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Andrzej AMBROZIAK*
Gdansk University of Technology

CHABOCHE MODEL – DEVELOPMENT AND FE APPLICATION

Summary. The aim of the paper is to present the development and numerical application of the elasto-viscoplastic Chaboche model. The detailed description of several variants of Chaboche model is given, with the material parameters specified. The dynamic analysis of the circular steel plate, compared with the experiments, illustrates the practical application of the described model. For the sake of introducing the Chaboche model into the commercial program MSC.Marc system the user-defined subroutine UVSCPL has been applied.

MODEL CHABOCHE'A – ROZWÓJ I NUMERYCZNA APLIKACJA

Streszczenie. Celem artykułu jest prezentacja rozwoju i numerycznej aplikacji sprężysto-lepkoplastycznego modelu Chaboche`a. Przedstawione są szczegółowe opisy kilku wariantów modelu wraz z parametrami materiałowymi. Dynamiczna analiza kołowej płyty stalowej, która jest porównywana z wynikami eksperymentalnymi, ilustruje praktyczne zastosowanie opisywanego modelu. Do wprowadzenia modelu Chaboche`a do komercyjnego programu MSC.Marc użyta jest procedura UVSCPL.

1. Introduction

The constitutive models in material modelling are nowadays very promising branches of science and a highly developed discipline with a great many applications. High progress in computational tools as well as the development of new analytical concepts gives the new perspectives to the evolution of constitutive models. The Chaboche model [4] belongs to these developing group. Besides of the Chaboche law, a lot of elasto-viscoplastic models is being developed, see e.g. [10]. It is indicated that the universal elasto-viscoplastic models does not exist. The Chaboche model has been taken for the present investigation, since it gives good approximation of material behaviour in numerous cases.

*Opiekun naukowy: Dr hab. inż. Paweł Kłosowski, prof. Politechniki Gdańskiej

2. Description of Chaboche model equations

At the beginning it should be noted that the additive decomposition of the strain rate into the elastic and inelastic parts is taken, according to the equation

$$\dot{\mathbf{E}} = \dot{\mathbf{E}}^E + \dot{\mathbf{E}}^I. \tag{1}$$

The relation between the stress and elastic strain rates for isotropic material is assumed as

$$\dot{\mathbf{S}} = \mathbf{D} : \dot{\mathbf{E}}^E = \mathbf{D} : \left(\dot{\mathbf{E}} - \dot{\mathbf{E}}^I\right), \tag{2}$$

where $\mathbf{D}$ is the elastic tensor. The inelastic strain rate in the simple variant of the Chaboche model may be written as follows

$$\dot{\mathbf{E}}^I = 1.5\, \dot{p} \frac{\mathbf{S}' - \mathbf{X}'}{J\left(\mathbf{S}' - \mathbf{X}'\right)}, \tag{3}$$

where the rate of the equivalent plastic strain is described by the formula

$$\dot{p} = \left\langle \frac{J\left(\mathbf{S}' - \mathbf{X}'\right) - R - k}{K} \right\rangle^n. \tag{4}$$

The parameters k, R and K, n are the initial yield stress, isotropic hardening and material constants, respectively. The tensors $\mathbf{S}'$; $\mathbf{X}'$ and scalar $J\left(\mathbf{S}' - \mathbf{X}'\right)$ are the deviatoric parts of stress; back stress tensors and their invariant, respectively. Consequence, the kinematic hardening rate $\dot{\mathbf{X}}$ is defined by

$$\dot{\mathbf{X}} = 1.5\, a\, \dot{\mathbf{E}}^I - c\, \mathbf{X}\, \dot{p}, \tag{5}$$

and isotropic hardening rate $\dot{R}$ is derived from the equation

$$\dot{R} = b\left(R_1 - R\right)\dot{p}. \tag{6}$$

The following parameters for steel at room temperature are given by Kłosowski [7]: E=223000.0 MPa, ν=0.3, k=210.15 MPa, n=9.51, K=14.085 MPas$^{1/n}$, c=38840, a=611700 MPa, b=16.74, R_1=-138.48 MPa.

