_{1}+3_{c_{1}}+\mu)}{\mu_{1}\mu_{2}(3+3_{1}+3_{c_{1}}+\mu)+\mu_{1}3} c_{0}^{(3+3_{c_{1}}+\mu)+3_{1}} c_{\mu_{2}}^{(3+3_{c_{1}}+\mu)+3_{1}} c_{\mu_{2}}^{(3+3_{c_{1}}+\mu)+$$

<u>Special case Model 2</u>: For n = 2,  $g(x) = \mu_1^2 x e^{-\mu_1 x}$  and  $h(x) = \mu_2^2 x e^{-\mu_2 x}$ . The Laplace transform of the instantaneous availability of the two-unit parallel system si

$$AV_{p}(s) = \frac{F_{1}s^{5} + (F_{2} + H_{1})s^{4} + (F_{3} + H_{2})s^{3} + (F_{4} + H_{3})s^{2} + (F_{5} + H_{4})s + F_{6} + H_{5}}{s \sum_{i=1}^{b} G_{i} s^{6-i}}, \quad (15)$$

$$\begin{split} F_{1} &= 1, \ F_{2} &= a_{2} + 2\mu_{2}, \ F_{3} &= a_{3} + 2\mu_{2}a_{2} + \mu_{2}^{2}, \ F_{4} &= a_{4} + 2\mu_{2}a_{3} + \mu_{2}^{2}a_{2}, \\ F_{5} &= 2\mu_{2}a_{4} + \mu_{2}^{2}a_{3}, \ F_{6} &= a_{4}\mu_{2}^{2}, \ H_{1} &= \lambda_{1}, \ H_{2} &= 2\lambda_{1}(\mu_{1} + \mu_{2}), \\ H_{3} &= \lambda_{1}(\mu_{1}^{2} + 4\mu_{1}\mu_{2} + \mu_{2}^{2}), \ H_{4} &= 2\lambda_{1}\mu_{1}\mu_{2}(\mu_{1} + \mu_{2}), \ H_{5} &= \lambda_{4}\mu_{1}^{2}\mu_{2}^{2}, \ G_{1} &= 1, \\ G_{2} &= a_{2} + a_{5}, \ G_{3} &= a_{3} + a_{2}a_{5} + a_{6} - \lambda_{1}\mu, \ G_{4} &= a_{4} + a_{3}a_{5} + a_{2}a_{6} + a_{7} - 2\lambda_{1}\mu(\mu_{1} + \mu_{2}), \\ G_{5} &= a_{4}a_{5} + a_{3}a_{6} + a_{2}a_{7} - \lambda_{1} \left[\mu(\mu_{1}^{2} + 4\mu_{1}\mu_{2} + \mu_{2}^{2}) + \lambda\mu_{1}^{2} + \lambda_{c}_{1}\mu_{2}^{2}\right], \ G_{6} &= a_{4}a_{6} + a_{3}a_{7} - 2\lambda_{1}\mu_{1}\mu_{2} \left[\mu((\mu_{1} + \mu_{2}) + \lambda\mu_{1} + \lambda_{c}_{1}\mu_{2}^{2}), \ a_{1} &= \lambda + \lambda_{c} + \mu, \end{split}$$

$$a_2 = a_1 + 2\mu_1, a_3 = 2\mu_1 a_1 + \mu_1^2, a_4 = a_1 \mu_1^2, a_5 = \lambda_1 + \lambda_{c_0} + 2\mu_2$$

$$a_6 = 2\mu_2(x_1 + x_c_0) + \mu_2^2, a_7 = x_1\mu_2^2.$$

The steady state availability of the two unit parallel system is given by

$$AV_{BB} = \frac{\mu_{1}^{2}\mu_{2}^{2}(\lambda+\lambda_{1}+\lambda_{c_{1}}+\mu)}{\frac{2}{\mu_{1}^{2}\mu_{2}^{2}(\lambda+\lambda_{1}+\lambda_{c_{1}}+\mu)+2\mu_{1}^{2}\mu_{2}\lambda_{c_{0}}(\lambda+\lambda_{c_{1}}+\mu)+2\mu_{1}\mu_{2}\lambda_{1}(\lambda\mu_{2}+\lambda_{c_{1}}\mu_{1})}$$
(16)

