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OBSERVERS FOR DESCRIPTOR SYSTEMS*

Summary. In this paper, for a non-causal descriptor system, full-order, and reduced-order, observers are considered. The suggested procedure is based on the transformation of the system into a singular value decomposition co-ordinate form. Also, the concept of generalized matrix inverses is applied. The method does not presuppose the observer structure. Illustrative examples are included.

OBSERWATORY DLA UKŁADÓW SINGULARNYCH

Streszczenie. W pracy przedstawiono teorię obserwatorów zredukowanych oraz pełnego rzędu dla układów singularnych. Proponowana procedura jest oparta na transformacji układu do tzw. singularnej postaci kanonicznej. W pracy zastosowano również uogólnione macierze odwrotne. Metoda nie wymaga zakładania z góry struktury obserwatora. Podano również przykłady obliczeniowe.

НАБЛЮДАТЕЛИ ДЛЯ СИНГУЛЯРНЫХ СИСТЕМ

Резюме. В работе представлена теория наблюдателей редуцированной степени и наблюдателей полного порядка для сингулярных систем. Предлагаемая процедура основана на преобразовании системы в т.н. канонический сингулярный вид. Применяются также обобщенные обратные матрицы. Метод не требует априорной предпосылки о структуре наблюдателя. Дан численный пример.

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1. INTRODUCTION

Consider the continuous-time descriptor system described by

$$E\dot{x} = Ax + Bu \tag{1a}$$
$$y = Cx \tag{1b}$$

where $x \varepsilon R^n$ is the descriptor vector, $u \varepsilon R^q$ is the input vector, and $y \varepsilon R^m$ is the output vector. The matrices A,B and C are compatibly dimensional and E is square and singular matrix and rank(E) = p.

Descriptor (singular or generalized state space) systems have recently received considerable effort [see, e.g., 4, 5, 6, 8, 9, 13].

The observer desing problem for system (1) has been studied by using several approaches [see, e. g. 2, 3, 10, 11, 12].

In this paper a simple method to desing full-order and reduced-order observers for non-causal descriptor system is given. The suggested procedure does not presuppose the observer structure and is based on the singular value decomposition and the generalized matrix inverses.

2. OBSERVER CONSTRUCTION

Performing a singular value decomposition (SVD) [7] of E

$$P^T E Q = \begin{bmatrix} \Sigma_p & 0 \\ 0 & 0 \end{bmatrix}$$

where $\Sigma_p = \mathrm{diag}\,\{\sigma_1, \sigma_2, \ldots, \sigma_p\}$ and σ_i (i = 1, 2,...p) are the non-zero singular values of E, P and Q are orthogonal matrices.

Defining

$$P^T A Q = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, P^T B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, CQ = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, Q^T x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

with x_1 being px1 and x_2 being (n-p)x1. Then system (1) can be rewritten as

$$\dot{x}_1 = \Sigma_p^{-1} A_1 x_1 + \Sigma_p^{-1} A_2 x_2 + \Sigma_p^{-1} B_1 u \tag{2}$$

$$O = A_3 x_1 + A_4 x_2 + B_2 u \tag{3}$$

$$y = C_1 x_1 + C_2 x_2 \tag{4}$$

Equations (3) and (4) can be compressed into the equation

$$\begin{bmatrix} A_4 \\ C_2 \end{bmatrix} x_2 + \begin{bmatrix} A_3 \\ C_1 \end{bmatrix} x_1 + \begin{bmatrix} B_2 u \\ -y \end{bmatrix} = 0 \tag{5}$$

Clearly, (5) can be uniquely solved for x_2 if and only if the matrix $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ has full column rank (n-p). It should be noted that if system (1) is strongly observable, i.e.: using an allowed transformation to bring the model observabiliuty matrix $\begin{bmatrix} sE - A \\ C \end{bmatrix}$ to the from $\begin{bmatrix} sE_1 - A_1' \\ A_2' \\ C \end{bmatrix}$ with E_1 of full row rank then the following conditions will be satesfied [13]

$$\operatorname{rank} \begin{bmatrix} sE - A \\ C \end{bmatrix} = n, \ \forall \text{ finite values of s, and} \tag{6a}$$

$$\operatorname{rank} \begin{bmatrix} E_1 \\ A_2' \\ C \end{bmatrix} = n \tag{6b}$$

Then $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ has rank (n-p).

