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APPROXIMATE CONTROLLABILITY OF CERTAIN DISTRIBUTED SYSTEM

Summary. In the paper the analysis of mathematical models describing a class of elastic mechanical system is presented. The mathematical models of such systems have the form of partial differential equations of higher orders. On the base of the theory of linear, unbounded, differential operators it was made transformation from partial differential equation describing the system to infinite dimensional, linear, abstract equation of state in Hilbert space. On the ground of the general conditions for approximate controllability it was formulated necessary and sufficient conditions for approximate controllability of investigated system. Finally some general remarks concerning controllability of distributed parameter systems are presented.

APROKSYMACYJNA STEROWALNOŚĆ PEWNEGO UKŁADU O PARAMETRACH ROZŁOŻONYCH

Streszczenie. W ramach pracy dokonano analizy modeli opisujacych pewną klasę elastycznych układów mechanicznych. Modele matematyczne takich układów mają postać równań różniczkowych cząstkowych wyższych rzędów. Na podstawie teorii liniowych, nieograniczonych operatorów różniczkowych dokonano przejścia do opisu układu w postaci nieskończenie wymiarowego, liniowego, abstrakcyjnego równania stanu w odpowiedniej przestrzeni Hilberta. Na podstawie ogólnych warunków aproksymacyjnej sterowalności sformułowano warunki konieczne i wystarczające aproksymacyjnej sterowalności dla badanego układu. W ostatniej części pracy przedstawiono kilka uwag ogólnych, dotyczących sterowalności układów o parametrach rozłożonych.

1. Introduction

We consider in this paper certain type of control system described by linear, partial, differential equation. We treat the case when the control inputs appear in partial differential equation as distributed inputs. The paper is devoted to so called approximate controllability of certain distributed system. Approximate controllability generally means, that the system can be steered from an arbitrary initial state to an arbitrary small vicinity of a final state.

Using the spectral theory for linear, unbounded, differential, operators the mathematical model describing the dynamic behavior of the control system is transformed into first order abstract, evolution equation in an appropriate Hilbert space. To this equation are applied conditions for approximate controllability. Finally the necessary and sufficient conditions for approximate contollability of investigated system are presented.

2. System description

Let us consider dynamical control system described by the linear partial differential equation:

$$\frac{\partial^2 u(t,x)}{\partial t^2}+\frac{\partial^4 u(t,x)}{\partial x^4}+2a\frac{\partial^5 u(t,x)}{\partial x^4 \partial t}-2b\frac{\partial^3 u(t,x)}{\partial x^2 \partial t}-g\frac{\partial^2 u(t,x)}{\partial x^2}=b(x)\,f(t) \tag{2.1}$$

for $x\in(0,l)$, $t>0$

with initial conditions

$$u(0,x)=u_0(x)$$
$$\frac{\partial u(0,x)}{\partial t}=u_1(x) \qquad for\ x\in(0,l) \tag{2.2}$$

and boundary conditions

$$u(t,0)=\frac{\partial^2 u(t,0)}{\partial x^2}=0$$
$$u(t,l)=\frac{\partial^2 u(t,l)}{\partial x^2}=0, \qquad for\ t>0 \tag{2.3}$$

where : $b(x)=[b_1(x)\ b_2(x)\ \dots\ b_p(x)]$ and $f(t)=[f_1(t)\ f_2(t)\ \dots\ f_p(t)]^T$, $f_i(t)$, $i=1,\dots,p$ are Hölder continuous control functions, $\alpha>0$, $\beta\in[0,1/2)$, $\gamma\in R$.

Equation (2.1) describes the transverse motion in the X-Y plane of an elastic beam which occupies the interval $[0,l]$ of the X axis in the reference state. The function u(t,x) denotes the displacement from the reference state to the Y- direction at position $x\in(0,l)$ and time $t>0$. The third and the fourth term of (2.1) represent internal structural damping [3],[6],[8] and the fifth term represents the effect of axial force on the beam [1],[14]. The boundary conditions correspond to hinged ends of the beam.

