ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1998
Seria: AUTOMATYKA z. 125 Nr kol. 1391

Tadeusz SAWIK
Akademia Górniczo-Hutnicza, Kraków

DISTRICT HEATING NETWORK TOPOLOGY AND CAPACITY EXPANSION PLANNING MODEL[1]

Summary. The paper presents a mixed integer programming formulation for the multi-period district heating capacity expansion problem. A number of valid inequalities are introduced to strengthen the formulation.

MODEL MATEMATYCZNY ZADANIA PLANOWANIA ROZWOJU MIEJSKIEJ SIECI CIEPŁOWNICZEJ

Streszczenie. W pracy przedstawiono model programowania liniowego mieszanego dla zadania planowania rozwoju miejskiej sieci ciepłowniczej. Wyznaczono dodatkowe nierówności zastępcze, odcinające rozwiązania potencjalnie nieoptymalne.

1. Introduction

Multiperiod expansion of district heating facilities is important for accomodating growing demand for heat supplies in urban areas. The driving force on the demand side might be increasing number of inhabitants in cities and their requirements for cheaper energy supplies, whereas on the supplier side, technology innovations reducing the costs of providing and maintaining central heating services as well as reducing the environmental emissions. A multiperiod network topology and capacity expansion policy attempts to optimize the tradeoffs between investing in network expansion versus the revenue producing opportunities and reduction in system maintenance costs, and in particular, costs of environmental losses due to pollutions associated with production and conversion of energy ([3]). A mixed integer programming formulation proposed in this paper for the network capacity expansion problem is intended to be applied for computer aided expansion planning of Cracow district heating system.

[1]This work was partially supported by a CEEC/NIS research grant of FNRS, Switzerland

2. Problem description

Let F be the given forest-like graph over which the district heating network expansion problem is defined. The nodes of this network represent the existing and potential heat sources and destinations and its edges correspond to existing and potential pipeline sections. The number of nodes and their locations are known in advance.

In the system there are two types of sources: central thermal stations and local thermal stations. A central thermal station supplies a number of destination nodes. Each central station is associated with a tree rooted at the central station, where the other nodes are destinations. On the other hand, the local thermal stations are associated with a subset of isolated nodes in graph F. An isolated node represents either a pair of the local source and destination supplied by this source or a potential destination node.

Given a planning horizon and the projected demand for thermal energy consumption at each destination node over the horizon, the problem objective is to find a district heating network expansion schedule which satisfies the demand and minimizes the present worth of the total investment, operating and maintenance cost incurred during the planning horizon.

The investment costs include cost of installing new local thermal stations and new pipeline sections of the network, cost of eliminating old local thermal stations and cost of capacity expansion of new local thermal stations and the existing sections of pipeline.

The maintenance cost of local thermal stations includes also fines paid for the environmental emissions. The latter cost can be decreased by extending the district heating network to eliminate the existing old local thermal stations and/or by modernizing the existing thermal stations.

The network planner needs to decide:

- from which central source to supply each destination node in the network;
- where and when to locate new local sources, and with what capacity;
- where and when to install new edges (pipeline sections) of the network, and with what capacity;
- which new local source to expand, when and by how much;
- which edges of the network to expand, when and by how much;
- which initially existing old local source to eliminate from the system, and when;
- in a more general model, which existing old local source to modernize (e.g., change from coal or coke to gas or fuel oil), and when.

2.1. Notation and assumptions

The notation used to describe the problem formally is presented below where all costs coefficients are the present values of the associated costs discounted at an appropriate rate.

Let us note that the network expansion costs vary by edge (depending on the length, diameter, and location of pipeline section) and has both a fixed and variable component. Similarly, local thermal station location costs also have location dependent fixed and variable components.

Sets

E	–	the set of all (existing and potential) edges (k,l) in the district heating network F.
$I=\{1,\ldots,m\}$	–	the index set of central source nodes representing central thermal stations.
$J=\{1,\ldots,n\}$	–	the index set of all (existing and potential) destination nodes, $J = J^{(c)} \bigcup J^{(l)} \bigcup J^{(p)}$.
$J^{(c)}$	–	the subset of existing destination nodes connected to central sources
$J^{(l)}$	–	the subset of isolated nodes representing pairs of existing destinations with local sources.
$J^{(p)}$	–	the subset of potential new destination nodes to be connected to central or new local sources.
P_{ij}	–	the unique path in the tree connecting source node i and destination node j.
SD_{kl}	–	the set of all ordered node pairs (i,j) (source i,destination j) whose connecting path P_{ij} contains edge (k,l).
$T=\{1,\ldots,h\}$	–	the set of time periods in the planning horizon. Period t refers to unit period from time t to $(t+1)$.