In the work [6] by Imatani the modification of the kinematic and isotropic hardening equations is proposed. This concept assumed division of the kinematics back stress $\mathbf{X}$ into two parts, by the formula

$$\dot{\mathbf{X}} = \dot{\mathbf{X}}_{(1)} + \dot{\mathbf{X}}_{(2)} \Rightarrow \begin{cases} \dot{\mathbf{X}}_{(1)} = 1.5\, a_1\, \dot{\mathbf{E}}^I - c_1\, \dot{p}\, \mathbf{X}_{(1)} - \beta_1 \left(J_2\left(\mathbf{X}_{(1)}\right)\right)^{r_1 - 1} \mathbf{X}_{(1)} \\ \dot{\mathbf{X}}_{(2)} = 1.5\, a_2\, \dot{\mathbf{E}}^I \end{cases}. \tag{7}$$

The evolution of the isotropic variable is represented by

$$\dot{R} = b\left(R_1 - R\right)\dot{p} - q_1\, R^{q_2}. \tag{8}$$

The material parameters for SUS304 stainless steel at 650°C appearing in the model are as follows [6]: E=145000 MPa, ν=0.3, n=8.03, K=103.0 MPas$^{1/n}$, k=129.0 MPa, c_1=133.0, a_1=11743.9 MPa, $\beta_1 = 8.71 \cdot 10^{-12}$, $r_1 = n = 8.03$, a_2=491.0 MPa, b=25.0, R_1=73.6 MPa, $q_1 = 1.1 \cdot 10^{-18}$, $q_2 = n = 8.03$.

Yaguchi et al. [11] proposed the next variant of Chaboche model. In this case, the rate of the equivalent plastic strain is expressed by the formula

$$\dot{p} = \left\langle \frac{J\left(\mathbf{S}' - \mathbf{X}'\right)}{K} \right\rangle^n. \tag{9}$$

where the kinematic hardening equations is described as follows

$$\dot{\mathbf{X}} = 1.5\, a\, \dot{\mathbf{E}}^I - c\, \mathbf{X}\, \dot{p} - \beta_1 \left(J_2\left(\mathbf{X}\right)\right)^{r_1 - 1} \mathbf{X}, \tag{10}$$

where β_1 and r_1 are material constants. The last term of the above equation depicts the static recovery property of the back stress using the power law function. For example, the material constants for IN738LC at 850°C are given by Yaguchi et al. [11]: E=164000.0 MPa, n=4.75, K=1510.0 MPa, a=175000 MPa, c=500.0, $\beta_1 = 3.54 \cdot 10^{-18}$, r_1=6.08. In the same work, see [11], the authors considered the anisotropic property of deformation and proposed the second rank tensor $\mathbf{Y}$, which takes influence on the back stress, by the formula

$$\dot{\mathbf{X}} = 1.5\, a\, \dot{\mathbf{E}}^I - c\left(\mathbf{X} - \mathbf{Y}\right)\dot{p} - \beta_1 \left(J_2\left(\mathbf{X}\right)\right)^{r_1 - 1} \mathbf{X}. \tag{11}$$

The Eq. (11) is the same expression as the kinematic hardening rule proposed by Chaboche and Nouailhas [3]. The evolution of the variable $\mathbf{Y}$ is expressed as

$$\mathbf{Y} = -\alpha \left(Y_{st} \frac{\mathbf{X}}{J\left(\mathbf{X}\right)} + \mathbf{Y} \right) \left(J\left(\mathbf{X}\right)\right)^{\eta}, \tag{12}$$

where material constant α and Y_{st} specific the evolution rate of the variable $\mathbf{Y}$ and the saturation value of $\mathbf{Y}$. In this case the following material parameters for IN738LC at 950°C are given by Yaguchi et al. [11]: E=164000.0 MPa, n=5.645, K=1156.0 MPa, a=175000 MPa, c=500.0, Y_{st}=100 MPa, r_1=4.275, $\beta_1 = 5.507 \cdot 10^{-14}$, $\alpha = 5.507 \cdot 10^{-15}$.

The following variant of Chaboche model is developed in the framework of thermodynamics, see e.g. Chellapandi and Alwar [5]. The general expressions of this model introduce the modification of the viscoplastic flow rate as

$$\dot{\mathbf{E}}^I = 1.5\, \dot{p} \exp\left(\alpha \left\langle \frac{J(\mathbf{S}' - \mathbf{X}') - R^* - k}{K(R)} \right\rangle^{n+1} \right) \frac{\mathbf{S}' - \mathbf{X}'}{J(\mathbf{S}' - \mathbf{X}')}, \tag{13}$$

where

$$\dot{p} = \left\langle \frac{J(\mathbf{S}' - \mathbf{X}') - R^* - k}{K(R)} \right\rangle^{n}. \tag{14}$$

