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ROBUSTIFICATION PROCEDURE FOR JLQ PROBLEM

Summary: This paper deals with the problem of robustification procedure for jump linear quadratic (JLQ) control. Mainly, we present sufficient conditions for quadratic stabilization of uncertain jump linear system using state feedback control. The proposed control law contains two components. The first one is a JLQ control law, while the second is a nonlinear bounded robustification term whose cost is not included in the performance index.

PROCEDURA UODPARNIAJĄCA STEROWANIE W PROBLEMIE JLQ

Streszczenie: W pracy przedstawiono procedurę pozwalającą na zapewnienie odpornej stabilności w problemie liniowo-kwadratowym ze skokami. W szczególności podano warunki wystarczające kwadratowej stabilizowalności systemu liniowego z niepewnością, w którym występują markowskie skoki. Przy założeniu dostępności do stanu systemu i realizacji markowskiego procesu o stanach dyskretnych zaproponowano sterowanie zawierające dwie składowe: pierwszą, która jest strategią JLQ, drugą, która jest nieliniowym ograniczonym prawem uodparniającym.

ROBUSTIFIZIERUNGVERFAHREN FUR DIE JLQ – AUFGABE

Zusammenfassung: In der Arbeit wird ein Verfahren, welches sichert die robuste Stabilitat im linear – quadratischen Problem mit Sprungen zu, vorgestellt. Im besonderen werden die genugende Bedingungen der quadratischen Stabilisierbarkeit im linearen System mit Unsicherheit und Markoff – Sprungen formuliert. Fur solche Voraussetzungen wird die Steuerung mit zwei Komponenten : erste : die JLQ – Strategie, zweite : nichtlineares, beschranktes, robustifiziertes Gesetz vorgeschlagen.

1. Introduction

In many practical situations, the natural state space is hybrid: to the usual plant state in R^n we append a discrete variable taking values in $\mathcal{B} = \{1, 2, ...s\}$ called the mode that describes sudden changes in the plant characteristics. It is typical case in the complex large scale systems, such as manufacturing systems (see for example [1]), power systems (see for example [2]) or redundant multiplex control systems ([3]).

In this paper we consider systems which are linear in the continuous plant state and whose mode dynamics is described via random jumps modelled by a discrete-state Markov chain. One way of stabilizing the linear stochastically stabilizable system with markovian jumps is to solve the JLQ problem (see for example [4], [5], [6]). However the optimality of the solution as well as the stability of the system is guaranteed only for the perfectly measurable state variables and complete information about the system parameters. Moreover an optimal controller uses all the state variables to construct a control vector. This is an overidealization especially in the case of a complex system containing many subsystems interconnected by incompletely known crosscoupling. The situation becomes especially complex for the piecewise deterministic processes when the controller is designed under the assumption of the complete access to the mode i.e. discrete random state variables representing the form process.

To overcome at least a part of these difficulties we propose to combine JLQ approach with nonlinear control design method used by some authors (see for example [7], [8], [9]) to ensure practical stability of uncertain systems. The uncertainty is described by deterministic inequality model and the main assumption is the well-known matching conditions.

The paper is organized as follows. In section 2, we present models used in the paper, assumptions to be satisfied and formulation of the control task. In section 3, we give sufficient conditions of robust stability of the system under consideration. The results are summarized in section 4.

2. Problem statement

In this section, we consider jump linear systems described by

$$\dot{x}(t) = A(\xi(t))x(t) + B(\xi(t))u(t) \tag{1}$$

$$x(0) = x_0 \tag{2}$$

where the n-dimensional vector $x(t) \in \mathbb{R}^n$ stands for the state of the system and the m-dimensional vector $u(t) \in \mathbb{R}^m$ is the control. The parameter $\xi(t)$ represents a continuous discrete-state Markov process taking values in a finite set $\mathcal{B} = \{1, 2, .., s\}$ with transition probability $Pr\{\xi(t+\delta t) = \beta | \xi(t) = \alpha\}$ given by:

$$Pr\{\xi(t+\delta t) = \beta | \xi(t) = \alpha\} = \begin{cases} q_{\alpha\beta}\delta t + o(\delta t), & \text{if } \alpha \neq \beta \\ 1 + q_{\alpha\alpha}\delta t + o(\delta t), & \text{if } \alpha = \beta \end{cases} \tag{3}$$

In this relation, $q_{\alpha\beta}$ stands for the transition probability rate from mode α to mode β and satisfies the following relations:

$$q_{\alpha\beta} \geq 0 \tag{4}$$

$$q_{\alpha\alpha} = -\sum_{\beta \in \mathcal{B}, \alpha \neq \beta} q_{\alpha\beta} \tag{5}$$

In eq. (1), $A(\xi(t))$, $B(\xi(t)$ are appropriately dimensioned matrices. These matrices are constant for a given $\xi(t)$ value.