Special case Model 3: for n = 3, g(x) =  $\frac{\mu_1^3 x^2 e^{-\mu_1 x}}{2!}$  and h(x) =  $\frac{\mu_2^3 x^2 e^{-\mu_2 x}}{2!}$ ,

The Laplace transform of the instantaneous availability of the two-unit parallel system is given by

$$AV_{p}(s) = \sum_{i=1}^{3} O_{i} s^{8-i} / s \sum_{j=1}^{8} L_{j} s^{8-j}, \qquad (17)$$

$$b_1 = \lambda + \lambda_{c_1} + \mu_* \ b_2 = 3(\mu_1 + \mu_2), \ b_3 = 3(\mu_1^2 + 3\mu_1\mu_2 + \mu_2^2),$$

$$\begin{split} b_{4} &= \mu_{1}^{3} + \mu_{2}^{3} + 9\mu_{1}\mu_{2}(\mu_{1} + \mu_{2}), \ b_{5} &= \mu_{1}\mu_{2}b_{3}, \ b_{6} &= 3\mu_{1}^{2}\mu_{2}^{2}(\mu_{1} + \mu_{2}), \\ b_{7} &= \mu_{1}^{3}\mu_{2}^{3}, \ b_{8} &= b_{1} + b_{1} + b_{c_{0}}, \ b_{9} &= b_{1}(h_{1} + h_{c_{0}}), \ b_{10} &= h_{c_{0}}\mu_{2}^{3}, \\ K_{1} &= 1, \ K_{2} &= b_{1} + b_{2}, \ K_{3} &= b_{1}b_{2} + b_{3}, \ K_{4} &= b_{1}b_{3} + b_{4}, \ K_{5} &= b_{1}b_{4} + b_{5}, \\ K_{6} &= b_{1}b_{5} + b_{6}, \ K_{7} &= b_{1}b_{6} + b_{7}, \ K_{8} &= b_{1}b_{7}, \ L_{1} &= 1, \ L_{2} &= b_{2} + b_{8}, \\ L_{3} &= b_{9} + b_{2}b_{8} + b_{3} - h_{1}\mu, \ L_{4} &= b_{2}b_{9} + b_{3}b_{8} + b_{4} - h_{3}\mu b_{2}, \\ L_{5} &= b_{3}b_{9} + b_{4}b_{8} + b_{5} - (h_{3}\mu b_{3} + b_{10}), \\ L_{6} &= b_{4}b_{9} + b_{5}b_{8} + b_{6} - \left[h_{1}(\mu b_{4} + h_{1}\mu_{1}^{3} + h_{c}\mu_{2}^{3}) + b_{10}(b_{1} + 3\mu_{1})\right], \end{split}$$

Availability modeling of man-machine systeme...

$$\begin{split} \mathsf{L}_7 &= \mathsf{b}_5 \mathsf{b}_9 + \mathsf{b}_6 \mathsf{b}_8 + \mathsf{b}_7 - \left[ \aleph_1 (\mu \mathsf{b}_5 + 3 \aleph \mu_2 \mu_1^3 + 3 \aleph_{c_1} \mu_1 \mu_2^3) + \mathsf{b}_{10} (3 \mu_1 \mathsf{b}_1 + 3 \mu_1^2) \right], \\ \mathsf{L}_8 &= \mathsf{b}_6 \mathsf{b}_9 + \mathsf{b}_7 \mathsf{b}_8 - \left[ \aleph_1 (\mu \mathsf{b}_6 + 3 \aleph \mu_2^2 \mu_1^3 + 3 \aleph_{c_1} \mu_2^2 \mu_2^3) + \mathsf{b}_{10} (3 \mu_1^2 \mathsf{b}_1 + \mu_1^3) \right], \\ \mathsf{O}_1 &= \mathsf{K}_1, \ \mathsf{O}_2 &= \mathsf{K}_2 + \aleph_1, \ \mathsf{O}_3 &= \mathsf{K}_3 + \aleph_1 \mathsf{b}_2, \ \mathsf{O}_4 &= \mathsf{K}_4 + \aleph_1 \mathsf{b}_3, \ \mathsf{O}_5 &= \mathsf{K}_5 + \aleph_1 \mathsf{b}_4, \\ \mathsf{O}_6 &= \mathsf{K}_6 + \aleph_1 \mathsf{b}_5, \ \mathsf{O}_7 &= \mathsf{K}_7 + \aleph_1 \mathsf{b}_6, \ \mathsf{O}_8 &= \mathsf{K}_8 + \aleph_1 \mathsf{b}_7. \end{split}$$