3. Preliminary results

Let $H=L^2(0,l)$ be the complex Hibert space of all square integrable functions on the open interval $(0,l)$ with the inner product and the norm :

$$\langle f,g\rangle_H=\int_0^l f(x)\overline{g(x)}dx \quad for\, f,g\in H, \tag{3.1}$$

$$\|f\|_H=\left(\int_0^l |f(x)|^2 dx\right)^{\frac{1}{2}} \quad for\, f\in H. \tag{3.2}$$

We define a linear, unbounded, differential operator $A : D(A)\subset H\to H$ by

$$Au(x)=\frac{\partial^4 u(x)}{\partial x^4}=u''''(x) \quad , \quad u\in D(A) \tag{3.3}$$

with domain

$$D(A)=\left\{u(x)\in H^4(0,l):\int_0^l u^2(x)dx<+\infty,\right.$$
$$u(0)=u''(0)=0, \tag{3.4}$$
$$\left.u(l)=u''(l)=0\right\}$$

where $H^4(0,l)$ denotes fourth order Sobolev space on interval $(0,l)$.

The linear operator A has the following properties [5],[9] :

a) A is self-adjoint and positive definite operator with dense domain D(A) in H,

b) A has the spectral representation

$$Au(x)=\sum_{i=1}^{+\infty}\lambda_i\langle u,\phi_i\rangle_H\,\phi_i(x)\ , \quad u\in D(A),$$

where λ, ϕ are eigenvalues and eigenfunctions of A, respectively, and

$$\lambda_i=\left(\frac{\pi i}{l}\right)^4 \quad , \quad \phi_i(x)=\left(\frac{2}{l}\right)^{\frac{1}{2}}\sin\left(\frac{\pi i x}{l}\right) \quad for\ x\in(0,l) \tag{3.5}$$

c) The set of eigenfunctions of A $\{\phi_i;\ i\in N\}$ forms the complete, orthonormal system in H

d) A has only pure discrete point spectrum $\sigma(A)$ consisting entirely with eigenvalues:

$$\sigma(A)=\{\lambda_i\ ;\ i\in N\}$$

e) There exists compact inverse A^{-1} and the resolvent of A is compact

f) For the operator A fractional power A^α, $\alpha\in(0,1)$ can be defined by

$$A^{\alpha}u=\sum_{i=1}^{+\infty}\lambda_i^{\alpha}\left\langle u,\phi_i\right\rangle_H\phi_i \quad , \quad u\in D(A^{\alpha})$$

$$D(A^{\alpha})=\left\{u\in H:\sum_{i=1}^{+\infty}\lambda_i^{2\alpha}\left|\left\langle u,\phi_i\right\rangle_H\right|^2<+\infty\right\},$$

which is also self-adjoint and positive definite operator with dense domain in H.

Particularly, $A^{1/2}$ can be definite by

$$A^{\frac{1}{2}}u=-\frac{\partial^2 u}{\partial x^2}=-u''$$

$$D(A^{1/2})=\{u\in H^2(0,l):u(0)=u(l)=0\},$$

where $H^2(0,l)$ denotes the second order Sobolev space on interval $(0,l)$ and $D(A)\subset D(A^{1/2})$. It should be stressed, that fractional power of A may have quite different nature. In spite of A is the differential operator its fractional power generally may be quite different, nondifferential operator.

Applying operator A to partial differential equation (2.1) with boundary conditions (2.3) we obtain the following abstract, ordinary second order differential equation with respect to t in Hilbert space H:

$$\ddot{u}(t)+Au(t)+2\alpha A\dot{u}(t)+2\beta A^{\frac{1}{2}}\dot{u}(t)+\gamma A^{\frac{1}{2}}u(t)=bf(t) \quad ,t>0, \tag{3.6}$$

where: $\ddot{u}(t), \dot{u}(t), u(t)\in H$.

4. First order equation

The purpose of this section is to transform the second order abstract differential equation (3.6) to first order one by using procedure proposed by Sakawa [8] and developed by Kunimatsu, Ito [5]. We make two additional assumptions on the coefficients of (2.1)

$$\alpha\lambda_1+\beta\lambda_1^{\frac{1}{2}}<(1-\beta^2)/2\alpha$$

$$\lambda_i\neq(1-\beta)^2/\alpha^2 \quad , \quad i=1,2,\ldots$$

Let Hilbert space **X** be the product space **HxH** with inner product

$$\left\langle [u_1,v_1]^T,[u_2,v_2]^T\right\rangle_X=\left\langle u_1,u_2\right\rangle_H+\left\langle v_1,v_2\right\rangle_H \quad \text{for } u_1, u_2, v_1, v_2\in H$$

and corresponding norm $\|\circ\|_X$.