Remark 1: the system described by Eqs. (1)-(5) is an hybrid system with state $(x, \xi(t))$. The n-dimensional vector $x(t)$ is continuous while $\xi(t)$ is discrete. In the rest of this paper, we will refer to the second component of the state vector as the mode.

Remark 2: the system described by Eqs. (1)-(2) represents the nominal model for the real system.

In order to control the nominal system as described by Eq. (1), we use the optimal control approach. The optimization problem consists of minimizing the cost function

$$J = \mathbf{E}\left\{\int_0^\infty x'(t)Q(\xi(t))x(t) + u'(t)R(\xi(t))u(t)dt\right\} \tag{6}$$

subject to the constraints (1)-(2).

In relation (6), the cost weighting matrices $R(\xi(t))$ and $Q(\xi(t))$ are symmetric with $R(\xi(t))$ positive definite and $Q(\xi(t))$ positive semi definite for each $\xi(t)$.

In the remainder of this paper, we assume that the nominal system described by Eq. (1) is stochastically stabilizable and the first component of the state vector, i.e. $x(t)$ in each mode is assumed to be available for feedback. The Markov process $\{\xi(t), t \geq 0\}$ is also assumed to be irreductible. Moreover observability of each pair $(C(\alpha), A(\alpha))$ where $Q(\alpha) = C'(\alpha)C(\alpha)$ is assumed.

Under the previous assumptions it was established (see Ji and Chiseck (1990)) that the solution of the optimization problem is given by

$$u(\alpha,t) = -L(\alpha)x(t), \text{ when } \xi(t) = \alpha \tag{7}$$

where

$$L(\alpha) = R^{-1}(\alpha)B'(\alpha)K(\alpha) \tag{8}$$

The matrix $K(\alpha)$ is the solution of the following coupled algebraic Riccati equations:

$$\begin{aligned} A'(\alpha)K(\alpha) &+ K(\alpha)A(\alpha) - K(\alpha)B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha) + q_{\alpha\alpha}K(\alpha) \\ &+ \sum_{\beta=1,\beta\neq\alpha}^{s} q_{\alpha\beta}K(\beta) + Q(\alpha) = 0, \qquad \alpha \in \mathcal{B} \end{aligned} \tag{9}$$

In the standard formulation of the previous optimization problem, only the nominal model of the system is considered. In real life, the matrices $A(\xi(t))$ and $B(\xi(t))$ for each $\xi(t)$ value in $\mathcal{B}$ are not precisely known. These uncertainties can made the feedback controller not efficient and in worst case the real system can become unstable. In order to avoid these problems, we need to take into account the system's uncertainties.

Let assume that the system's uncertainties be additive and let the real system be described by the following differential equations:

$$\dot{x}(t) = [A(\xi(t)) + \Delta A(\xi(t)]x(t) + [B(\xi(t)) + \Delta B(\xi(t))](u(t) + \Delta u(t)) \tag{10}$$

$$x(0) = x_0 \tag{11}$$

where the matrices $\Delta A(\xi(t))$ and $\Delta B(\xi(t))$ represent respectively the uncertainties on the $A(\xi(t))$ matrix components and $B(\xi(t))$ matrix components for each $\xi(t)$ value, and $\Delta u(t)$ is a robustification term which will be specified subsequently. $x(t)$ and $u(t)$ have the same meaning as previous.

Let assume that the parameters uncertainties satisfy the following matching conditions (see e.g. [8],[9]).

$$\Delta A(\xi(t)) = B(\xi(t))D(\xi(t)) \tag{12}$$

$$\Delta B(\xi(t)) = B(\xi(t))F(\xi(t)) \tag{13}$$

The uncertainties are norm bounded by some scalars for each $\xi(t) = \alpha$:

$$\|D(\alpha)\| \leq d(\alpha) \tag{14}$$

$$\|F(\alpha)\| \leq f(\alpha) < 1 \tag{15}$$

where $\|.\|$ represents the matrix norm induced by euchlidean norm in $\mathbb{R}^n$.