The non-linear kinematic hardening, like in the paper [6], has been divided into two parts, the particular elements was assigned as

$$\begin{aligned} \dot{\mathbf{X}}_{(1)} &= 1.5\, a_1\, \dot{\mathbf{E}}^I - c_1\, \Phi(p)\, \mathbf{X}_{(1)}\, \dot{p} - \beta_1 \left(J_2\left(\mathbf{X}_{(1)}\right) \right)^{r_1 - 1} \mathbf{X}_{(1)} \\ \dot{\mathbf{X}}_{(2)} &= 1.5\, a_2\, \dot{\mathbf{E}}^I - c_2\, \Phi(p)\, \mathbf{X}_{(2)}\, \dot{p} - \beta_2 \left(J_2\left(\mathbf{X}_{(2)}\right) \right)^{r_2 - 1} \mathbf{X}_{(2)} \end{aligned}, \tag{15}$$

where the function $\Phi(p)$ is defined by

$$\Phi(p) = \phi_S + (1 - \phi_S) \exp(-b\, p). \tag{16}$$

The expression R^* is specified by product of the material constant α_R and isotropic hardening R as

$$R^* = \alpha_R\, R, \tag{17}$$

where the isotropic hardening rate ($R(0) = 0$) is written in the form

$$\dot{R} = b(Q - R)\, \dot{p} + \gamma\, |Q - R|^m \operatorname{sgn}(Q_R - R) \tag{18}$$

and the parameter Q_R is defined as

$$Q_R = Q - Q_R^* \left[1 - \left((Q_{\max} - Q)/Q_{\max} \right)^2 \right]. \tag{19}$$

The material parameter $K(R)$ and the plastic strain memory $\dot{Q}$ are determined as

$$K(R) = K_0 + \alpha_K R, \quad \dot{Q} = 2\,\mu\,(Q_{\max} - Q)\,\dot{q}, \tag{20}$$

where $Q(0) = Q_0$ and $\dot{q}$ is the internal variable corresponding to the radius of the memory surface F, and ξ its centre as

$$F = I\left(\dot{\mathbf{E}}^I - \xi\right) - q \le 0. \tag{21}$$

The material constants for (SS316 LN [5]) stainless steel at $600^\circ C$ are: $\alpha = 2\cdot 10^6$, n=24, k=10, K_0=116, c_1=45, c_2=1300, ϕ_S=0.5, α_K=2, a_1=3600.0, a_2=87750.0, b=12, α_R=0.0, $\beta_1 = 0.5\cdot 10^{-14}$, $\beta_2 = 0.9\cdot 10^{-11}$, $\gamma = 0.2\cdot 10^{-6}$, μ=19, $r_1 = r_2$=4, m=2, η=0.6, $Q_{\max}$=455, Q_0=30, Q_R^*=200. In general, all the 23 parameters, mentioned above, depend of temperature in the range $0^\circ C$ to $600^\circ C$ according to Rive et al. [8].

3. Numerical example

The example of dynamic analysis of the circular steel plate is presented, with the Chaboche model subjected to the description of the behaviour of the steel plate. The results of the numerical calculations are compared with the laboratory test. The experiment was carried out at the Department of General Mechanics of RWTH Aachen, see Fig. 1. The material parameters for Chaboche model were taken from Kłosowski [7]. Basic equations of the model have been presented in the beginning of the Section 2.

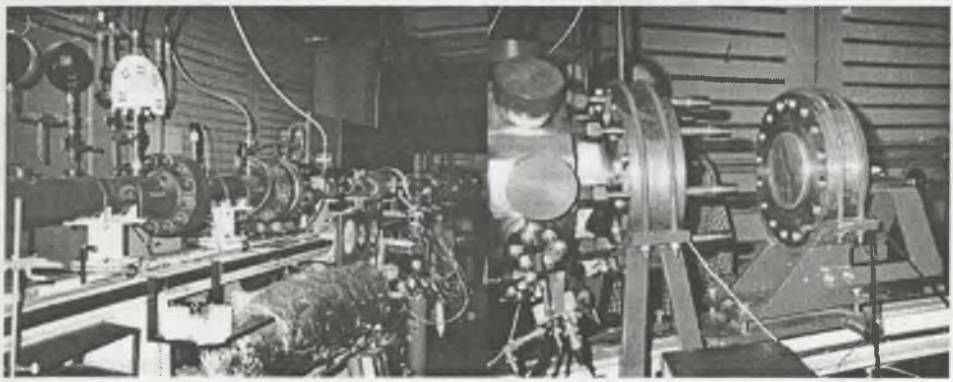
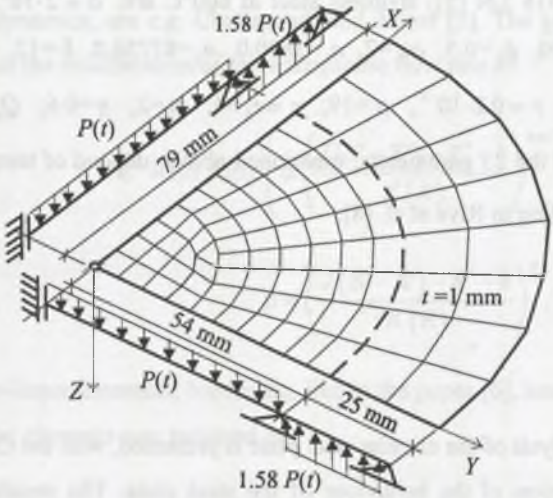