We can convert second order abstract equation (3.6) into the following first order one in Hilbert space **X**= **H**x**H**

$$\zeta(t) = \Lambda \zeta(t) + Bf(t) \quad , t > 0, \tag{4.1}$$

where:

$$\zeta(t) = \begin{bmatrix} \xi(t) \\ \mu(t) \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} -A^{+} - T & -T \\ T & -A^{-} + T \end{bmatrix} \qquad \Lambda: D(\Lambda) = D(A) \times D(A) \subset X \to X$$

$$B = \begin{bmatrix} g(A)^{-1} b \\ -g(A)^{-1} b \end{bmatrix} \qquad B: D(B) = R^{p} \to X$$

Components of state vector ζ have the form:

$$\xi(t) = u(t) + v(t), \; \mu(t) = u(t) - v(t)$$

$$v(t) = g(A)^{-1}\left\{\dot{u}(t) + \alpha A u(t) + \beta A^{\frac{1}{2}} u(t)\right\} \quad , t > 0, u(t) \in D(A).$$

Components of state operator Λ are expressed by:

$$A^{\pm} = \alpha A + \beta A^{\frac{1}{2}} \mp g(A) \quad A^{\pm}: D(A^{\pm}) = D(A) \subset H \to H$$

$$T = \frac{\gamma}{2} g(A)^{-1} A^{\frac{1}{2}} \qquad T: D(T) = D(A^{\frac{1}{2}}) \subset H \to H.$$

Components of control operator B are given by:

$$g(A)u = \sum_{i=1}^{+\infty} g(\lambda_i) \langle u, \phi_i \rangle_H \, \phi_i \qquad D(g(A)) = \left\{ u \in H : \sum_{i=1}^{+\infty} \left| g(\lambda_i) \langle u, \phi_i \rangle_H \right|^2 < +\infty \right\} = D(A) \tag{4.2}$$

$$g(\lambda) = \left\{ \left(\alpha \lambda + \beta \lambda^{\frac{1}{2}} \right)^2 - \lambda \right\}^{\frac{1}{2}} \quad , \lambda > 0$$

$$g(A)^{-1} u = \sum_{i=1}^{+\infty} g(\lambda_i)^{-1} \langle u, \phi_i \rangle_H \, \phi_i \qquad D\left(g(A)^{-1}\right) = \left\{ u \in H : \sum_{i=1}^{+\infty} \left| g(\lambda_i)^{-1} \langle u, \phi_i \rangle_H \right|^2 < +\infty \right\} = H. \tag{4.3}$$

All of introduced operators (Λ, B, $A^{\pm}$, T, g(A), $g(A)^{-1}$) are of course linear. Moreover, operators B, $g(A)^{-1}$ are bounded.(for details see [5])

Equation (4.1) with initial condition

$$\zeta_0 = \zeta(0) = \left[\xi(0), \mu(0)\right]^T = \left[\xi_0, \mu_0\right]^T$$

can be interpreted as the state equation of dynamical system (2.1). In consideration of the fact, that investigated system (2.1) is described by partial differential equation it is infinite dimensional system. As a result of that state equation (4.1) describing system (2.1) is also infinite dimensional. State matrix which occurs in state equations describing finite dimensional systems, in this case, is replaced with linear, unbounded, differential state operator Λ definite in an appropriate Hilbert space X.

The state operator Λ has the following properties (Kunimatsu, Ito [5])

a) Λ has the spectral representation

$$\Lambda\zeta = \sum_{i=1}^{+\infty}\left\{a_i \nu_i^+ \begin{bmatrix}\phi_i \\ p_i\phi_i\end{bmatrix} + b_i \nu_i^- \begin{bmatrix}p_i\phi_i \\ \phi_i\end{bmatrix}\right\}, \quad \zeta \in D(\Lambda),$$

where $\nu_i^\pm$, $\{\psi_i^+=[\phi_i, p_i\phi_i]^T, \psi_i^-=[p_i\phi_i, \phi_i]^T ; i\in N\}$ are eigenvalues and eigenfunctions of Λ, respectively, and

$$\nu_i^\pm = -\left(\alpha\lambda_i + \beta\lambda_i^{\frac{1}{2}}\right) \pm h(\lambda_i)$$

$$p_i = \frac{\gamma\lambda_i^{\frac{1}{2}}}{2g(\lambda_i)\{g(\lambda_i)+h(\lambda_i)\} - \gamma\lambda_i^{\frac{1}{2}}} \tag{4.4}$$