By introducing (12) and (13) into the state equation (10), the system may be described for each $\xi(t) = \alpha$ by:

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)\Big[u(\alpha,t) + \Delta u(\alpha,t) + e(\alpha,t)\Big] \tag{16}$$

$$x(0) = x_0 \tag{17}$$

where $e(\alpha,t)$ represents the uncertainty acting in the range of the input and is defined as follows:

$$e(\alpha,t) = D(\alpha)x(t) + F(\alpha)\Big[u(\alpha,t) + \Delta u(\alpha,t)\Big] \tag{18}$$

Let assume that the robustification term $\Delta u(\alpha,t)$ is bounded by $\rho(\alpha)$ which is upper bound for the norm of $e(\alpha,t)$, i.e:

$$\|\Delta u(\alpha,t)\| \leq \rho(\alpha) \tag{19}$$

The bound $\rho(\alpha)$ may be defined by the following estimation procedure which takes into account the form of the control law (7):

$$\begin{aligned}\|e(\alpha,t)\| &\leq \|D(\alpha) - F(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)\|\|x(t)\| + \|F(\alpha)\|\rho(\alpha) \\ &\leq \Big[d(\alpha) + f(\alpha)\|R^{-1}(\alpha)B'(\alpha)K(\alpha)\|\Big]\|x(t)\| + f(\alpha)\rho(\alpha)\end{aligned} \tag{20}$$

If $\rho(\alpha)$ is introduced by the equation:

$$\rho(\alpha) = \Big[d(\alpha) + f(\alpha)\|R^{-1}(\alpha)B'(\alpha)K(\alpha)\|\Big]\|x(t)\| + f(\alpha)\rho(\alpha) \tag{21}$$

then its value is given as:

$$\rho(\alpha) = [1 - f(\alpha)]^{-1}[d(\alpha) + f(\alpha)\|R^{-1}(\alpha)B'(\alpha)K(\alpha)\|]\|x(t)\| \tag{22}$$

The uncertainty matrices $D(\alpha)$, $F(\alpha)$ could be time or even state dependent assuming only that they are bounded in norm by scalars $d(\alpha)$, $f(\alpha)$.

3. Main results

In this section, we present conditions which enable robustification of the JLQ control by introducing the term $\Delta u(\alpha, t)$ defined below. Robustness of the system will be understood both in the sense of robust stability. The following control law is proposed for $\Delta u(\alpha, t)$:

$$\Delta u(\alpha, t) = \begin{cases} \frac{R(\alpha)u(\alpha,t)}{\|R(\alpha)u(\alpha,t)\|}\rho(\alpha) & \text{if } u(\alpha, t) \neq 0 \\ 0 & \text{if } u(\alpha, t) = 0 \end{cases} \tag{23}$$

To find the sufficient conditions for robust stability, let assume Lyapunov function candidate in the form:

$$V(x, \alpha) = x'K(\alpha)x = S(x, \alpha) \tag{24}$$

where $S(x, \alpha)$ is an optimal cost to go for the nominal model.

The following theorem gives the required conditions.

Theorem 3.1. Assume that the system described by the state equation (10) meets the matching conditions (12) - (15) and is governed by the control law (7) with the robustification term (23). Then the system remains stochastically stable in the whole ranges of uncertainty.

Proof: Consider weak infinitesimal operator $\tilde{A}$ of the joint process $(\xi(t), x(t))$ which is the natural analogue of the deterministic derivative of the Lyapunov function and is defined as follows:

$$\begin{aligned} \tilde{A}V(x, \alpha) = & \lim_{h \to 0} \frac{1}{h} \Big[E\Big\{ V(x(t+h), \xi(t+h)) | x(t), \xi(t) \\ & = \alpha \Big\} - V(x(t), \xi(t) = \alpha) \Big] \end{aligned} \tag{25}$$

The weak infinitesimal operator is then given by:

$$\begin{aligned} \tilde{A}V(x, \alpha) = & x'(t) \Big\{ [A(\alpha) - B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)]'K(\alpha) \\ & + K(\alpha)[A(\alpha) - B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)] + \sum_{\beta \in B} q_{\alpha\beta}K(\beta) \Big\} x(t) \\ & + x'(t)K(\alpha)B(\alpha)[e(\alpha, t) + \Delta u(\alpha, t)] \\ & + [e(\alpha, t) + \Delta u(\alpha, t)]'B'(\alpha)K(\alpha)x(t) \end{aligned} \tag{26}$$

Using the form of the control law (23) and assumptions regarding the bounds imposed on the uncertainty, the last term in (26) can be estimated as follows:

$$[e(\alpha, t) + \Delta u(\alpha, t)]'B'(\alpha)K(\alpha)x(t) = \left[e(\alpha, t) + \frac{R(\alpha)u(\alpha, t)}{\|R(\alpha)u(\alpha, t)\|}\rho(\alpha) \right]' B'(\alpha)K(\alpha)x(t)$$

$$= e'(\alpha,t)B'(\alpha)K(\alpha)x(t) - \|B'(\alpha)K(\alpha)x(t)\|\rho(\alpha)$$
$$\leq \|e'(\alpha,t)\|\|B'(\alpha)K(\alpha)x(t)\| - \|B'(\alpha)K(\alpha)x(t)\|\rho(\alpha) \leq 0 \qquad (27)$$