Parameters

b_{it} – the capacity of central thermal station i in period t.

$c_{it}^{(c)}$ – the variable operating and maintenance cost of the heat distribution network connected to central station i per unit of thermal energy supplied by i in period t. This cost represents the unit price of thermal energy bought from source i in period t and the expenses associated with distribution of thermal energy from i to its destinations.

$c_{jt}^{(l)}$ – the variable operating and maintenance cost per unit of utilized capacity of local thermal station j in period t. This cost represents the operating and maintenance expenses of thermal generator modules as well as a fine paid for the environmental emissions.

d_{jt} – the projected demand of destination node j in period t.

e_{jt} – the investment cost of augmenting a unit capacity for the new local thermal station $j \in J^{(p)}$ in period t.

f_{klt} – the investment cost of augmenting a unit capacity for the existing edge (k, l) in period t.

$g_{jt}^{(p)}$ – the fixed cost of installing a new local thermal station at destination node $j \in J^{(p)}$ in period t. This cost represents land acquisition and infrastructure investments at node j.

$g_{jt}^{(l)}$ – the fixed cost of eliminating from the system an old local thermal station at destination node $j \in J^{(l)}$ in period t and connecting this node to the network.

h_{klt} – the fixed cost of installing a pipeline section on edge (k, l) in period t. This cost is incurred when putting a new pipeline section as well as each time the edge capacity is expanded.

t_j – the first period with a positive demand at node j, $t_j = \arg\min\{t : d_{jt} > 0\}$.

α_{ij} – the unit heat loss for path P_{ij}.

Assumptions

A1: The number and locations of all existing and potential destination nodes and edges of the heat distribution network are known in advance.

A2: The demand of all destination nodes must be satisfied over the entire planning horizon.

A3: Each destination node can be supplied only from one source, central or local.

A4: Each potential destination node can be either connected to the network as a new leaf node or suplied from a new local source located at this node.

A5: For each destination node to be connected to the network there exists only one potential edge to be installed for connecting this node with the network.

A6: An isolated node representing a pair of destination and its local source operates independently on the other nodes in the network.

A7: If a new local source has been located at some potential destination node during the planning horizon, this source cannot be eliminated from the system during the entire horizon.

A8: Each isolated destination node initially supplied from its local source can be connected to the network via some new edge while the old local source is eliminated from the system.

A9: Once connected to some central source, a destination node cannot be disconnected from that source during the entire planning horizon.

A10: No new local source and edge capacity contractions can take place during the planning horizon, whereas the capacities of the exisiting local sources are not expanded.

A11: Considering the time value of money and the technological innovations, the unit investment costs e_{jt}, and f_{klt} are assumed to be decreasing over time.

3. Network expansion planning model

The model proposed in this section minimizes the total cost of installing new or eliminating old local sources, installing new capacities in new local sources and expanding the topology and capacity of the district heating network to meet the projected demand over a planning horizon (for similar models used in telecommunications, see [1, 2])

The following decision variables are introduced to model the district heating network topology and capacity expansion planning problem:

Continuous Variables

The local source capacity variable:

$u_{jt} \geq 0$ – the capacity of local source j in period t.

It is assumed that new local source capacities are nondecreasing over time,

i.e., $u_{jt} \leq u_{jt+1},\ j \in J^{(p)}$.

The edge capacity variable:

$v_{klt} \geq 0$ – the capacity of edge (k,l) in period t.

It is assumed that edge capacities are nondecreasing over time,

i.e., $v_{klt} \leq v_{klt+1},\ (k,l) \in E$.

Discrete Variables

The assignment variable:

$x_{ijt} = 1$, if a central $(i \in I)$ or a local $(i = j)$ source node supplies destination node $j \in J$ in period t; otherwise $x_{ijt} = 0$.

Let us note that $x_{jjt} = 1,\ j \in J^{(p)}$ denotes that a new local source is located at destination node j in period t. By definition, the following relations hold: $x_{ijt} \leq x_{ijt+1},\ i \in I,\ x_{jjt} \leq x_{jjt+1},\ j \in J^{(p)},\ x_{jjt} \geq x_{jjt+1},\ j \in J^{(l)}$

i.e., there is no disconnection of a destination node from a central source or a new local source, whereas the initially existing local source can be eliminated from the system.