The steady state availability of the two-unit parallel system is given by

$$AV_{BB} = \frac{\mu_{1}^{3}\mu_{2}^{3}(\lambda+\lambda_{1}+\lambda_{c_{1}}+\mu)}{\mu_{1}^{3}\mu_{2}^{3}(\lambda+\lambda_{1}+\lambda_{c_{1}}+\mu)+3\mu_{1}^{3}\mu_{2}^{2}\lambda_{c_{0}}(\lambda+\lambda_{c_{1}}+\mu)+3\mu_{1}^{2}\mu_{2}^{2}\lambda_{1}(\lambda\mu_{2}+\lambda_{c_{1}}\mu_{1})}$$
(18)

Special case Model 4: For n = 4, g(x) =  $\frac{\mu_1^4 x^3 e^{-\mu_1 x}}{3!}$  and h(x) =  $\frac{\mu_2^4 x^3 e^{-\mu_2 x}}{3!}$ .

The Laplace transform of the instantaneous availability of the two-unit parallel system is given by

$$AV_{p}(s) = \sum_{i=1}^{10} Y_{i}s^{10-i}/s \sum_{j=1}^{10} V_{j}s^{10-j}, \qquad (19)$$

$$d_{1} = 4(\mu_{1}+\mu_{2}), d_{2} = 6\mu_{1}^{2}+16\mu_{1}\mu_{2}+6\mu_{2}^{2}, d_{3} = 4(\mu_{1}^{3}+6\mu_{1}^{2}\mu_{2}+6\mu_{1}\mu_{2}^{2}+\mu_{2}^{3}),$$

$$d_{4} = \mu_{1}^{4}+16\mu_{1}^{3}\mu_{2}+36\mu_{1}^{2}\mu_{2}^{2}+16\mu_{3}\mu_{2}^{3}+\mu_{2}^{4}, d_{5} = \mu_{1}\mu_{2}d_{3}, d_{6} = \mu_{1}^{2}\mu_{2}^{2}d_{2},$$

$$d_{7} = \mu_{1}^{3}\mu_{2}^{3}d_{1}, d_{8} = \mu_{1}^{4}\mu_{2}^{2}, d_{9} = 2 + 2\mu_{1}^{2}\mu_{2}d_{3}, d_{6} = \mu_{1}^{2}\mu_{2}^{2}d_{2},$$

$$d_{12} = d_{9}d_{10}, d_{13} = 2\mu_{0}^{4}\mu_{2}, u_{1} = 1, u_{2} = d_{9}+d_{1}, u_{3} = d_{1}d_{9}+d_{2}, u_{4} = d_{2}d_{9}+d_{3},$$

$$u_{5} = d_{3}d_{9}+d_{4}, u_{6} = d_{4}d_{9}+d_{5}, u_{7} = d_{5}d_{9}+d_{6}, u_{8} = d_{6}d_{9}+d_{7}, u_{9} = d_{7}d_{9}+d_{8},$$

$$u_{10} = d_{8}d_{9}, v_{1} = 1, v_{2} = d_{11}+d_{1}, v_{3} = d_{12}+d_{1}d_{11}+d_{2} - 2\mu_{1}\mu_{4},$$

$$v_{4} = d_{1}d_{12}+d_{2}d_{11}+d_{3} - 2\mu_{1}\mu_{4}, v_{5} = d_{2}d_{12}+d_{3}d_{11}+d_{4} - 2\mu_{4}\mu_{2},$$

