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REDUCED-ORDER OBSERVERS FOR DESCRIPTOR SYSTEMS*

Summary. In this paper, a reduced-order observer for descriptor system is considered. The given system is decomposed into slow and fast subsystem, and then observers are desingned for each subsystem. The singular value decomposition and the generalized inverses of matrices will be used to design the observers. Illustrative numerical examples are also given.

OBSERWATORY ZREDUKOWANEGO RZĘDU DLA UKŁADÓW SINGULARNYCH

Streszczenie. W pracy rozpatrzono obserwatory zredukowanego rzędu dla układów singularnych. Dany układ dekomponowany jest na dwa podukłady; "wolny" i "szybki", a nastepnie dla każdego z nich projektowany jest oddzielny obserwator.

Przy projektowaniu obserwatorów wykorzystuje się uogólnione macierze odwrotne oraz dekompozycję macierzy z wykorzystaniem wartości singularnych. Podano również ilustracyjne przykłady numeryczne.

НАБЛЮДАТЕЛИ РЕДУЦИРОБАННОЙ СТЕПЕНИ ДЛЯ СИНГУЛЯРНЫХ СИСТЕМ

Резюме. В равоте рассматриваются наблюдатели редуцированной степени для сингулярных систем. Данная система разбивается ма 2 отдельные подсистемы: "

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"медленную" и "быструю". Для каждой из них отдельно проектируется наблюдатель. Во время проектирования используются обобщенные обратные матрицы и разбиение матриц с использованием сингулярных значений. Даны иллюстративные численные примеры.

1. INTRODUCTION

Consider the continuous-time descriptor system described by

$$E\dot{x} = Ax + Bu \tag{1a}$$

$$y = Cx \tag{1b}$$

where $x \in R^n$ is the descriptor vector, $u \in R^p$ is the input vector, and $y \in R^m$ is the output vector. The matrices A,B and C are respectively dimensional and E is square and singular matrix.

Throughout we will assume that:

(a) System (1) is regular, i.e. det (sE-A) $\neq$ 0.

(b) System (1) is generalized observable, which means

$$\operatorname{rank}\begin{bmatrix} sE - A \\ C \end{bmatrix} = n \tag{2a}$$

for all finite values of s, and

$$\operatorname{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n \tag{2b}$$

Descriptor (singular or generalized state space) systems have recently received considerable effort [see, e.g., 5, 6, 7, 9, 10, 14].

The observer design problem for system (1) has been studied by using several approaches [see, e.g. 3, 4, 11, 12, 13].

In this paper a simple method to design reduced-order observers for slow and fast subsystems of the given continuous descriptor system is considered. The suggested procedure does not presuppose the observer structure and is based on the singular value decomposition (SVD) and the generalized inverses of matrices.

2. OBSERVER CONSTRUCTION

Under the assumption of regularity, there exist two nonsingular matrices Y and T, such that system (1) is restricted system equivalent (rse) to [2]

$$\dot{x}_1 = A_1 x_1 + B_1 u \tag{3a}$$

$$y_i = C_1 x_1 \tag{3b}$$

$$N\dot{x}_2 = x_2 + B_2 u \tag{4a}$$

$$y_2 = C_2 x_2 \tag{4b}$$

$$y = y_1 + y_2$$

where $x \in R^{n1}$, $x \in R^{n2}$, $n_1 = \deg |sE\text{-}A|$, $n_1 + n_2 = n$, $x = T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$,

$YET = \mathrm{diag}\{I_{n1}, N\}$; $YAT = \mathrm{diag}\{A_1, I_{n2}\}$; $YB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ and $CT = [C_1, C_2]$.

2.1. Slow Subsystem Observer

Using the generalized matrix inverses [1], the general solution of (3b) is given by

$$x_1 = C_1^g y_1 + \left(I_{n1} - C_1^g C_1\right) f \tag{5}$$

with consistency condition

$$\left(I_m - C_1 C_1^g\right) y_1 = 0 \tag{6}$$

where C_1^g is an $n_1 x m$ generalized inverse of C_1 and f is an $n_1 x 1$ vector whose elements are arbitrary functions of time. It should be noted that if C_1 has a full row rank, then condition (6) is always satisfied. Let us take the SVD [8] od C_1 which is

$$C_1 = U\begin{bmatrix} \Sigma_d & 0 \\ 0 & 0 \end{bmatrix} V^T;\ \text{then}\ C_1^g = V\begin{bmatrix} \Sigma_d^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^T$$

