ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2007
Seria: BUDOWNICTWO z. 112 Nr kol. 1763

Łukasz PYRZOWSKI*
Gdansk University of Technology

IZOTROPIC DAMAGE IN VISCOPLASTIC FLOW CONDITIONS

Summary. The damage mechanics is a very important branches of solid mechanics. Although it is still developing, it has already been applied to many engineering problems. This paper includes information about the viscoplastic type of constitutive modeling, the presentation of mechanical representation of damage by variable D, effective stress concept, equations of the chosen damage model and the Chaboche viscoplastic constitutive model including damage effects. Also the identification procedure of damage parameters with practical example and MES applications in the commercial program MSC.Marc is presented.

IZOTROPOWE ZNISZCZENIE W WARUNKACH LEPKOPLASTYCZNEGO PŁYNIĘCIA

Streszczenie. Mechanika zniszczenia jest obecnie jedną z ważniejszych gałęzi mechaniki ciał stałych. Jest to cały czas rozwijająca się dziedzina nauki, która znalazła swoje zastosowania w problemach inżynierskich. W artykule zaprezentowano podstawowe informacje na temat lepkoplastyczności i jej zastosowania, koncepcję mechanicznej reprezentacji zniszczenia przez zmienną D, koncepcję naprężeń efektywnych, równania opisujące zniszczenie oraz lepkoplastyczny model Chaboche'a z uwzględnieniem zniszczenia. Ponadto, w artykule przedstawiono procedurę identyfikacji parametrów zniszczenia dla przyjętej koncepcji wraz z przykładem praktycznym oraz przykład prezentujący wykorzystanie przedstawionych praw w MES w programie MSC.Marc.

1. Viscoplastic modeling coupled with damage, basic information, applications

The viscoplastic theory describes rheological phenomenon, which depends on time and which effects does not vanish after reloading [1]. For such materials as metals and alloys, which are the main subject of this paper, it corresponds to mechanisms linked to the movement of dislocations in grains witch superposed effects of intercrystalline gliding. These

*Opiekun naukowy: Dr hab. inż. Paweł KŁOSOWSKI, prof. ndzw. Politechniki Gdańskiej.

mechanisms begin to arise as soon as the temperature is greater then one third of absolute melting temperature.

Characteristic tests in viscoplastic problem domain are hardening, creep, relaxation and cyclic tests. The hardening test with a constant strain rate (Fig. 1 a) shows considerable influence of the strain rate on the test result, as the strain rate is higher, the plastic yield limit and maximum stress rise.

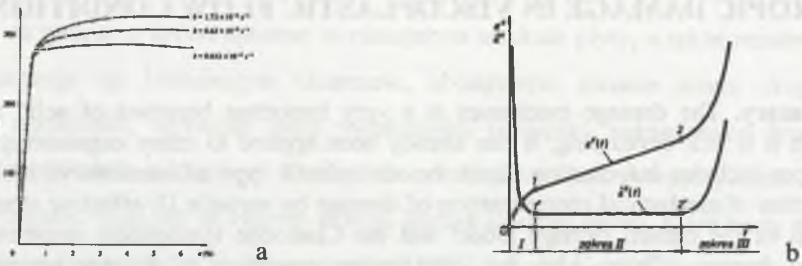

Fig. 1. a) Hardening test with different strain rates AU4G, 200°C [1]; b) creep curve and creep velocity curve at constant strain state [2]

Rys. 1. a) Test wzmocnienia przy różnych prędkościach odkształceń AU4G, 200°C [1]; b) krzywa pełzania i krzywa prędkości pełzania przy stałym obciążeniu [2]

The creep test (Fig. 1 b) shows substantial dependence deformation from time. We can divide such a test into three phases. The first, is one in which, strains are nonlinearly and the hardening of a material leads to decrease of the creep rate, which is initially very high. In the second one, the rate of flow stabilizes and is almost constant, strain change linearly. During the last one the rate of flow rises, the damage phenomenon occurs, strain accrues nonlinearly up to rapture. Among these three flow phases especially the third one is main object of this paper.