Fig. 1. Experimental stand
Rys. 1. Stanowisko badawcze

The MSC.Marc system has been used in the geometric non-linear numerical calculations with the four-node thin-shell element (Element 139 [9]). To apply the elasto-viscoplastic Chaboche model into the MSC.Marc system the user-defined subroutines UVSCPL [9] were used. Detailed description of applying this subroutine in static analysis with Chaboche model employed has been shown by the author in [1].



Time t [s]	Pressure $P(t)$ [kPa]
0.0000	0.0
0.0032	0.0
0.0035	34.0
0.0041	34.0
0.0045	40.0
0.0060	45.0
0.0113	46.0
0.0142	56.0
0.0160	58.0
0.0240	60.5

Fig. 2. Steel plate visualization – geometry and boundary conditions
Rys. 2. Wizualizacja płyty stalowej – geometria i warunki brzegowe

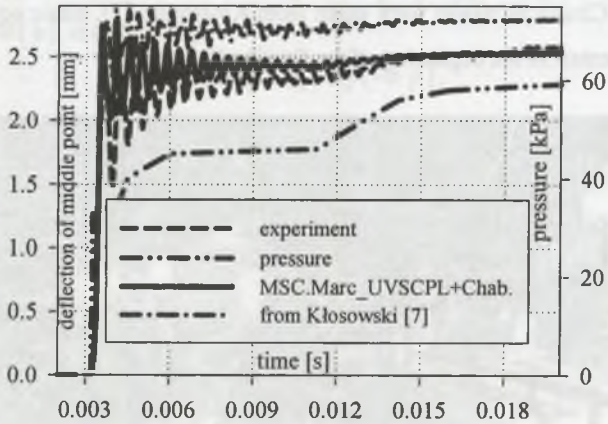


Fig. 3. Inelastic vibrations of the middle of the plate – numerical simulation and experiment
Rys. 3. Niesprężyste drgania środka płyty – symulacja numeryczna i eksperyment

A quarter of the plate with proper symmetry boundary conditions is investigated, see Fig. 2. The value of the experimental pressure $P(t)$ was estimated by the straight-line

approximation. The Newmark method, with the time step $\Delta t = 5 \cdot 10^{-7}$ s, was applied to integrate the non-linear equations of motions

$$\mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{C}\Delta\dot{\mathbf{q}} + (\mathbf{K}_1 + \mathbf{K}_2)\Delta\mathbf{q} = {}^{t+\Delta t}\mathbf{R} - \mathbf{M}\,{}^{t}\ddot{\mathbf{q}} - \mathbf{C}\,{}^{t}\dot{\mathbf{q}} - {}^{t}\mathbf{Q}. \tag{22}$$

The calculations were performed assuming the proportional damping matrix with the Rayleigh damping multipliers $\alpha = 3.46 \cdot 10^{-6}$ (stiffness matrix multiplier) and $\beta = 27.32$ (mass matrix multiplier). The concept of specifying these multipliers is proposed in the work [2], its the author used Bodner-Partom model in the viscoplastic analysis of damped vibrations of circular plate.

The result of the inelastic vibrations of the circular plate is given in Fig. 3. The numerical calculation is compared with the FE analysis, performed by Kłosowski [7] and with the experimental tests. Good agreement of the deflection of the middle point of the plate in time domain has been obtain. It should be noted that Kłosowski [7] in his work used the nine-node isoparametric plate elements with different damping assumption. The simple Rayleigh damping model applied to the calculations and the validity of material parameters may explain small differences, obtained between the numerical analysis and experimental results.

4. Conclusions

In this paper the compact review of the several variants of the Chaboche model is presented with the examples of the material parameters. In the dynamic calculations the UVSCPL procedure was used, with the equations of Chaboche model described the inelastic behaviour of steel. The author showed how the Chaboche model can be applied into the MSC.Marc commercial code. The proposed FE procedure is open and flexible and may be implemented in many industrial applications.

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