where Σ_d is a d x d nonsingular matrix, d = rankC_1, and U and V are square orthogonal matrices of order m and n_1 respectively. Letting $V = [V_1, V2]$, where V_2 is an n_1 x $(n_1 - d)$ full column rank matrix, then (5) becomes

$$x_1 = C_1^g y_1 + V_2 h \tag{7}$$

where $h = V_2^T f$ is an $(n_1 - d)$ vector. Substituting (7) into (3a), yields

$$V_2 \dot{h} = A_1 V_2 h + A_1 C_1^g y_1 + B_1 u - C_1^g \dot{y}_1 \tag{8}$$

So the generalized matrix inverses can be used to uniquely solve (8), since V_2 has full column rank, and then by using the solution with its consistency condition, an observer may be constructed. However, in order to work with matrices of smaller dimensions which reduce and simplify the computational effort, the following manipulation is applied.

Since V_2 has full column rank, there exists an $n_1 \times n_2$ nonsingular matrix M such that $MV_2 = \begin{bmatrix} M_1 \\ 0 \end{bmatrix}$, where M_1 is an $(n_1 - d) \times (n_1 - d)$ nonsingular matrix. Premultiplying (8) by M, yields

$$\begin{bmatrix} M_1 \\ 0 \end{bmatrix} \dot{h} = \begin{bmatrix} A_{11} \\ A_{12} \end{bmatrix} h + \begin{bmatrix} P_{11} \\ P_{12} \end{bmatrix} y_1 + \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix} u - \begin{bmatrix} D_{11} \\ D_{12} \end{bmatrix} \dot{y}_1 \tag{9}$$

where A_{11}, B_{11}, D_{11} and P_{11} have $(n_1 - d)$ rows and $(n_1 - d)$, p, $(n_1 - d)$ and m columns, respectively. Let us denote

$$w_1 = M_1 h + D_{11} y_1 \tag{10}$$

Then (9) can be splitted into the follwing two equations

$$\dot{w}_1 = A_{11} M_1^{-1} w_1 + \left(P_{11} - A_{11} M_1^{-1} D_{11}\right) y_1 + B_{11} u \tag{11}$$

$$A_{12} M_1^{-1} w_1 = \left(A_{12} M_1^{-1} D_{11} - P_{12}\right) y_1 - B_{12} u + D_{12} \dot{y}_1 \tag{12}$$

Equations (11) and (12) can ben interpreted as a dynamical system, where w_1 is the state vector, $\begin{bmatrix} y_1 \\ u \end{bmatrix}$ is the input vector and the right hand side of (12) is the output vectore.

An observer of order $(n_1 - d)$ can be initially constructed for system (11) and (12) as follows

$$\dot{\overline{w}}_1 = \left(A_{11}M_1^{-1} - K_1A_{12}M_1^{-1}\right)\overline{w}_1 + \\ + \left(P_{11} - A_{11}M_1^{-1}D_{11} + K_1A_{12}M_1^{-1}D_{11} - K_1P_{12}\right)y_1 + \\ + \left(B_{11} - K_1B_{12}\right)u + K_1D_{12}\dot{y}_1 \tag{13}$$

where K_1 is a $(n_1 - d) \times d$ arbitrary matrix which must be chosen such that the matrix $\left(A_{11}M_1^{-1} - K_1A_{12}M_1^{-1}\right)$ has arbitrarily specified eigenvalues. Clearly, this can be done if and only if the pair od matrices $\left(A_{12}M_1^{-1}, A_{11}M_1^{-1}\right)$ is observable [15].

Theorem 1: If system (1) satisfies observability condition (2a), then $\left(A_{12}M_1^{-1}, A_{11}M_1^{-1}\right)$ is observable pair of matrices.

Proof: Using suitable matrix operation non (2a), yields

$$n = n_1 + n_2 = \text{rank}\begin{bmatrix} Y & O \\ O & I_m \end{bmatrix}\begin{bmatrix} sE - A \\ C \end{bmatrix}\begin{bmatrix} T \end{bmatrix} = \text{rank}\begin{bmatrix} sI_{n1} - A_1 & O \\ O & sN - I_{n2} \\ C_1 & C_2 \end{bmatrix}$$

$$= \text{rank}\begin{bmatrix} I_{n1} & O & O \\ O & I_{n2} & O \\ O & O & U^T \end{bmatrix}\begin{bmatrix} sI_{n1} - A_1 & O \\ O & sN - I_{n2} \\ U\begin{bmatrix} \Sigma_d & O \\ O & O \end{bmatrix}V^T & C_2 \end{bmatrix}\begin{bmatrix} V & O \\ O & I_{n2} \end{bmatrix}$$

$$= \text{rank}\begin{bmatrix} (sI_{n1} - A_1)V & O \\ O & sN - I_{n2} \\ \begin{bmatrix} \Sigma_d & O \\ O & O \end{bmatrix} & C_2 \end{bmatrix}$$