If equation (5) is consistence, it has unique solution for x_2. So, the generalized matrix inverse can be used to solve (5). Substituting x_2 into (2) and then using the resulting equation with the consistency condition of (5), an observer may be constructed to estimate x_1, once x_1 obtained, x_2 can be found. However, in order to work with matrices of smaller dimensions which reduce and simplify the computational effort, the following manipulation is applied.

Since $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix}$ has full column rank, there exists an (n-p+m) x (n-p+m) nonsingular matrix T such that $T\begin{bmatrix} A_4 \\ C_2 \end{bmatrix} = \begin{bmatrix} T_1 \\ 0 \end{bmatrix}$, where T_1 is an (n-p) x (n-p) nonsingular matrix. Premultiplying (5) by T and letting

$$T\begin{bmatrix} A_3 \\ C_1 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}; \; T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}; \; T\begin{bmatrix} B_2 u \\ -y \end{bmatrix} = \begin{bmatrix} T_{11}B_2 u - T_{12}y \\ T_{21}B_2 u - T_{22}y \end{bmatrix}$$

we get

$$T_1 x_2 = -M_1 x_1 - T_{11}B_2 u + T_{12}y \tag{7}$$

$$M_2 x_1 = -T_{21}B_2 u + T_{22}y \tag{8}$$

Substituting of (7) into (2), yields

$$\dot{x}_1 = \overline{A}_1 x_1 + \overline{B}_1 u + \overline{C}_1 y \tag{9}$$

where

$$\overline{A}_1 = \Sigma_p^{-1} A_1 - \Sigma_p^{-1} A_2 T_1^{-1} M_1$$

$$\overline{B}_1 = \Sigma_p^{-1} B_1 - \Sigma_p^{-1} A_2 T_1^{-1} T_{11} B_2$$

$$\overline{C}_1 = \Sigma_p^{-1} A_2 T_1^{-1} T_{12} .$$

Now, the following two different-order observers may be constructed for system (8) and (9).

I. Full-order observer:

This observer can be directly constructed for system (8) and (9) to estimate x_1 as follows

$$\dot{\hat{x}}_1 = \left(\overline{A}_1 - \overline{K}M_2\right)\hat{x}_1 + \left(\overline{B}_1 - \overline{K}T_{21}B_2\right)u + \left(\overline{C}_1 + \overline{K}T_{22}\right)y \qquad (10)$$

where $\overline{K}$ is a p x m arbitrary matrix which must be chosen such that the matrix $\left(\overline{A}_1 - \overline{K}M_2\right)$ has arbitrarily specified eigenvalues. Obviously, this can be done if and only if $(M_2, \overline{A}_1)$ is observable pair of matrices [14].

Theorem 1: If system (1) satisfies the observability condition (6) then $(M_2, \overline{A}_1)$ is observable pair of matrices.

Proof: Using suitable matrix operations on (6a), yields

$$n = \mathrm{rank}\begin{bmatrix} sE - A \\ C \end{bmatrix} = \mathrm{rank}\begin{bmatrix} I_p & O \\ O & T \end{bmatrix}\begin{bmatrix} s\Sigma_p - A_1 & -A_2 \\ \begin{bmatrix} A_3 \\ C_1 \end{bmatrix} & \begin{bmatrix} A_4 \\ C_2 \end{bmatrix} \end{bmatrix} =$$

$$= \mathrm{rank}\begin{bmatrix} \Sigma_p^{-1} & O & O \\ O & I_{n-p} & O \\ O & O & I_m \end{bmatrix}\begin{bmatrix} s\Sigma_p - A_1 & -A_2 \\ M_1 & T_1 \\ M_2 & O \end{bmatrix}\begin{bmatrix} I_p & O \\ -T_1^{-1}M_1 & I_{n-p} \end{bmatrix} =$$

$$= \mathrm{rank}\begin{bmatrix} sI_p - \left(\Sigma_p^{-1}A_1 - \Sigma_p^{-1}A_2T_1^{-1}M_1\right) & -\Sigma_p^{-1}A_2 \\ O & T_1 \\ M_2 & O \end{bmatrix}$$