The edge installation variable:

$y_{klt} = 1$, if a pipeline section has been installed on edge (k,l) by period t; otherwise $y_{klt} = 0$.

By definition, the relation $y_{klt} \leq y_{klt+1},\ (k,l) \in E$ holds

The edge capacity expansion variable:

$z_{klt} = 1$, if the capacity of edge (k,l) is augmented during period t; otherwise $z_{klt} = 0$.

The network topology and capacity expansion planning problem has the following basic mixed integer programming formulation.

Objective function

$$\min \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} c_{it}^{(c)}(1+\alpha_{ij})d_{jt}x_{ijt} + \sum_{t\in T}\sum_{j\in J} c_{jt}^{(l)}d_{jt}x_{jjt}$$
$$+\sum_{t\in T}\sum_{j\in J^{(p)}} g_{jt}^{(p)}(x_{jjt}-x_{jjt-1}) + \sum_{t\in T}\sum_{j\in J^{(l)}} g_{jt}^{(l)}(x_{jjt-1}-x_{jjt})$$
$$+\sum_{t\in T}\sum_{j\in J^{(p)}} e_{jt}(u_{jt}-u_{jt-1}) + \sum_{t\in T}\sum_{(k,l)\in E}(f_{klt}(v_{klt}-v_{klt-1}) + h_{klt}(y_{klt}-y_{klt-1}+z_{klt})) \quad (1)$$

The objective function (1) minimizes the sum of all costs of investment in the new local thermal stations and the existing and new pipeline sections as well as operating and maintenance costs of the entire district heating system.

Constraints

Assignment constraints:

$$\sum_{i\in I} x_{ijt} + x_{jjt} = 1; \quad j\in J^{(l)}\bigcup J^{(p)},\ t\in T,\ t\geq t_j \quad (2)$$
$$x_{ijt} \geq x_{ijt-1}; \quad i\in I,\ j\in J^{(c)}\bigcup J^{(p)},\ t\in T \quad (3)$$
$$x_{jjt} \geq x_{jjt-1}; \quad j\in J^{(p)},\ t\in T \quad (4)$$
$$x_{jjt} \leq x_{jjt-1}; \quad j\in J^{(l)},\ t\in T \quad (5)$$

Constraints (2) ensure that each destination node $j \in J^{(l)}\bigcup J^{(p)}$ can be assigned to exactly one central ($i \in I$) or local (exisiting $j \in J^{(l)}$ or new $j \in J^{(p)}$) source node in each period $t \geq t_j$. Relations (3) and (4) state that there is no disconnection of a destination node from a central source or a new local source, whereas (2) and (5) specify that initially exisiting local source can be eliminated from the system and the corresponding destination node $j \in J^{(l)}$ connected to some central source.

Contiguity constraints:

$$x_{ijt} \leq x_{i,k_{ij},t}; \ i\in I,\ j\in J,\ t\in T \quad (6)$$

where k_{ij} is the destination node, the immediate predecessor of node j on path P_{ij}.

If central source i supplies destination node j in period t, then node j's immediate predecessor k_{ij} on the connecting path P_{ij} must also be supplied from source i in period t.

Source capacity constraints:

$$\sum_{j\in J}(1+\alpha_{ij})d_{jt}x_{ijt} \le b_{it}; \qquad i \in I,\ t \in T \tag{7}$$

$$d_{jt}x_{jjt} \le u_{jt}; \qquad j \in J,\ t \in T \tag{8}$$

$$u_{jt} \ge u_{jt-1}; \quad j \in J^{(p)},\ t \in T \tag{9}$$

Constraints (7) and (8) ensure that the total heat consumed from central and local sources in each period t must not exceed these sources capacity in this period. Relations (9) indicates that no new local source capacity contractions can take place.

Network capacity constraints:

$$\sum_{(i,j)\in SD_{kl}} d_{jt}x_{ijt} \le v_{klt}; \ (k,l) \in E,\ t \in T \tag{10}$$

The left hand side of the network capacity constraints (10) expresses the total flow on edge (k,l) in terms of the destination-to-source assignments that use this edge. The edge capacity must satisfy this flow.