2. Mechanical representation of damage

Damage in metals is mainly the process of the initiation and growth of microcracks and cavities in material's structure. The first, who introduce a continuous variable related to the density of such defects is Kachanov in 1958 [3].

The variable D_n in any material point of damaged body is defined by equation [4]:

$$D_n = \frac{S_D}{S}, \tag{1}$$

where S is the overall cross-section area of undamaged material, defined by the normal n, S_D is the effective area of the intersections of all microcracks or cavities, which interfere with analyzed section in damaged material. Assuming isotropy of damage, which means uniform distribution of cracks and cavities in all directions, D_n does not depend upon n and becomes to be a scalar value D.

Using the concept of effective stress [5], the effective stress $\tilde{\sigma}$ can be specified by the damage variable D:

$$\tilde{\sigma} = \frac{\sigma}{1-D}. \tag{2}$$

According the hypothesis of strain equivalence [5] each constitutive equation of a damaged material can be derived in the same way as for a undamaged material with replacing the stress components by their effective values.

3. Constitutive equations for isotropic damage

The constitutive equations of an isotropic material used in this paper are based on additive decomposition of the strain rate into its elastic $\dot{\varepsilon}^E$ and plastic $\dot{\varepsilon}^I$ parts:

$$\dot{\varepsilon} = \dot{\varepsilon}^E + \dot{\varepsilon}^I. \tag{3}$$

The elastic part of the free energy function can be expressed as follows [4]:

$$\psi^E = \frac{(1-D)}{2\rho}(a_{ijkl}\varepsilon_{ij}^E\varepsilon_{kl}^E), \tag{4}$$

where ρ is density of a material, a_{ijkl} are components of the elasticity tensor.

According to the strain equivalence principle the stress component can be calculated as:

$$\sigma_{ij} = \rho\frac{\partial \psi^E}{\partial \varepsilon_{ij}^E} = (1-D)a_{ijkl}\varepsilon_{kl}^E. \tag{5}$$

The elastic strain components can be calculated by reversing Equation (5):

$$\varepsilon_{ij}^E = \frac{1+\nu}{E}\frac{\sigma_{ij}}{1-D} - \frac{\nu}{E}\frac{\sigma_{kk}}{1-D}\delta_{ij}. \tag{6}$$

The damage strain energy is defined as follows [4]:

$$Y = \rho\frac{\partial \psi^E}{\partial D} = -\frac{1}{2}a_{ijkl}\varepsilon_{ij}^E\varepsilon_{kl}^E. \tag{7}$$

Considering the definition of the elastic strain energy density W_e:

$$dW_e = \sigma_{ij}d\varepsilon_{ij}^E \tag{8}$$

substituting Equation (6) and then integrating, we obtain the result:

$$W_e = \frac{1}{2}(1-D)a_{ijkl}\varepsilon_{ij}^E\varepsilon_{kl}^E, \tag{9}$$

which helps establish the relation between the damage strain energy Y and the density of elastic strain energy W_e:

$$-Y = \frac{W_e}{1-D}. \tag{10}$$

Splitting the density of the elastic strain energy W_e into two parts: the shear energy part and the hydrostatic energy part finely we get the function Y [4]:

$$-Y = \frac{\sigma_{eq}^2}{2(1-D)^2E}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right], \tag{11}$$

where ν is the Poisson's ratio, E is the Young's modulus of undamaged material, σ_{eq} is the Huber-Misses equivalent stress and σ_H is the hydrostatic stress expressed as:

$$\sigma_{eq} = \left[\frac{3}{2}(\sigma_{ij}-\sigma_H)(\sigma_{ij}-\sigma_H)\right]^{1/2}, \quad \sigma_H = \frac{1}{3}\sigma_{kk} \tag{12}$$