Concequently,

$$\mathrm{rank}\begin{bmatrix} sI_p - \overline{A}_1 \\ M_2 \end{bmatrix} = p \qquad (11)$$

which completes the proof. ■

So, once $\hat{x}_1$ is obtained, $\hat{x}_2$ can be found by using (7), which is

$$\hat{x}_2 = -T_1^{-1}M_1\hat{x}_1 - T_1^{-1}T_{11}B_2u + T_1^{-1}T_{12}y \tag{12}$$

II. Reduced-order observer:

Putting (8) in the following form

$$\bar{y} = M_2 x_1 \tag{13}$$

where $\bar{y} = -T_{21}B_2u + T_{22}y$, and letting rank $M_2 = d$. Then an observer of order (p-d) is constructed as follows.

Using the generalized matrix inverses [1], the general solution of (13) is

$$x_1 = M_2^g \bar{y} + \left(I_p - M_2^g M_2\right) f \tag{14}$$

with consistency condition

$$\left(I_m - M_2 \; M_2^g\right) \bar{y} = 0 \tag{15}$$

where M_2^g is a p x m generalized inverse of M_2 and f is a px1 vector whose elements are arbitrary functions of time. Taking the (SVD) of M_2 which is

$$M_2 = U\begin{bmatrix} \Sigma_d & O \\ O & O \end{bmatrix} V^T \; ; \quad \text{then } M_2^g = V\begin{bmatrix} \Sigma_d^{-1} & O \\ O & O \end{bmatrix} U^T$$

where Σ_d is a d x d nonsingular matrix, and U and V are square orthogonal matrices of order m and p respectively. Letting $V = [V_1, V_2]$, where V_2 is a p x (p-d) full column rank matrix, then (14) becomes

$$x_1 = M_2^g \bar{y} + V_2 h \tag{16}$$

where $h = V_2^T f$ is a (p-d) vector. Substituting of (16) into (9), yields

$$V_2 \dot{h} = \bar{A}_1 V_2 h + \bar{Q}_1 y + \bar{L}_1 u - M_2^g \dot{\bar{y}} \tag{17}$$

where $\overline{Q}_1 = \overline{A}_1 M_2^g T_{22} + \overline{C}_1$; $\overline{L}_1 = \overline{B}_1 - \overline{A}_1 M_2^g T_{21} B_2$

Again, since V_2 has full column rank, there exists a p x p nonsingular matrix $\overline{P}$ such that $\overline{P}V_2 = \begin{bmatrix} \overline{V}_2 \\ O \end{bmatrix}$, where $\overline{V}_2$ is a (p-d) x (p-d) nonsingular matrix. Premultiplying (17) by $\overline{P}$, yields

$$\begin{bmatrix} \overline{V}_2 \\ O \end{bmatrix} \dot{h} = \begin{bmatrix} \overline{A}_{11} \\ \overline{A}_{21} \end{bmatrix} h + \begin{bmatrix} \overline{Q}_{11} \\ Q_{21} \end{bmatrix} y + \begin{bmatrix} \overline{L}_{11} \\ \overline{L}_{21} \end{bmatrix} u - \begin{bmatrix} \overline{D}_1 \\ \overline{D}_2 \end{bmatrix} \dot{\overline{y}} \tag{18}$$