Notice that we have the same results for the term $x'(t)K(\alpha)B(\alpha)[e(\alpha,t)+\Delta u(\alpha,t)]$ which is just the tranpose of the previous one. Thus from (9), (26) and (27) follows:

$$\tilde{A}V(x,\alpha) \leq -x'(t)[Q(\alpha)+K(\alpha)B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)]x(t) \qquad (28)$$

Since

$$V(x,\alpha) \leq \lambda_{max}[K(\alpha)]\|x\|^2 \qquad (29)$$

and

$$x'(t)[Q(\alpha) + K(\alpha)B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)]x(t) \geq \lambda_{min}[Q(\alpha)+K(\alpha)B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)]\|x\|^2 \qquad (30)$$

hold then

$$\frac{\tilde{A}V(x,\alpha)}{V(x,\alpha)} \leq -\frac{\lambda_{min}[Q(\alpha)+K(\alpha)B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)]}{\lambda_{max}(K(\alpha))} \qquad (31)$$

Then by Dynkin's formula and the Bellman-Gronwall lemma for all $\alpha \in \mathcal{B}$, it follows that:

$$E[V(x,\alpha)] \leq exp(-\gamma t)V(x_0,\alpha) \qquad (32)$$

where

$$\gamma = \frac{\lambda_{min}[Q(\alpha)+K(\alpha)B(\alpha)R^{-1}(\alpha)B'(\alpha)K(\alpha)]}{\lambda_{max}[K(\alpha)]} > 0 \qquad (33)$$

Thus

$$\lim_{T\to\infty} E\left[\int_0^T x'(t)K(\alpha)x(t)dt|x_0,\xi(0)=\alpha\right] \leq \frac{1}{\gamma}x_0'K(\alpha)x_0 \qquad (34)$$

Since $K(\alpha) > 0$ for each $\alpha \in \mathcal{B}$, thus

$$\lim_{T\to\infty} E[\int_0^T x'(t)x(t)dt|x_0,\xi(0)=\alpha] \leq x_0' \max_{\alpha} \frac{K(\alpha)}{\gamma\|K(\alpha)\|}x_0 \qquad (35)$$

which proves the theorem.

□

4. Conclusion

In this paper we have proposed a robustification procedure for the JLQ problem in the presence of unknown but bounded uncertainties satisfying the matching conditions. By introducing the additional component into the control law we ensure robustness in the sense of robust stability and guaranteed cost property that may be easily proved. The results have been obtained under usual assumptions of irreductibility of discrete state Markov process $\xi(t)$, perfect observability of the continuous state $x(t)$, and stochastic stabilizability and observability of the nominal model of the system.

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Streszczenie

Liniowe systemy ze skokami markowskimi są obecnie przedmiotem badań, zwłaszcza jeśli chodzi o syntezę sterowania tego typu procesami. To duże zainteresowanie wynika z potrzeby modelowania rzeczywistych procesów produkcyjnych, których struktura i parametry mogą podlegać nagłym zmianom na skutek awarii i remontów, zmiennych oddziaływań środowiska lub połączeń między podsystemami. Dla tej klasy systemów można znaleźć w literaturze rozwiązanie problemu optymalizacji dla kwadratowego wskaźnika jakości zarówno przy skończonym, jak i nieskończonym horyzoncie. W tym ostatnim przypadku znane są warunki konieczne i wystarczające istnienia rozwiązania stacjonarnego zapewniającego skończoną wartość wskaźnika jakości i stabilizującego układ. Rezultaty te były jednak uzyskane przy założeniu pełnej znajomości parametrów modelu i dostępu do zmiennych stanu zarówno ciągłych, jak i dyskretnych. Urealnienie problemu syntezy wymaga wprowadzenia modelu niepewności, np.: poprzez uwzględnienie dopuszczalnych tolerancji macierzy systemowej i wejściowej w równaniach stanu. Przy założeniu znajomości ograniczeń norm zmiennych niepewnych reprezentujących te tolerancje oraz warunku dopasowania proponujemy syntezę prawa sterowania zapewniającego zarówno odporną stabilność, jak i gwarantowany koszt sterowania JLQ dla układu z niepewnością. Procedura uodparniająca polega na uzupełnieniu liniowego prawa sterowania JLQ o nieliniowy ograniczony składnik, którego wielkość jest zależna od ograniczeń na zmienne niepewne.