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SPECIAL MODEL FOR COATED WOVEN FABRICS

Summary. The paper discusses the fundamental equations of the dense net model and gives the material parameters for two chosen fabrics. The special finite element procedure in the plane stress state is described to introduce effects typical for woven fabrics. Application of this model will be extended to consider the influence of coating in the proposed layered dense net model.

SPECJALNY MODEL DLA TKANINY TECHNICZNEJ

Streszczenie. W pracy przedstawiono podstawowe równania konstytutywne modelu sieci gęstej oraz podano parametry materiałowe dla dwu wybranych tkanin technicznych. Opisano specjalny model elementu skończonego w płaskim stanie naprężenia, który opisuje typowe właściwości tkaniny technicznej. Zastosowanie modelu będzie rozszerzone w celu uwzględnienia wpływu pokrycia w zaproponowanym warstwowym modelu sieci gęstej.

1. Dense net model

1.1. Introduction

The coated woven fabrics consist of the thread's net (usually two families of fabrics, named the warp and the weft) which is both sides coated with the material like PVC (polyvinyl chloride) or PTFE (polytetrafluoroethylene). These fabrics are complex composite non-linear materials that work in a manner characterized by SCHOCK in [6]. Coated woven fabrics are used in various branches of industry nowadays. In civil engineering they are applied mainly in membrane, hanging and pneumatic constructions (including tents, prestressed and air supported shells, or sails). Basic information on the materials used for these structures has been presented in [3] by HOUTMAN and ORPARA.

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1.2. Description of traditional dense net model

The dense net model was described by BRANICKI [2] and developed by KŁOSOWSKI and BRANICKI [1]. The following two assumptions are made undertaken in the model description: (1) the threads work in uniaxial tension forces state only; (2) in a given family of threads the force depends on strains in the same direction only. These assumptions do not give a full description of the fabric, but they enable creation of the fabric numerical model, which has at least the most important properties of this material. The analyzed fabric structure is modelled by membrane finite elements working in the plane stress state. In each element a special substructure modelling threads and coating is applied. The geometrically nonlinear calculations are considered in the range of large displacements and small strains. Also initial prestressing forces must be included. The disadvantage of this model is that it does not take into account in the interaction between both families of threads. On other hand, it is possible to include a change the angle between threads families in the deformation process. The derivation of the elasticity matrix is based on the assumption that one family of threads ξ_1 is parallel to x_1 axis of the local coordinate system, and the second family ξ_2 is sloped by the angle α with reference to the first family (Fig.1).

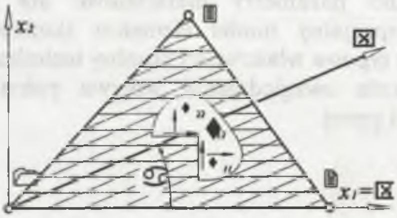

Fig. 1. The dense net model of three nodes finite element
Rys. 1. Model sieci gęstej w trójwęzłowym elemencie skończonym

The relation between strains γ_{11}, γ_{22} along threads' families ξ_1, ξ_2 and strain $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ in the local coordinates x_1, x_2 is defined as:

$$\mathbf{g}=\begin{Bmatrix}\gamma_{11}\\ \gamma_{22}\end{Bmatrix}=\begin{bmatrix}1 & 0 & 0\\ \cos^2\alpha & \sin^2\alpha & \sin\alpha\cos\alpha\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{Bmatrix}=\mathbf{Ce} \tag{1}$$

The threads forces (specific forces) T_{11}, T_{22} in fibres depend on elongation in the same direction only, hence:

$$\mathbf{T}=\begin{Bmatrix} T_{11} \\ T_{22} \end{Bmatrix}=\begin{bmatrix} F_1(\gamma) & 0 \\ 0 & F_2(\gamma) \end{bmatrix}\begin{Bmatrix} \gamma_{11} \\ \gamma_{22} \end{Bmatrix}=\mathbf{F}(\mathbf{g})\mathbf{g} \tag{2}$$

where: $F_1(\gamma), F_2(\gamma)$ – the components of the elasticity matrix, are experimentally determined from the uniaxial tension tests in the warp and the weft direction.