Letting

$$w = \overline{V}_2 h + \overline{D}_1 \overline{y} \tag{19}$$

Then (18) can be splitted into the following two equations

$$\dot{w} = \overline{A}_{11}\overline{V}_2^{-1} w + \left(\overline{Q}_{11} - \overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1 T_{22}\right) y + \left(\overline{L}_{11} + \overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1 T_{21} B_2\right) u \tag{20}$$

$$\overline{A}_{21}\overline{V}_2^{-1} w = \overline{D}_2 \dot{\overline{y}} + \left(\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_2 T_{22} - \overline{Q}_{21}\right) y - \left(\overline{L}_{21} + \overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1 T_{21} B_2\right) u \tag{21}$$

Equations (20) and (21) can be interpreted as a dynamical system, where w is the state vector, $\begin{bmatrix} y \\ u \end{bmatrix}$ is the input vector and the right hand side of (21) is the output vector.

An observer of order (p-d) can be initially constructed for system (20) and (21) as follows

$$\begin{aligned} \dot{\overline{w}} &= \left(\overline{A}_{11}\overline{V}_2^{-1} - \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\right)\overline{w} + \\ &+ \left(\overline{Q}_{11} - \overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1 T_{22} - \tilde{K}\overline{Q}_{21} + \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1 T_{22}\right) y \\ &+ \left(\overline{L}_{11} + \overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1 T_{21} B_2 - \tilde{K}\overline{L}_{21} - \tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1 T_{21} B_2\right) u + \tilde{K}\overline{D}_2 \dot{\overline{y}} \end{aligned} \tag{22}$$

where $\bar{K}$ is a (p-d) x d arbitrary matrix which must be chosen such that the matrix $\left(\bar{A}_{11}\bar{V}_2^{-1} - \bar{K}\bar{A}_{21}\bar{V}_2^{-1}\right)$ has arbitrarily specified eigenvalues. As before, this is satisfied of and only if the pair of matrices $\left(\bar{A}_{21}\bar{V}_2^{-1}, \bar{A}_{11}\bar{V}_2^{-1}\right)$ is observable.

Theorem 2: If condition (11) is satisfied, then $\left(\bar{A}_{21}\bar{V}_2^{-1}, \bar{A}_{11}\bar{V}_2^{-1}\right)$ is observable pair of matrices.

Proof: Using the followig matrix operation on (11), yields

$$p = \text{rank}\begin{bmatrix} sI - \bar{A}_1 \\ M_2 \end{bmatrix} = \text{rank}\begin{bmatrix} I & O \\ O & U^T \end{bmatrix}\begin{bmatrix} sI - \bar{A}_1 \\ U\begin{bmatrix} \Sigma_d & O \\ O & O \end{bmatrix}V^T \end{bmatrix}\begin{bmatrix} V_1 & V_2 \end{bmatrix} =$$

$$= \text{rank}\begin{bmatrix} \left[sI - \bar{A}_1\right]V_1 & \left[sI - \bar{A}_1\right]V_2 \\ \Sigma_d & O \end{bmatrix}$$

Clearly, rank $\left(sI - \bar{A}_1\right)V_2 = $ p-d $=$ rank $\begin{bmatrix} s\bar{V}_2 - \bar{A}_{11} \\ \bar{A}_{21} \end{bmatrix}$ which completes the proof

■

Returning to (22), the derivative of $\bar{y}$ can be eliminated by defining another new variable as follows

$$z = \bar{w} - \bar{K}\bar{D}_2\bar{y} \tag{23}$$

and then the final form of (22) may be written as

$$\dot{z} = Fz + Gy + Su \tag{24}$$

where

$$F = \bar{A}_{11}\bar{V}_2^{-1} - \bar{K}\bar{A}_{21}\bar{V}_2^{-1}$$

$$G = \bar{Q}_{11} - \bar{A}_{11}\bar{V}_2^{-1}\bar{D}_1T_{22} - \bar{K}\bar{Q}_{21} + \bar{K}\bar{A}_{21}\bar{V}_2^{-1}\bar{D}_1T_{22} +$$

$$+ \bar{A}_{11}\bar{V}_2^{-1}\bar{K}\bar{D}_2T_{22} - \bar{K}\bar{A}_{21}\bar{V}_2^{-1}\bar{K}\bar{D}_2T_{22}$$

$$S=\overline{L}_{11}+\overline{A}_{11}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2-\tilde{K}\overline{L}_{21}-\tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\overline{D}_1T_{21}B_2-$$
$$-\overline{A}_{11}\overline{V}_2^{-1}\tilde{K}\overline{D}_2T_{21}B_2+\tilde{K}\overline{A}_{21}\overline{V}_2^{-1}\tilde{K}\overline{D}_2T_{21}B_2 \quad .$$