$$\begin{aligned} \mathbf{v}_{6} &= \mathbf{d}_{3}\mathbf{d}_{12} + \mathbf{d}_{4}\mathbf{d}_{11} + \mathbf{d}_{5} = (\mathfrak{H}_{3}\mu\mathbf{d}_{3} + \mathbf{d}_{13}), \\ \mathbf{v}_{7} &= \mathbf{d}_{4}\mathbf{d}_{12} + \mathbf{d}_{5}\mathbf{d}_{11} + \mathbf{d}_{6} = \left[\mathfrak{H}_{1}\left(\mu\mathbf{d}_{4} + \mathfrak{H}_{1}^{4} + \mathfrak{H}_{6}\mu_{2}^{4}\right) + \mathbf{d}_{13}\left(\mathbf{d}_{9} + \mathbf{d}_{11}\right)\right], \\ \mathbf{v}_{8} &= \mathbf{d}_{5}\mathbf{d}_{12} + \mathbf{d}_{6}\mathbf{d}_{11} + \mathbf{d}_{7} = \left[\mathfrak{H}_{1}\left(\mu\mathbf{d}_{5} + 4\mathfrak{H}_{2}\mu_{2}^{4} + 4\mathfrak{H}_{6}\mu_{2}^{4}\mu_{1}\mu_{2}^{4}\right) + \mathbf{d}_{13}\left(4\mu_{1}\mathbf{d}_{9} + 6\mu_{1}^{2}\right)\right], \\ \mathbf{v}_{9} &= \mathbf{d}_{6}\mathbf{d}_{12} + \mathbf{d}_{7}\mathbf{d}_{11} + \mathbf{d}_{8} = \left[\mathfrak{H}_{1}\left(\mu\mathbf{d}_{6} + 6\mathfrak{H}_{2}\mu_{2}^{4}\mu_{1}^{4} + 6\mathfrak{H}_{6}\mu_{2}^{2}\mu_{1}^{4} + 6\mathfrak{H}_{6}\mu_{2}^{2}\mu_{1}^{4}\right) + \mathbf{d}_{13}\left(6\mu_{1}^{2}\mathbf{d}_{9} + 4\mu_{1}^{3}\right)\right], \\ \mathbf{v}_{10} &= \mathbf{d}_{7}\mathbf{d}_{12} + \mathbf{d}_{8}\mathbf{d}_{11} - \left[\mathfrak{H}_{1}\left(\mu\mathbf{d}_{7} + 4\mathfrak{H}_{2}\mu_{2}^{3}\mu_{1}^{4} + 4\mathfrak{H}_{6}\mu_{1}^{3}\mu_{2}^{4}\right) + \mathbf{d}_{13}\left(4\mu_{1}^{3}\mathbf{d}_{9} + \mu_{1}^{4}\right)\right], \\ \mathbf{v}_{1} &= \mathbf{u}_{1}, \ \mathbf{v}_{2} = \mathbf{u}_{2} + \mathfrak{H}_{1}, \ \mathbf{v}_{3} = \mathbf{u}_{3} + \mathfrak{H}_{1}\mathbf{d}_{1}, \ \mathbf{v}_{4} = \mathbf{u}_{4} + \mathfrak{H}_{1}\mathbf{d}_{2}, \ \mathbf{v}_{5} = \mathbf{u}_{5} + \mathfrak{H}_{1}\mathbf{d}_{3}, \\ \mathbf{v}_{6} &= \mathbf{u}_{6} + \mathfrak{H}_{1}\mathbf{d}_{4}, \ \mathbf{v}_{7} = \mathbf{u}_{7} + \mathfrak{H}_{1}\mathbf{d}_{5}, \ \mathbf{v}_{8} = \mathbf{u}_{8} + \mathfrak{H}_{1}\mathbf{d}_{6}, \ \mathbf{v}_{9} = \mathbf{u}_{9} + \mathfrak{H}_{1}\mathbf{d}_{7}, \ \mathbf{v}_{10} = \mathbf{u}_{10} + \mathfrak{H}_{1}\mathbf{d}_{8} \\ \mathbf{u}_{1}\mathbf{u}_{1}\mathbf{u}_{1}\mathbf{u}_{2} + \mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2} + \mathfrak{H}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{3}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{3}\mathbf{u}_{2}\mathbf{u}_{1}\mathbf{u}_{2}\mathbf{u}_{2}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{4}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{3}\mathbf{u}_{4}\mathbf{u$$

is given by:

S11

$$AV_{BS} = \frac{\mu_{1}^{4}\mu_{2}^{4}(h+h_{1}+h_{c_{1}}+\mu)}{\mu_{1}^{4}\mu_{2}^{4}(h+h_{1}+h_{c_{1}}+\mu) + 4\mu_{1}^{4}\mu_{2}^{2}h_{c_{0}}(h+h_{c_{1}}+\mu) + 4\mu_{1}^{3}\mu_{2}^{3}h_{1}(h_{1}\mu_{2}+h_{c_{1}}\mu_{1})}$$
(20)

For all these special case models, the system instantaneous availability expressions in time domain can be obtained by inverting the Laplace transform expressions given in equations (13), (15), (17) and (19).