Function $h(\lambda_i)$ has the form:

$$h(\lambda_i) = \left\{\left(\alpha\lambda_i + \beta\lambda_i^{\frac{1}{2}}\right)^2 - \lambda_i - \gamma\lambda_i^{\frac{1}{2}}\right\}^{\frac{1}{2}}.$$

Coefficients a_i, b_i are given by:

$$a_i = \frac{\left\langle \zeta, [\phi_i, p_i\phi_i]^T \right\rangle_X}{\left\| [\phi_i, p_i\phi_i]^T \right\|_X^2}, \quad b_i = \frac{\left\langle \zeta, [p_i\phi_i, \phi_i]^T \right\rangle_X}{\left\| [p_i\phi_i, \phi_i]^T \right\|_X^2}.$$

b) The set of eigenfunctions of Λ $\{\psi_i^+=[\phi_i, p_i\phi_i]^T, \psi_i^-=[p_i\phi_i, \phi_i]^T ; i\in N\}$ forms the complete system in X.

c) Λ has only pure discrete point spectrum σ(Λ) consisting entirely with eigenvalues:

$$(\Lambda) = \{\nu_i^+, \nu_i^-; i \in N\} \cup \{-\tfrac{1}{2\alpha}\}$$

$$\lim_{i\to+\infty} \nu_i^+ = -\frac{1}{2\alpha}$$

$$\lim_{i\to+\infty} \nu_i^- = -\infty$$

d) Operator Λ is the infinitesimal generator of the analytic semigroup S(t): X→X, t>0, represented by formula

$$S(t)\zeta = \sum_{i=1}^{+\infty}\left\{a_i \exp(\nu_i^+)\begin{bmatrix}\phi_i \\ p_i\phi_i\end{bmatrix} + b_i \exp(\nu_i^-)\begin{bmatrix}p_i\phi_i \\ \phi_i\end{bmatrix}\right\}, \quad \zeta \in X.$$

Kunimatsu and Ito [5] proofed the following theorem:

Theorem 4.1

The Cauchy problem associated with (4.1) has for each $t_1>0$ the unique global solution $\zeta(t):[0,t_1]\rightarrow X$, which satisfies the following conditions:

$$\zeta(t)\in C^1([0,t_1],X),$$

$$\zeta(t)\in D(\Lambda) \text{ for any } t\in(0,t_1].$$

5. Basic definitions

We may define many different notions of controllability for distributed parameter systems. We shall concentrate on so called approximate controllability. Let us introduce the attainable set for dynamical system (4.1) defined at time t>0 from zero initial conditions by formula:

$$K_t = \left\{ \zeta = \int_0^t S(t-\tau)Bf(\tau)d\tau : f \in F, i = 1,\dots,p \right\}, \tag{5.1}$$

where F denotes the set of admissible controls. Moreover the attainable set for dynamical system (4.1) is defined as:

$$K_\infty = \bigcup_{t>0} K_t \tag{5.2}$$

Definition 5.1

Dynamical system (4.1) is said to be approximately conrollable in the time interval [0,T] in the set of admissible controls F if

$$\overline{K}_T = X = H \times H, \tag{5.3}$$

where $\overline{K}$ denotes the closure of K.

Definition 5.2

Dynamical system (4.1) is said to be approximately conrollable in the set of admissible controls F if

$$\overline{K}_\infty = X = H \times H. \tag{5.4}$$

Generally approximate controllability in [0,T] is stronger notion than approximate controllability.

On the basis of definition 5.1 we can formulate more understandable corollary concerning approximate controllability in interval [0,T].

Corollary 5.1

The dynamical system (4.1) is said to be approximately controllable from any initial state $\zeta \in X$ to any final state $\zeta_k \in X$ in interval [0,T] if for any $\varepsilon > 0$ there exists control $f \in F$, such that the solution of (4.1) satisfies

$$\left\| \zeta(T) - \zeta_k \right\|_X < \varepsilon \quad , \quad \varepsilon > 0 .$$

The necessary condition for approximate controllability of (4.1) is

$$\sup_i n_i < \infty ,$$

where n_i is the multiplicity of eigenvalue $\nu_i^{\pm}$ of Λ.