Also, the estimated state $\hat{x}_1$ can be obtained by using (16), (19), and (23) as follows

$$\hat{x}_1=V_2\overline{V}_2^{-1}z+\overline{R}y+\tilde{R}u \tag{25}$$

where

$$\overline{R}=\left(M_2^g+V_2\overline{V}_2^{-1}\tilde{K}\overline{D}_2-V_2\overline{V}_2^{-1}\overline{D}_1\right)T_{22}$$

$$\tilde{R}=\left(V_2\overline{V}_2^{-1}\overline{D}_1-M_2^g-V_2\overline{V}_2^{-1}\tilde{K}D_2\right)T_{21}B_2$$

Remark: If system (1) is causal (i.e. deg $|sE-A|$ = rank E, and then A_4 is nonsingular matrix [3]), then elimination of x_2 by using (3) leads to a system of the form (8) and (9), but, of course, coeficient matrices will be different from those defined in (8) and (9).

3. ILLUSTRATIVE EXAMPLES

Example 1

Consider the following descriptor system

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\dot{x}=\begin{bmatrix}1&-1&1&0&0\\0&1&0&0&0\\1&0&1&2&0\\1&0&3&5&1\\0&0&0&-3&3\end{bmatrix}x+\begin{bmatrix}1&-1\\0&2\\-1&1\\2&-1\\1&0\end{bmatrix}u$$

$$y=\begin{bmatrix}0&0&1&2&1\\1&0&0&0&0\end{bmatrix}x$$

Clearly,

$$\begin{bmatrix} A_4 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 1 \\ 0 & -3 & 3 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ has full column rank } (n-p) = 3$$

Then the matrix T can be evaluated by using elementary tranformations as follows

$$T = \begin{bmatrix} 9 & 0 & 2 & -6 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & -3 & 0 \\ -3 & 1 & -0,333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and then

$$T\begin{bmatrix} A_3 \\ C_1 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ -1 & 0 \\ 3 & 0 \\ -2 & 0 \\ 1 & 0 \end{bmatrix}; \; T_1 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & 0 \end{bmatrix}$$

Also, equations (8) and (9) are as follows

$$\begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} x_1 = \begin{bmatrix} -4,6666 & 4 \\ 0 & 0 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} y$$

$$\dot{x}_1 = \begin{bmatrix} -2 & -1 \\ 0 & 1 \end{bmatrix} x_1 + \begin{bmatrix} 3,3333 & -4 \\ 0 & 2 \end{bmatrix} u + \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix} y$$

I. Full-order observer:

Assuming the required observer eigenvalues are - 2 and -3, then

$$\overline{K} = \begin{bmatrix} 0 & 4 \\ 6 & 0 \end{bmatrix} \qquad \text{and (10) becomes}$$

$$\dot{\hat{x}}_1 = \begin{bmatrix} -6 & -1 \\ 12 & 1 \end{bmatrix} \hat{x}_1 + \begin{bmatrix} 3,3333 & -4 \\ -28 & 26 \end{bmatrix} u + \begin{bmatrix} -2 & 4 \\ 0 & 0 \end{bmatrix} y$$

Moreover,

$$\hat{x}_2 = \begin{bmatrix} -3 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \hat{x}_1 + \begin{bmatrix} 2,3333 & -3 \\ 0,6666 & 1 \\ 1 & 1 \end{bmatrix} u + \begin{bmatrix} -2 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} y$$