With the aid of equations (14), (16), (18) and (20), the following general formula for the two-unit parallel system steady state availability is developed:

$$AV_{ss} = \frac{\mu_{1}^{n}\mu_{2}^{n}(\mathfrak{I}, +\mathfrak{I}_{1} + \mathfrak{I}_{c_{1}} + \mu)}{\mu_{1}^{n}\mu_{2}^{n}(\mathfrak{I}, +\mathfrak{I}_{1} + \mathfrak{I}_{c_{1}} + \mu) + n(\mu_{1}\mu_{2})^{n-1}\mathfrak{I}_{1}(\mathfrak{I}, \mu_{2} + \mathfrak{I}_{c_{1}} + \mu)},$$
(21)

where n is any positive integer.

The plots of the sbove equation are shown in Figures 2, 3 and 4. In Figures 2 and 3, plots show the impact of critical human error rates  $\alpha_{c_0}$  and  $\alpha_{c_1}$ , respectively, on the parallel system steady state availability

#### Availability modeling of man-machine systems....

for different values of the Erlangian parameter n. Similarly, the plots of Figure 4 show the effect of unit failure rate  $\lambda$  on system steady state availability.

## Model II

This model represents a two identical unit cold standby system. At time t = 0, one unit starts operating and the other one is on standby. The failure rate of the standby unit is zero. The operating unit can fail due to hardware failure/non-critical human error. As soon as teh operating unit fails, the standby unit is switched into operation. The failed unit is repaired back to its normal operational mode. The occurrence of critical human error can cause the total system-failure. The failed system is repaired back to normal state. The state space diagram of the model is shown in Figure 5.

The notations used to develop the availability analysis of this model are defined as follows: j is the jth state of the system; j = 0 (one unit is operating, other unit is kept as a cold standby), j = 1 (one unit failed due to hardware failures/non-critical human errors, other unit operating), j = 2 (both units failed due to hardware failures/non-critical human errors), j = 3 (system failure due to critical human error). Rest of the symbols are the same as in Model I.

By setting  $\vartheta_1 = \vartheta$  in equations (1) through (21) yield results pertaining to state probability equations, instantaneous availability and steady state availability of the two-unit cold standby system.

When the system repair time distributions are Erlangian with probability density functions  $g(x) = \frac{\mu_1(\mu_1 x)^{n-1} e^{-\mu_1 x}}{(n-1)!}$  and  $h(x) = \frac{\mu_2(\mu_2 x)^{n-1} e^{-\mu_2 x}}{(n-1)!}$ , the general formula for the system steady state availability is

$$AV_{ss} = \frac{(\mu_1 \mu_2)^n (2\lambda + \lambda_{c_1} + \mu)}{(\mu_1 \mu_2)^n (2\lambda + \lambda_{c_1} + \mu) + n(\mu_1 \mu_2)^{n-1} \lambda_{c_0} (\lambda + \lambda_{c_1} + \mu) + n(\mu_1 \mu_2)^{n-1} \lambda_{(\lambda \mu_2 + \lambda_{c_1} \mu_1)}}$$

(22)

The plots of equation (22) are shown in Figures 6, 7 and 8.

# 4. CONCLUSION

This paper presents two mathematical models to perform the availability analysis of two-unit parallel and standby redundant systems with critical human errors. For both the models instantaneous and steady state availability expressions are developed. For each model, a general system steady state availability formula is developed when the system repair time distributions are Erlangian. Specific plots are shown for both the models to demonstrate the impact of critical human errors on system steady state availability. Finally, it is contended that these pracitcally inclined models will be useful to reliability, maintainability and human factors engineers.

## 5. ACKNOWLEDGEMENT

The financial assistance from the Natural Sciences and Engineering Research Council of Canada is gratefully appreciated.