It must be point out that the case if $n_i < \infty$ for all i=1,2,... does not ensure in general that $\sup_i n_i < \infty$.

6. Approximate controllability

First of all we make an important remark. If state operator Λ is the infinitesimal generator of analytic semigroup S(t) (property (d) of Λ) then approximate controllability of dynamical system (4.1) is equivalent to approximate controllability in an arbitrary time interval [0,T].

The necessary and sufficient condition for approximate controllability of investigated system is given by formula:

$$(1-p_i)^2 \left\{ \left(\int_0^l \left(\sum_{j=1}^{+\infty} \left(g(\lambda_j)^{-1} \int_0^l b_1(x) \sin\frac{\pi j x}{l} dx \sin\frac{\pi j x}{l} \right) \sin\frac{\pi i x}{l} dx \right) \right)^2 + \cdots \right.$$

$$\left. \cdots + \left(\int_0^l \left(\sum_{j=1}^{+\infty} \left(g(\lambda_j)^{-1} \int_0^l b_p(x) \sin\frac{\pi j x}{l} dx \sin\frac{\pi j x}{l} \right) \sin\frac{\pi i x}{l} dx \right) \right)^2 \right\} \neq 0 \qquad \text{for all } i = 1,2,\ldots \tag{6.1}$$

Proof

The general necessary and sufficient conditions for approximate controllability have the form [7], [13]:

Theorem 6.1

The dynamical system described by state equation (4.1) is approximately controllable in an arbitrary time interval if and only if

$$rank\,W_i^+ = rank\,W_i^- = n_i \quad , \quad for\ all\ i = 1,2,\ldots, \tag{6.2}$$

where:

$$W_i^+ = \begin{bmatrix} \langle B_1, \Psi_{i1}^+ \rangle_X & \langle B_2, \Psi_{i1}^+ \rangle_X & \cdots & \langle B_p, \Psi_{i1}^+ \rangle_X \\ \vdots & \vdots & \ddots & \vdots \\ \langle B_1, \Psi_{in_i}^+ \rangle_X & \langle B_2, \Psi_{in_i}^+ \rangle_X & \cdots & \langle B_p, \Psi_{in_i}^+ \rangle_X \end{bmatrix},$$

$$W_i^- = \begin{bmatrix} \langle B_1, \Psi_{i1}^- \rangle_X & \langle B_2, \Psi_{i1}^- \rangle_X & \cdots & \langle B_p, \Psi_{i1}^- \rangle_X \\ \vdots & \vdots & \ddots & \vdots \\ \langle B_1, \Psi_{in_i}^- \rangle_X & \langle B_2, \Psi_{in_i}^- \rangle_X & \cdots & \langle B_p, \Psi_{in_i}^- \rangle_X \end{bmatrix}$$

and B_l are components of control operator B, $B_i=[\,g(A)^{-1}b_i\,;\,-g(A)^{-1}b_i]^T$, i=1,2,...,p and n_i denotes the multiplicity of eigenvalue $v_i^{\pm}$. If all eigenvalues $v_i^{\pm}$ of state operator Λ are single, as in our case, conditions (6.2) can be simplified to the following formulae:

$$\left.\begin{aligned} \langle B_1, \Psi_i^+ \rangle_X^2 + \langle B_2, \Psi_i^+ \rangle_X^2 + \cdots + \langle B_p, \Psi_i^+ \rangle_X^2 \neq 0 \\ \langle B_1, \Psi_i^- \rangle_X^2 + \langle B_2, \Psi_i^- \rangle_X^2 + \cdots + \langle B_p, \Psi_i^- \rangle_X^2 \neq 0 \end{aligned}\right\} \quad for\ all\ i = 1,2,\ldots \tag{6.3}$$

Applying conditions (6.3) to our dynamical system (4.1) we obtain

$$\begin{aligned} &\left\{ \int_0^l \left(g(A)^{-1}b_1\right)\phi_i(x)dx - \int_0^l \left(g(A)^{-1}b_1\right)p_i\,\phi_i(x)dx \right\}^2 + \cdots \\ &\cdots + \left\{ \int_0^l \left(g(A)^{-1}b_p\right)\phi_i(x)dx - \int_0^l \left(g(A)^{-1}b_p\right)p_i\,\phi_i(x)dx \right\}^2 \neq 0 \quad for\ all\ i = 1,2,\ldots \end{aligned} \tag{6.4}$$