II. Reduced order observer:

The singular value decomposition of M_2 is

$$M_2 = \begin{bmatrix} -0,8944 & 0,4472 \\ 0,4472 & 0,8944 \end{bmatrix} \begin{bmatrix} 2,2361 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Then,

$$M_2^g = \begin{bmatrix} -0,4 & 0,2 \\ 0 & 0 \end{bmatrix}; \quad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Letting the observer eigenvalue is -3, then by direct calculation, the reduced-order observer (24) is as follows

$$\dot{z} = -3z - 8y + 0,8y_2 - 46,9333u_1 + 30,8u_2$$

Moreover,

$$\hat{x}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} z + \begin{bmatrix} 0 & 0,2 \\ 0 & -0,8 \end{bmatrix} y \begin{bmatrix} 2,2666 & -1,6 \\ -9,0666 & 6,4 \end{bmatrix} u$$

Example 2:

Consider the following descriptor system [3,12]

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 1 & -1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 3 & -1 \\ -3 & -2 & 1 & 2 & 0 \\ 1 & 0 & 3 & 5 & 1 \\ -1 & 1 & 0 & -3 & 3 \end{bmatrix} x + \begin{bmatrix} 1- & 1 \\ 0 & 2 \\ -1 & 1 \\ 2 & -1 \\ 1 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -1 & 1 & 2 & 1 \\ -2 & 1 & 3 & -1 & 2 \end{bmatrix} x$$

Clearly, $\begin{bmatrix} A_4 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 1 \\ 0 & -3 & 3 \\ 1 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix}$ has full column rank (n-p) = 3.

The matrix T is

$$T = \begin{bmatrix} 9 & 0 & 2 & -6 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & -3 & 0 \\ -3 & 1 & -0,3333 & 0 & 0 \\ -8 & 0 & -2,3333 & 5 & 1 \end{bmatrix}$$

and then,

$$T\begin{bmatrix} A_3 \\ C_1 \end{bmatrix} = \begin{bmatrix} -35 & -10 \\ 4 & 1 \\ -13 & -2 \\ 10,333 & 5,6666 \\ 29,3333 & 9,6666 \end{bmatrix}$$

Obviously, $$M_2 = \begin{bmatrix} 10,3333 & 5,6666 \\ 29,3333 & 9,6666 \end{bmatrix}$$

Since M_2 is invertible, then (8) gives a static observer for x_1 as follows

$$\hat{x}_1 = \begin{bmatrix} 0,196 & 0,1004 \\ -1,181 & 0,5226 \end{bmatrix} u + \begin{bmatrix} 0,4271 & 0,0854 \\ -0,7788 & 0,1558 \end{bmatrix} y$$

And x_2 can be evaluated by using (7), which is

$$\hat{x}_2 = \begin{bmatrix} 0,6836 & -0,0859 \\ -0,7287 & 0,2163 \\ -0,6031 & 0,0756 \end{bmatrix} u + \begin{bmatrix} 0,3866 & 0,4771 \\ -0,3315 & 0,2662 \\ 0,0704 & -0,1854 \end{bmatrix} y$$

It should be noted that for this example, the estimation of descriptor vector may be evaluated by compressing equations (3) and (4) as follows.

$$\begin{bmatrix} A_2 & A_4 \\ C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -B_2 u \\ y \end{bmatrix}$$

Clearly the left hand side matrix is square and nonsingular, and then the vector x can be easily obtained.

So, dynamic observer is not needed for this example.

4. CONCLUSIONS

A straightforward method to design full-order and reduced-order observers for a non-causal descriptor system is presented. Firstly, the given descriptor system is transformed into singular value decomposition descriptor-space coordinates by performing a SVD of matrix E. And then, a full-order - rank E - observer may be designed. However, in order to desing a reduced-order observer the generalized matrix inverse and the SVD of matrices will be used. The suggested procedure does not presuppose the observer structure. Moreover, as shown in example 2, the necessity of dynamic observer for descriptor system may be checked easily.

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