Notcing the fact that (sN-In2) is invertible for any finite s,

$$\operatorname{rank}\begin{bmatrix} (sI_{n1}-A_1)\,V \\ [\Sigma_d \quad O] \end{bmatrix} = n_1 = \operatorname{rank}\begin{bmatrix} (sI_{n1}-A_1)\,V_1 & (sI_{n1}-A_1)\,V_2 \\ \Sigma_d & O \end{bmatrix}$$

Consequently,

$$\operatorname{rank}(sI_{n1}-A_1)\,V_2 = n_1 - d = \operatorname{rank}[M]\left(sI_{n1}-A_1\right)V_2 = \operatorname{rank}\begin{bmatrix} sM_1 - A_{11} \\ -A_{12} \end{bmatrix}$$

Then $\left(A_{11}M_1^{-1}, A_{12}M_1^{-1}\right)$ is observable pair of matrices. ■

Returning to (13), the derivative of y_1, can be eliminated by defining another new variable as follows

$$z_1 = \overline{w}_1 - K_1 D_{12} y_1 \tag{14}$$

and then the final form of (13) may be written as

$$\dot{z}_1 = F_1 z_1 + G_1 y_i + S_1 u \tag{15}$$

where

$$F_1 = A_{11}M_1^{-1} - K_1 A_{12} M_1^{-1}$$

$$G_1 = P_{11} - A_{11}M_1^{-1}D_{11} - K_1 P_{12} + K_1 A_{12} M_1^{-1} K_1 D_{12} - K_1 A_{12} M_1^{-1} K_1 D_{12}$$

$$S_1 = B_{11} - K_1 B_{12}$$

Also, the estimated state $\hat{x}_1$ can be obtained by using (7), (10), and (14) as follows

$$\hat{x}_1 = V_2 M_1^{-1} z_1 + \overline{R}_1 y_1 \tag{16}$$

where $\overline{R}_1 = \left(C_1^g - V_2 M_1^{-1} D_{11} + V_2 M_1^{-1} K_1 D_{12}\right)$

This completes the observer construction for the slow subsystem.

2.2. Fast Subsystem Observer

Here, the above procedure will be repeated with some differences to desing the observer. Using again, the generalized matrix inverses, the general solution of (4b) is given by

$$x_2 = C_2^g y_2 \left(I_{n2} - C_2^g C_2\right)\gamma \tag{17}$$

with consistency condition

$$\left(I_m - C_2 C_2^g\right) y_2 = 0 \tag{18}$$

where C_2^g is an n_2 x m generalized inverse of C_2 and γ is an n_2 x 1 arbitrary vector. Let us take the SVD of C_2 which is

$$C_2 = P\begin{bmatrix} \Sigma_1 & O \\ O & O \end{bmatrix} Q^T;\ \text{then}\ C_2^g = Q\begin{bmatrix} \Sigma_1^{-1} & O \\ O & O \end{bmatrix} P^T$$

where l = rank C_2. Letting Q = $[Q_1, Q_2]$ where Q_2 is an n_2 x $(n_2$-l) full column rank matrix, then (17) becomes

$$x_2 = C_2^g y_2 + Q_2 \sigma \tag{19}$$

where $\sigma = Q_2^T \gamma$ is an $(n_2$ - l)x1 vector. Substituting of (19) into (4a), yields

$$NQ_2 \dot{\sigma} = Q_2 \sigma + C_2^g y_2 + B_2 u - NC_2^g \dot{y}_2 \tag{20}$$

Equation (20) can be uniquely solved for $\dot{\sigma}$ if and only if the matrix NQ_2 has full column rank.

Theorem 2: If system (1) satisfies condition (2b), then the matrix NQ_2 has full column rank equal to $(n_2$ - l).