Through the constitutive equations my be derived from the dissipation function φ:

$$\dot{\varepsilon}_{ij}^I = \frac{\partial\varphi}{\partial\sigma_{ij}}, \quad \dot{D} = -\frac{\partial\varphi}{\partial Y}. \tag{13}$$

4. Isotropic damage in Chaboche model

In the present study the isotropic damage model proposed by Lemaitre [6] is used. The dissipation function for ductile plastic damage is written as a power function of Y and linear with respect to accumulated plastic strain rate $\dot{p}$:

$$\varphi = \frac{S}{s+1}\left(\frac{-Y}{S}\right)^{s+1}\dot{p}, \quad \dot{p} = \sqrt{\frac{2}{3}\varepsilon_{ij}^I\varepsilon_{ij}^I}, \tag{14}$$

where S and s are the damage material parameters.

The damage function is derived from Equation (13):

$$\dot{D} = \left(-\frac{Y}{S}\right)^s\dot{p}. \tag{15}$$

Among many viscoplastic constitutive models the Chaboche model has been chosen. In this model the inelastic strain rate has the following form [1]:

$$\dot{\varepsilon}^I = \frac{3}{2}\dot{p}\frac{\sigma' - X'}{J(\sigma' - X')}, \quad J(a_{ij}) = \sqrt{\tfrac{3}{2}a^{ij}a_{ij}}, \tag{16}$$

where σ' and X' are the deviatoric parts of the stress and kinematic hardening tensors, respectively.

The accumulated plastic strain rate is given by:

$$\dot{p} = \left\langle \frac{(J(\sigma' - X')/(1-D)) - R - k}{K} \right\rangle^n, \tag{17}$$

where k is the initial yield stress, K and n are the viscous material parameters. The angle brackets $\langle x \rangle$ are referred to the McCauley brackets: $\langle x \rangle = \frac{1}{2}(x + |x|)$.

The kinematic hardening tensor X and the isotropic hardening scalar R are expressed as:

$$\dot{X} = \frac{2}{3} a \dot{\varepsilon}^I - cX\dot{p}, \quad \dot{R} = b(R_1 - R)\dot{p}, \tag{18}$$

where a, b, R_1 are hardening material parameters.

5. Identification of damage material parameters

The easiest method of damage material parameters identification is to carry out suitable quantity of uniaxial tensile experiments. The single experiment consists of a set of constant strain rate cycles with constant amplitude of strain to obtain a weakening of the elastic modulus. This method assumes homogeneous character of damage.

According to hypothesis of effective stress the elastic strain can be expressed as follows [4]:

$$\varepsilon^E = \frac{\sigma}{E(1-D)}. \tag{19}$$

Using Equation (19) it is possible to obtain the effective elastic modulus $\bar{E}$:

$$\bar{E} = E(1-D). \tag{20}$$

Reversing Equation (20) the damage variable, depending from initial and effective elastic modulus emerges:

$$D = 1 - \frac{\bar{E}}{E}. \tag{21}$$

The next step of identification is deriving the equations, which allow to calculate damage material parameters S and s.

As the result of experiments, for each cycle, are recorded: elastic modulus, plastic strain and maximum value of the stress.

The damage function (15) and the damage strain energy function (11) in uniaxial loading conditions are expressed as:

$$\dot{D} = \left(\frac{-Y}{S}\right)^{s} \dot{p} = \left(\frac{-Y}{S}\right)^{s} \dot{\varepsilon}_{pl}, \tag{22}$$

$$-Y = \frac{1}{2(1-D)^2 E}\left(\frac{2}{3}(1+\nu)\sigma_{eq}^2 + 3(1-2\nu)\sigma_H^2\right) = \frac{\sigma^2}{2(1-D)^2 E}, \tag{23}$$

then substituting Equation (23) to Equation (22):

$$\dot{D} = \frac{dD}{dt} = \left(\frac{\sigma^2}{2E(1-D)^2 S}\right)^{s} \frac{d\varepsilon_{pl}}{dt} \tag{24}$$

and reducing dt the following formula is given:

$$\frac{dD}{d\varepsilon_{pl}} = \left(\frac{\sigma^2}{2E(1-D)^2 S}\right)^{s}. \tag{25}$$