Taking into account spectral representation of $g(A)^{-1}$ given by (4.3) we obtain

$$\begin{aligned} &(1-p_i)^2 \left\{ \left[\int_0^l \left(\sum_{j=1}^{+\infty} g(\lambda_j)^{-1} \int_0^l b_1(x)\phi_j(x)dx\,\phi_j(x) \right) \phi_i(x)dx \right]^2 + \cdots \right. \\ &\left. \cdots + \left[\int_0^l \left(\sum_{j=1}^{+\infty} g(\lambda_j)^{-1} \int_0^l b_p(x)\phi_j(x)dx\,\phi_j(x) \right) \phi_i(x)dx \right]^2 \right\} \neq 0 \quad for\ all\ i = 1,2,\ldots \end{aligned} \tag{6.5}$$

Taking into consideration the form of functions $\phi_i(x)$, i=1,2,... (6.5) implies (6.1).

The first term of condition (6.1) has form $(1-p_i)^2 \neq 0$. On the base of (4.4) we can say that system (2.1) can be approximately controllable if its coefficients satisfies condition

$$\left(\alpha\lambda_i + \beta\lambda_i^{\frac{1}{2}}\right)^2 - \lambda_i - \gamma\lambda_i^{\frac{1}{2}} \neq 0 \quad for\ all\ i = 1,2,...$$

As we see, approximate controllability depends significantly on the form of functions $b_i(x)$, i=1,2,...,p. For example if we take functions $b_i(x)=1$ for i=1,2,...,p, $x\in(0,l)$ it is easy to verify that the dynamical system (4.1) is not approximately controllable.

7. Conclusions

Remark 1

As it was mentioned we can define many different notions of controllability for distributed parameter systems. We discuss, in this paper, the approximate controllability of distributed systems. It can be also analyzed so called exact controllability of distributed systems when the system can be steered from the zero initial state to an arbitrary final state exactly. Of course, the exact controllability conditions are much more restrictive. It should be stressed that in most cases if distributed system is controllable it is only approximately controllable. This is the case of our dynamical system (2.1) which is only approximately controllable. It follows directly from the compactness of the operator B. There exist only very limited class of distributed systems which are exactly controllable.

Remark 2

We treated, in this paper, approximate controllability by distributed inputs. In this case control inputs appear in the partial differential equation. In some papers there is considered approximate controllability by boundary inputs. In this case control inputs appear in the boundary conditions. Boundary control is easier in physical realization but it is much more complicated in mathematical analysis than controllability by distributed inputs.

Remark 3

We analyzed approximate controllability of dynamical system with arbitrary unconstrained controls. It is possible to derive necessary and sufficient conditions for approximate controllability of distributed systems with existence of constraints on controls (for example with nonegative controls). Controllability conditions in this case are much more complicated and desire more refined technique.

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Abstract

In the paper the analysis of mathematical models describing a class of elastic mechanical systems is presented. In the case when elements of mechanical constructions undergo elastic deformation we have to treat them as the distributed parameter elements. Therefore the dynamics of the elastic mechanical systems has to be described by partial differential equations. This fact complicates the mathematical model and makes it's analysis difficult. Much more complicated and advanced mathematical methods have to be used to analyze the distributed parameter systems.

The paper is devoted to the analysis of distributed parameter system described by partial differential equation (2.1) fourth order with respect to spatial coordinate and second order with respect to time. Equation (2.1) was completed by two initial conditions and four boundary conditions. Equation of dynamics contains terms describing so called internal (structural) damping.

On the base of spectral theory of linear, unbounded, differential operators defined in appropriate Hilbert space the analysis of system is presented. The fourth order differential operator with respect to spatial coordinate with appropriate boundary conditions was introduced. It was solved eigenvalues and eigenfunctions problem for this operator. The main properties of the operator was presented. Using defined differential operator the partial differential equation of dynamics was transformed into linear equation of evolution (3.6) (an abstract ordinary differential equation) in infinite dimensional Hilbert space. The evolution equation was finally transformed into equation of state (4.1).On the ground of the analysis of state equation the necessary and sufficient conditions for approximate controllability of the investigated system were formulated.