Proof: Using suitable matrix operations on (2b), we get

$$n = n_1 + n_2 = \text{rank}\begin{bmatrix} E \\ C \end{bmatrix} = \text{rank}\begin{bmatrix} Y & O \\ O & I_M \end{bmatrix}\begin{bmatrix} E \\ O \end{bmatrix}[T] =$$

$$= \text{rank}\begin{bmatrix} I_{n_1} & O & O \\ O & I_{n_2} & O \\ -C_1 & O & I_m \end{bmatrix}\begin{bmatrix} I_{n1} & O \\ O & N \\ C_1 & C_2 \end{bmatrix} = n_1 + \text{rank}\begin{bmatrix} N \\ C_2 \end{bmatrix}$$

Then, $n_2 = \text{rank}\begin{bmatrix} N \\ C_2 \end{bmatrix} =$

$$= \text{rank}\begin{bmatrix} I_{n_2} & O \\ O & P^T \end{bmatrix}\begin{bmatrix} N \\ P\begin{bmatrix} \Sigma_l & O \\ O & O \end{bmatrix}Q^T \end{bmatrix}[Q] = \text{rank}\begin{bmatrix} NQ_1 & NQ_2 \\ \Sigma_l & O \end{bmatrix}$$

Obviously, rank $NQ_2 = n_2 - l$, which completes the proof.

■

Since NQ_2 has full column rank matrix, then there exists a nonsingular $n_2 x n_2$ matrix H such that $HNQ_2 = \begin{bmatrix} H_1 \\ O \end{bmatrix}$ where H_1 is an $(n_2 - l) \times (n_2 - l)$ nonsingular matrix. Premultiplying (20) by H and letting $HQ_2 = \begin{bmatrix} A_{21} \\ A_{22} \end{bmatrix}$, $HB_2 = \begin{bmatrix} B_{21} \\ B_{22} \end{bmatrix}$, $HC_2^g = \begin{bmatrix} P_{21} \\ P_{22} \end{bmatrix}$, and $HNC_2^g = \begin{bmatrix} D_{21} \\ D_{22} \end{bmatrix}$, where A_{21}, B_{21}, D_{21} and P_{21} have $(n_2 - l)$ rows and appropriate number of colums, then the following observer can be obtained after direct substitutions

$$\dot{z}_2 = F_2 z_2 + G_2 y_2 + S_2 u \tag{21}$$

where

$$F_2 = A_{21}H_1^{-1} - K_2 A_{22} H_1^{-1}$$

$$G_2 = P_{21} - A_{21}H_1^{-1}D_{21} - K_2 P_{22} + K_2 A_{22} H_1^{-1} D_{21} +$$
$$+ A_{21}H_1^{-1}K_2 D_{22} - K_2 A_{22} H_1^{-1} K_2 D_{22}$$

$$S_2 = B_{21} - K_2 B_{22}$$

Here K_2 is an $(n_2 - l)$ xl arbitrary matrix and can be selected such that the matrix F_2 has arbitrarily specified eigenvalues. Clearly, this can be done if and only if $\left(A_{22}H_1^{-1}, A_{21}H_1^{-1}\right)$ is observable pair of matrices. This can be easily proved.

Also the estimated vector $\hat{x}_2$ can be found as

$$\hat{x}_2 = Q_2 H_1^{-1} z_2 + \overline{R}_2 y_2 \tag{22}$$

where $\overline{R}_2 = \left(C_2^g - Q_2 H_1^{-1} D_{21} + Q_2 H_1^{-1} K_2 D_{22}\right)$

This completes the observer construction for the fast subsystem.

3. ILLUSTRATIVE EXAMPLES

Example 1

Consider the follwing descriptor system [11]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & - & -1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} u\,;$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x$$

Using the method of [5], we get

$$\dot{x}_1 = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix} x_1 + \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix} u; \quad y_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} x_1$$

and

$$0 = x_2\ [1\ 0]u;\ y_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} x_2$$

Letting, $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $x_2 = -u_1$, and $y_1 = y - y_2$ are known.

Thus, in this system observer only for slow subsystem is necessary. Its output equation can take the following form

$$y_{11} = [0\ \ 0\ \ 1]\, x_1$$

with $y_1 = [y_{11}\ \ y_{12}]^T$ and $y_{11} = y_{12}$.