In the following step the function, which will approximate the function $D(\varepsilon_{pl})$ is selected:

$$D(\varepsilon_{pl}) = D_0 + a\left(1 - \exp(-b\varepsilon_{pl})\right) \tag{26}$$

and then, on the basis of experiments data, using the least squares method, the a and b parameters are calculated.

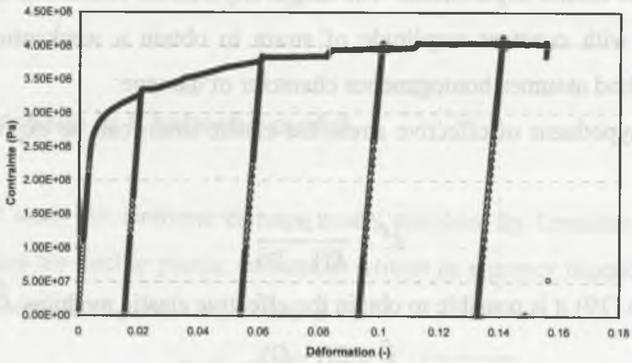

Fig. 2. Daudonet's cyclic tension test – stress vs. strain – Aluminium A12017 B [7]
Rys. 2. Próba cyklicznego rozciągania Daudonet'a – wykres naprężeń do odkształceń – Aluminium A12017B [7]

Table 1

Setting up experiments results and calculation of the damage variable D

A12017	początkowy	cykl 1	cykl 2	cykl 3	cykl 4
Moduł Younga E [GPa]	71,5	67	60,4	55,6	52
Odkształcenia plastyczne [-]		0,0154	0,0541	0,0933	0,1327
Naprężenia [MPa]		335,18	387,1	406,8	415,07
D	0	0,06294	0,15524	0,22238	0,27273

The last step of identification is comparison of the derivative function (26) with inelastic strain with formula (25) and using the least squares method for approximation of the damage material parameters *S* and *s*:

$$\frac{dD(\varepsilon_{pl})}{d\varepsilon_{pl}} = ab\exp(-b\varepsilon_{pl}) = \left(\frac{\sigma^2}{2E(1-D)^2 S}\right)^s \tag{27}$$

The approximation of the function $D(\varepsilon_{pl})$ gives: *a*= 0,3876; *b*= 8,0726, that leads to: $\frac{dD(\varepsilon_{pl})}{d\varepsilon_{pl}} = 3{,}1289\exp(-8{,}0726\cdot\varepsilon_{pl})$. Finally, using formula (27) the damage material parameters, which were sought, are: S = 2,9230; s = -0,877.

6. Practical example of isotropic damage implementation in MES

Numerical calculations presented in this paper has been performed using MSC.Marc system, which great advantage is possibility user subroutines application. The viscoplastic Chaboche model with damage has been applied to the program using UVSCPL subroutine.

In the presented example a hinged square plate of 5 m length and initial thickness 0,01 m is considered. The plate is loaded by uniformly distributed pressure increasing linearly from 0 to p_{max}=1,5MPa during 0,01 s, then the loading remains on the surface of the plate. In calculation the symmetry of the plate has been used.