We can find the description of the identification process of the material parameters in the paper [4] for the technical fabric "Panama" manufactured by Settler Company. It consists of polyester fibres (PES) and is double–sided coated with PVC. Elastic properties of the fabric in the warp and the weft direction are given in Table 1.

Table 1

Elasticity modulus of the "Panama" fabric

$F_1(\gamma)$ (warp)	$F_2(\gamma)$ (weft)		units
$0 \le \gamma_{11}$	$0 \le \gamma_{22} \le 0.035$	$0.035 \le \gamma_{22}$	–
932.00	122.00	253.00	kN/m

The relation between components of the membrane forces in the plane stress state $\mathbf{s}$ in local coordinates x_1, x_2 and obtained from (2) the threads forces $\mathbf{T}$ can be calculated from the equation:

$$\mathbf{s}=\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix}=\begin{bmatrix} 1 & \cos^2\alpha \\ 0 & \sin^2\alpha \\ 0 & \sin\alpha\cos\alpha \end{bmatrix}\begin{Bmatrix} T_{11} \\ T_{22} \end{Bmatrix}=\mathbf{C}^{\mathrm{T}}\,\mathbf{T} \tag{3}$$

Consequently, the relation between membrane forces $\mathbf{s}$ and strains $\mathbf{e}$ has the form:

$$\mathbf{s}=\mathbf{C}^{\mathrm{T}}\mathbf{F}(\mathbf{g})\mathbf{C}\,\mathbf{e}=\mathbf{D}\,\mathbf{e} \tag{4}$$

Therefore, using (1) and (2), the elasticity matrix can be expressed as:

$$\mathbf{D}=\begin{bmatrix} F_1(\gamma)+F_2(\gamma)\cos^4\alpha & F_2(\gamma)\sin^2\alpha\cos^2\alpha & F_2(\gamma)\sin\alpha\cos^3\alpha \\ F_2(\gamma)\sin^2\alpha\cos^2\alpha & F_2(\gamma)\sin^4\alpha & F_2(\gamma)\sin^3\alpha\cos\alpha \\ F_2(\gamma)\sin\alpha\cos^3\alpha & F_2(\gamma)\sin^3\alpha\cos\alpha & F_2(\gamma)\sin^2\alpha\cos^2\alpha \end{bmatrix} \tag{5}$$

The angle between families of threads α, changes during deformation and can be calculated on the basis of the current values of components σ_{22} and τ_{12} in the plane stress state.

$$\alpha=\operatorname{arctg}\left(\sigma_{22}/\tau_{12}\right) \tag{6}$$

1.3. Concept of angle between thread families α determination in the four node finite element

In this chapter the four-node isoparametric Lagrange–type element using the dense net concept is proposed. The local coordinate system for this element was chosen, as it is shown in Fig.2.

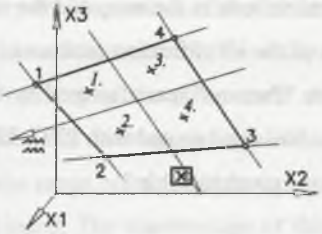

Fig. 2. Isoparametric four-node membrane finite element
Rys. 2. Czterowęzłowy izoparametryczny element membranowy

In the FEM approach the characteristic quantities are usually determined in the four integration points. In order that determine the current components of the elasticity matrix we have to know the actual value of the angle α. For that purpose we assume that in the initial and actual configurations, the warp and the weft directions are determined by the finite element edges (see Fig. 3). The disadvantage of this approach is that mesh of the finite elements must be chosen according to the initial direction of the warp and the weft. The advantage of this approach is simplicity of the angle α calculations. From definition of the scalar product, the angles β_i between two vectors $\mathbf{a}_i$ and $\mathbf{b}_i$ can be described as:

$$\beta_{(\mathbf{a}_i,\mathbf{b}_i)} = \arccos\left(\frac{\mathbf{a}_i \cdot \mathbf{b}_i}{|\mathbf{a}_i||\mathbf{b}_i|}\right), \qquad 0 \le \beta_i \le \pi, \quad i = 1,2,3,4 \tag{7}$$