The direct calculation gives

$$V_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix};\ M_1 = I_2\ ;\ A_{11} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix};\quad A_{12} = [-1\ \ 0];\ B_{11} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$B_{12} = [0\ \ 0];\quad D_{11} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};\ D_{12} = 1;\ P_{11} = -\begin{bmatrix} 0 \\ 0 \end{bmatrix};\ P_{12} = -1.$$

Assuming that the required observer eigenvalues are -3 and -4, then

$$F_1 = \begin{bmatrix} -6 & -1 \\ 6 & -1 \end{bmatrix};\ K_1 = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$$

The other observer parameters are as folows

$$G_1 = \begin{bmatrix} 25 \\ -36 \end{bmatrix};\ S_1 = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix};\ \overline{R}_1 = \begin{bmatrix} 5 \\ -6 \\ 1 \end{bmatrix}.$$

Example 2

Consider the following descriptor system [5]

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 2 & -2 & 0 \\ 2 & -1 & 0 \\ -2 & 2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} u;\ y = [2\ \ -1\ \ 1]\ x$$

The slow and fast subsystems are as follows [5]

$$\dot{x}_1 = 2x_1 + u \ ; \ y_1 = 2x_1$$

and

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \dot{x}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_2 + \begin{bmatrix} -1 \\ 3 \end{bmatrix} u \ ; \ y_2 = [1 \ 1] x_2$$

Clearly, the observer of order one is necessary only for fast subsystem. Then by direct calculation, we get $Q_2 = \begin{bmatrix} -0,7071 \\ 0,7071 \end{bmatrix}$; $H_1 = 0.7071$; $A_{21} = -0.7071$; $A_{22} = 0.7071$: $B_{21} = -1$; $B_{22} = 3$; $D_{21} = 0.5$; $D_{22} = 0.5$; $P_{21} = 0,5$; $P_{22} = 0$.

Letting that the observer eigenvalue is -3, the observer para,eters are as follows

$$K_2 = 2; \ \ G_2 = 1 \ ; \ \ S_2 = -7 \ ; \ \ \overline{R}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} .$$

4. CONCLUSIONS

A straightforward method to design reduced-order observers for slow and fast subsystems of the descriptor system is presented. The method is based on the singular value decomposition and the generalized inverses of matrices and does not presuppose the observer structure. It should be noted that, from theoretical point of view, under this decomposition, it is easily to desing reduced-order observers for the slow and fast subsystems and the sum of the observer order$[(n_1 - d) + (n_2 - l)]$ may be lower than that of one observer (n-rank C) [11]. However, from practical point of view, as in example 2, the calculation of the slow and fast subsystem outputs, y_1 and y_2 independently in terms of y, is sometimes very difficult. This problem may be overcommed by assuming a relation between y, y_1 and y_2. This point need more investigation.

REFERENCES

[1] S. Barnett, Matrices in Control Theory with Application to Linear Programming (Van Nostrand Reinhold, London 1971).

[2] L. Dai, Singular Control Systems, Lecture Notes in Control and Information Sciences, Spring-Verlog, Berlin 1989.

[3] M. El-Tohami, V. Lovass-Nagy and R. Mukundan, On the desing of observers for generalized state space systems using singular value decomposition, Internat. J. Control 38 (1983) 673-683.

[4] M.M. Fahmy and J. O'Reilly, Observers for descriptor systems, Internat. J. Control 49 (1989) 2013-2028.

[5] T. Kaczorek, Different forms of the description of generalized linear systems, Proceeding of 15-th Seminar on Fundamentals of Electrotechnics and Circuit Theory, Wisła, Poland, May (1992) 13-29.

[6] T. Kaczorek, Linear Control Systems, Vol. 1 entitled "Analysis of Multivariable Systems", John Wiley and Sons Inc., New York 1992.

[7] T. Kaczorek, Theory of Control and Systems, PWN, Warszawa 1993. (in Polish).

[8] V.G. Klema and A. J. Laub, The singular value decomposition: Its computation and some applications, IEEE Trans. Automat. Control 25 (1980) 164-176.

[9] F. L. Lewis, A survey of linear singular systems, Circuits Systems Signala Process, 5 (1986) 3-36.

[10] D.G. Lunberger, Dynamic equations in descriptor from, IEEE Trans. Automat. Control, 22 (1977) 312-321.

[11] A.-M. Mohamed, Observer design for descriptor systems, Foundation of Computing and Decision Sciences, 15 (1991) 173 - 180.

[12] B. Shafai and R.L. Caroll, Desing of minimal order observers for singular systems, Internat. J. Control 45 (1987) 1075-1081.

[13] D.N. Shields, Observers for descriptor systems, Interant. J. Control 55 (1992) 249 - 256.

[14] G.C. Verghese, B.C. Levy and T. Kailath, A generalized state-space for singular systems, IEEE Trans. Automat. Control 26 (1981) 811-831.

[15] W.H. Wonham, On pole assignment in multi-input controllable linear systems, IEEE Trans. Automat. Control 12 (1967) 660-665.

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