REFERENCES

- Williams H.L.: Reliability Evaluation of the Human Component in Man-Machine Systems, Electrical Manufacturing, April 1958.
- [2] Hagen E.W. (Editor): Human Reliability Analysis, Nuclear Safety, Vol. 17, pp. 315-326, 1976.
- [3] Dhillon B.S.: On Human Reliability Bibliography, Microelectronics and Reliability, Vol. 20, pp. 371-374, 1980.
- [4] Dhillon B.S., Rayapati S.N.: Analysis of Redundant Systems with Human Errors, Proceedings of Annual Reliability and Maintainability Symposium, pp. 315-321, 1985.
- [5] Dhillon B.S., Singh C.: Engineering Reliability: New Techniques and Applications, John Wiley & Sons, New York, 1981.
- [6] Dhillon B.S.: Human Reliability: With Human Factors, Pergamon Press, Inc., New York (in press).
- [7] Cox D.R.: The Analysis of Non-Markovian Stochastic Processes by the Inclusion of Supplementary Variables, Proceedings of Cambridge Philosophical Society, Vol. 51, pp. 433-441, 1955.

Recenzent: Doc. dr hab. inż. Jerzy FRACZEK

Wpłynęło do Redakcji: luty 1986 r.

Availability modeling of man-machine systems...





Fig. 1. State space diagram Model I Rys. 1. Wykres przestrzeni stanu dla modelu I







Fig. 3. Steady state availability plots for Model I (9, \_\_\_\_\_) Rys. 3. Wykreey ustalonego stanu





Fig. 4. Steady state availability plots for Model I (Ω)
Rys. 4. Wykresy ustalonego stanu dyspozycyjności dla modelu I (Ω)



Fig. 5. State space diagram for Model II Rys. 5. Wykres przestrzeni stanu dla modelu II



Fig. 6. Steady state availability plots for Model II (λ<sub>c</sub>) Rys. 6. Wykresy ustalonego stanu dyspozycyjności dla modelu II (λ<sub>c</sub>)



Fig. 7. Steady state availability plots for Model II (?, )





UNIT FAILURE RATE  $(\lambda)$ 

Fig. 8. Steady state availability plots for Model II (λ)
Rys. 8. Wykresy ustalonego stanu dyspozysyjności dla modelu II (λ)

# Availability modeling of man-machine systems. .

MODELOWANIE DYSPOZYCYJNOŚCI W SYSTEMACH CZŁOWIEK – MASZYNA Z UWZGLĘDNIENIEM KRYTYCZNEGO BŁEDU LUDZKIEGO

## Streszczenie

Referat przedstawia dwa nowe rozwinięte matematyczne modele reprezentujące łatwo naprawialne równoległe i będące w stanie pogotowia redundancyjne systemy, w których występuje krytyczny błąd ludzki. Dla obu modeli rozwinięto transformacje prawdopodobieństwa równań Laplace'a i wyrażenia dla ustalonego stanu dyspozycyjności dla przypadków, kiedy szybkości napraw systemu nie są stałe. Przeanalizowano także wyrażenia chwilowej dyspozycyjności. Dla każdego modelu przytoczono specjalne wykresy, aby zademonstrować wpływ krytycznego i niekorzystnego błędu ludzkiego na ustalony stan dyspozycyjności. W opracowaniu zdefiniowano krytyczny błąd ludzki jako taki, który przerywa ciągłość pracy całego systemu.

W podsumowaniu autor podaje, że modele ukierunkowane na praktykę będą użyteczne w rozważaniach niezawodności oraz wpływu remontów i czynnika ludzkiego na bezawaryjną pracę maszyn.

МОДЕЛИРОВАНИЕ ДИСПЕТЧЕРИЗАЦИИ В СИСТЕМАХ ЧЕЛОВЕК - МАШИНА С УЧЕТОМ КРИТИЧЕСКОЙ ОШИБКИ ЧЕЛОВЕКА

## Резюме

В докладе представлены две по-новому решенные математические модели, репрезептирующие легко поправимые параллельные, находящиеся в состоянии готовности компенсационные системы с критической опибкой человека. Для обенх моделей были развёрнуты трансформации правдоподобия уравнений Лапласа и выражения для установленного состояния диспетчпризации в случаях, когда скорость наладки не является постояния диспетчпризации в случаях, когда сковенной диспетчеризации. Для каждой модели приложены специальные диаграммы с целью демонстрации влияния критической и неблагоприятной ошибки человека на установленное состояние диспетчеризации. За критическую ошибку человека в работе принимается та, которая нарушает непрерывность работы целой системы.

В заключении авторы делают вывод, что представленными моделями можно руководствоваться на практике при решении проблем надежности и влияния наладки и человеческого фактора на безаварийную работу мажин.

when has he said have write 10 her working ht man