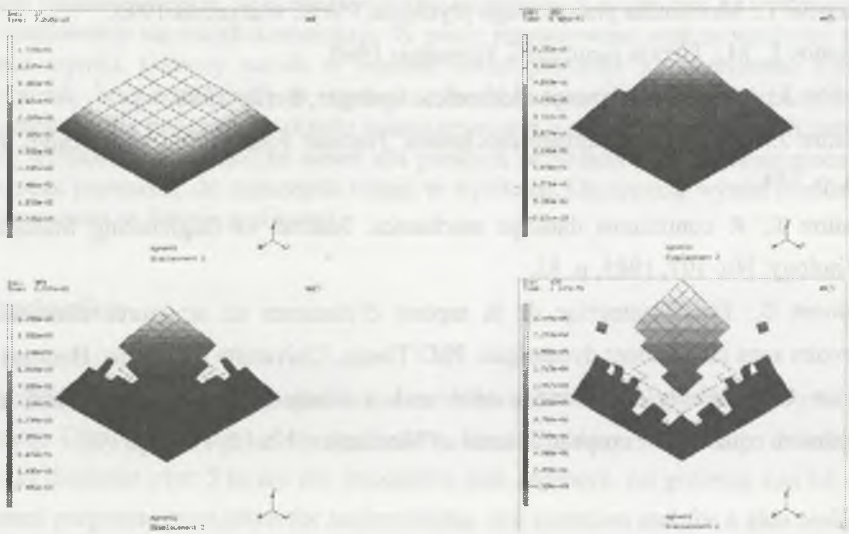

Fig. 3. Vertical displacement of plate ($t_1 = 7.2\cdot10^{-3}$s, $t_2 = 9.52\cdot10^{-2}$s, $t_3 = 1.05\cdot10^{-1}$s, $t_4 = 1.15\cdot10^{-1}$s)
Rys. 3. Przemieszczenia pionowe płyty ($t_1 = 7.2\cdot10^{-3}$s, $t_2 = 9.52\cdot10^{-2}$s, $t_3 = 1.05\cdot10^{-1}$s, $t_4 = 1.15\cdot10^{-1}$s)

The following material parameters were taken (INCO alloy in temp. 627°C) for the Chaboche model [7]: E=162GPa, n=0.3, k=501MPa, b=15, R_l=165.4MPa, a=80GPa, c=200, n=2.4, $K=12790(\mathrm{MPa{\cdot}s})^{1/n}$, S=4.48MPa, s=3 and the mass density r=7900kg/m^3.

As the results, screenshots of vertical displacement for the plate's quarter in four time moments are presented in Fig. 3.

In calculations the four-node thin-shell elements (Element 139) divided into five layers were used. Dynamic, geometrically non-linear analysis using the Newmark integration algorithm (a time step dt = $2{,}66{\cdot}10^{-4}$s) has been performed. Additionally, the re-meshing feature with the equivalent plastic strain criteria (the value of 0,05) to subdivide elements into four parts and deactivation of elements, which has been performed if the value of damage parameter D at all integration points of the element is greater than 0,5 were used.

Acknowledgements

Calculations presented in the paper were performed at the Academic Computer Centre in Gdansk (TASK).

LITERATURE

1. Chaboche J. L., Lemaitre J.: Mechanics of solid materials. Cambridge University Press, Cambridge 1990.
2. Bednarski T.: Mechanika plastycznego płynięcia. PWN, Warszawa 1995.
3. Kachanov L. M.: Tieoria pazuczesti. Fizmatgiz 1960.
4. Lemaitre J.: A course on damage mechanics. Springer, Berlin 1996.
5. Lemaitre J.: How to use damage mechanics. Nuclear Engineering and Design, No. 80, 1984, p. 234.
6. Lemaitre J.: A continuous damage mechanics. Journal of Engineering Materials and Technology, No. 107, 1985, p. 83.
7. Daudonet B.: Etude numeriqe de la rapture d'elements de structures constituant des reservoirs sous chargement dynamique. PhD Thesis, Universite d'Orleans, Bourges 2006.
8. Ammar G., Dufailly J.: Identification and validation of viscoplastic and damage constitutive equations. European Journal of Mechanics, No.12, 1993, p. 197.

Recenzent: Prof. dr hab. inż. Jan Kubik