During deformation process the change of nodes coordinates is accompanied by the change of angles β_i. If we know the actual value of angles β_i, we can determine the angles α_i between families of threads (Fig.3, Fig.4).

$$\begin{array}{ll} \alpha_1 = 180^\circ - \beta_1 & \alpha_4 = 180^\circ - \beta_4 \\ \alpha_2 = \beta_2 & \alpha_3 = \beta \end{array} \tag{8}$$

The four-node finite elements we can transform into three-node triangular elements by contracting the third node into the fourth one. Thus for the three-node triangular elements only one angle α is determined (see Fig.5). In most woven fabrics in the initial state, both families of threads are perpendicular (e.g. in the technical fabric "Panama").

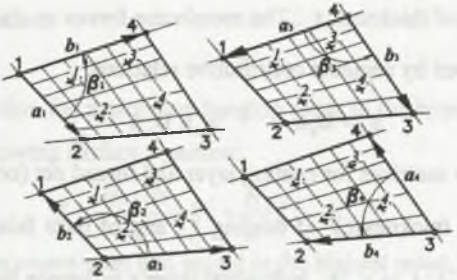

Fig. 3. Current angle β_i between finite element edges

Rys. 3. Aktualny kąt pomiędzy krawędziami elementu skończonego

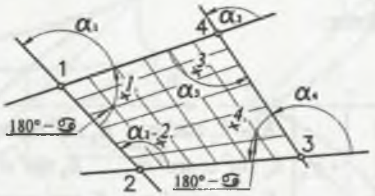

Fig. 4. Orientation of current angle α_i in finite element

Rys. 4. Orientacja aktualnego kąta α_i w elemencie skończonym

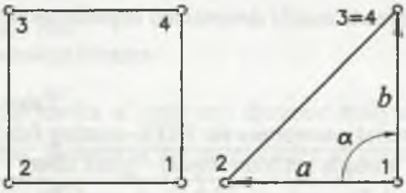

Fig. 5. Four-node and three-node isoparametric element

Rys. 5. Czterowęzłowy i trójwęzłowy izoparametryczny element

1.4. Layered dense net model

The layering cross-section of the technical woven fabric is proposed as the method of taking the coating into account. In the considered model the fabric is modelled by the threads' net (core layer), symmetrically sandwiched by the PVC coating films of small uniform thickness $t_c/2$. Behaviour of the core layer is described by the dense net model. It is assumed that the coating films on both surfaces are made of the same isotropic, linear elastic material. In the membrane structures, the components of strain tensor e do not change their values along the thickness. Therefore, in our approach the influence of both coating layers can be combined into one layer of thickness t_c. The membrane forces in coating film $\mathbf{s}_c$ and in the core layer $\mathbf{s}_n$ are expressed by separate constitutive relations:

$$\mathbf{s}_c = \mathbf{D}_c \mathbf{e} \qquad \mathbf{s}_n = \mathbf{D}_n \mathbf{e} \tag{9}$$

where: $\mathbf{D}_c, \mathbf{D}_n$ – elasticity matrices for coating layer and thread net (core layer).

We assume that both thicknesses: of coating t_c, and of base fabric t_n, are related to the total thickness t_t, $t_c = \chi_c \cdot t_t; t_n = \chi_n \cdot t_t$. Individual layer's influence factors χ_c, χ_n, have to be established in the experimental way. Under this assumption, finally, we derive the constitutive matrices of the core layer and the coating films individually. The total constitutive matrix can be expressed in the compact form:

$$\mathbf{D} = \chi_c \mathbf{D}_c + \chi_n \mathbf{D}_n =$$

$$= \frac{\chi_c E_c}{1-\nu_c^2} \begin{bmatrix} 1 & \nu_c & 0 \\ \nu_c & 1 & 0 \\ 0 & 0 & \frac{1-\nu_c}{2} \end{bmatrix} + \chi_n \begin{bmatrix} E_1 + E_2 \cos^4\alpha & E_2 \sin^2\alpha\cos^2\alpha & E_2 \sin\alpha\cos^3\alpha \\ E_2 \sin^2\alpha\cos^2\alpha & E_2 \sin^4\alpha & E_2 \sin^3\alpha\cos\alpha \\ E_2 \sin\alpha\cos^3\alpha & E_2 \sin^3\alpha\cos\alpha & E_2 \sin^2\alpha\cos^2\alpha \end{bmatrix} \tag{10}$$

where: E_c, ν_c – the elasticity modulus and Poisson's ratio for the coating films, E_1, E_2 – the elasticity modulus of each threads family determined experimentally from the uniaxial tension tests in proper direction.

Table 2

Material parameters for PTFE–coating fabrics

threads net (core layer) – glass fibers		
E_1	70.0	GPa
E_2	70.0	GPa
t_n	0.64	mm
coating film – PTFE		
E_c	420	MPa
ν_c	0.30	–
t_c	0.24	mm

Experiments leading to identification of material parameters for layered dense net model are out of the author's interest, so far. Therefore values taken from the literature will be applied in the following examples. KUWAZURU and YOSHIKAWA [6], gave the material parameters (see Table 2) for technical woven fabric made of fiberglass coated with PTFE film.

2. Numerical calculation

For numerical calculation the membrane hanging roofs in the hyperbolic paraboloid shape was chosen, with the following surface equation:

$$Z = \frac{H_1 - H_2}{A^2} X^2 + \frac{H_1}{A^2} Y^2 - H_1 \tag{11}$$

where: H_1, H_2 – height in centre span and height in the highest point, $2A$ – diagonal span.

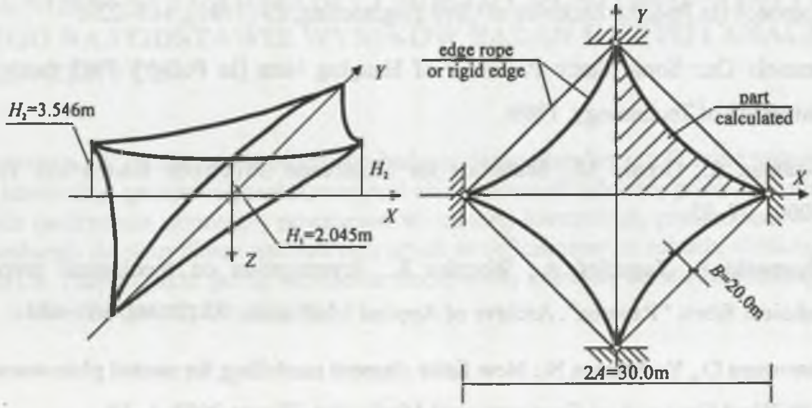

Fig. 6. Membrane suspension roof
Rys. 6. Membranowa konstrukcja wisząca

During the Conference the results of static and dynamic analysis of two models described in the paper will be presented, as well as the possibilities of implementation of the material model into commercial MSC.Marc system. The results obtained from the MSC.Marc software will be compared with those from the self–developed Finite Element Method (FEM) code. A good correlation of the results between both computations confirms the proper application of the model.

3. Conclusion

A new kind of the constitutive model is proposed to analyse a coated woven fabric. This model is based on the traditional dense net approach. The presented constitutive equations seem to be useful in the design of the hanging, membrane and pneumatic structures. For the proposed model the linear, non–linear and viscoplastic material parameters can be introduced in the direct way. Further investigation can be focused on the effects of temperature and viscous behaviour according to the proposed model.

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