Edge capacity expansion-forcing constraints:

$$v_{klt} - v_{klt-1} \le M_{kl}z_{klt}; \qquad (k,l) \in E,\ t \in T \tag{11}$$

$$z_{kls} \le y_{klt}; \quad (k,l) \in E,\ s,t \in T,\ s > t \tag{12}$$

$$v_{klt} \ge v_{klt-1}; \qquad (k,l) \in E,\ t \in T \tag{13}$$

where the edge capacity bound M_{kl} is given as the maximum total demand of all destination nodes j whose path P_{ij} connecting j with a central source i contains the edge (k,l). That is M_{kl} is the maximum total demand of all nodes (including node l) in the subtree rooted at l: $M_{kl} = \max_{t\in T}\{\sum_{j:\,(i,j)\in SD_{kl}} d_{jt}\}$.

Constraints (11) imply that a fixed charge is associated with a network positive capacity expansion. Relations (12) state that if there is no pipeline section installed on edge (k,l) by period t, then no capacity expansion can take place on that edge by period t. Relations (13) ensure that there is no edge capacity contraction.

Edge installation-forcing constraints:

$$v_{klt} \leq M_{kl} y_{klt}; \quad (k,l) \in E, \ t \in T \tag{14}$$

$$y_{klt} \geq y_{klt-1}; \quad (k,l) \in E, \ t \in T \tag{15}$$

Constraints (14) state that a positive edge capacity in period t implies the installation of a pipeline section by period t, if it was not installed at the beginning ($y_{kl0} = 0$). The new edge (k,l) capacity bound M_{kl} is given as the maximum demand of the newly connected destination node l and all its potential successors. Relation (15) state that there is no disconnection of the installed edge.

Integrality/nonnegativity constraints:

$$x_{ijt}, \ y_{klt}, \ z_{klt} = 0 \ \text{or} \ 1; \quad i \in I \bigcup J^{(l)} \bigcup J^{(p)}, \ j \in J, \ (k,l) \in E, \ t \in T \tag{16}$$

$$u_{jt}, \ v_{klt} \geq 0; \quad j \in J, \ (k,l) \in E, \ t \in T \tag{17}$$

Let us note that nonzero initial values of variables represent the beginning topology and capacity of the district heating network. For example, given nonzero values of x_{ij0} represent the beginning assignments of destinations to sources (central or local), and nonzero values of y_{kl0} – the beginning topology of the network. Nonzero values of u_{j0} are initial production capacities of the existing local sources, and nonzero values of v_{kl0} are initial capacities of the existing pipeline sections of the network.

3.1. Model enhancements

The mixed integer programming model proposed can be strengthened in a number of ways. They include the addition of valid inequalities to the base model, reduction of coefficients (e.g., M_{kl}) or the incorporation of redundant cut constraints. For example, it is possible to generate valid inequalities by relating different types of variables directly such as the assignment variables to edge installation variables or the edge installation variables to edge capacity expansion variables.

The following valid inequalities can be incorporated into the model.

Edge installation-forcing edge capacity expansion inequalities

$$y_{klt} - y_{klt-1} \leq z_{klt}; \ (k,l) \in E, \ l \in J^{(l)} \bigcup J^{(p)}, \ t \in T \tag{18}$$

Inequalities (18) ensure that installing a new edge in period t goes together with expanding this edge capacity (from the initial zero capacity) in this period.

Local source location or edge installation inequalities:

$$x_{llt_l} + y_{klt_l} = 1,\ (k,l) \in E,\ l \in J^{(p)} \tag{19}$$

For each isolated destination node l with the first positive demand in period t_l either a local source is located at l or a new edge (k,l) is installed by period t_l connecting l with some central source i $((i,l) \in SD_{kl})$ via node k.

Assignment-forcing edge installation inequalities

$$x_{ilt} \le y_{klt},\ (i,l) \in SD_{kl},\ (k,l) \in E,\ l \in J^{(l)} \bigcup J^{(p)},\ t \in T \tag{20}$$

Inequalities (20) ensure that connecting a new destination node l to the network in period t requires the corresponding new edge (k,l) to be installed by this period.

4. Conclusion

The network topology and capacity expansion planning model can be further extended to include, for example alternative connections of destination nodes to different central sources in different periods or possibility of modernization the existing local sources instead of their complete elimination. The computational results for Cracow district heating system obtained using the AMPLPLUS/CPLEX software package are very promising and prove practical value of the modelling approach proposed.

BIBLIOGRAPHY

1. Balakrishnan A., Magnanti T.L., Wong R.T.: A decomposition algorithm for local access telecommunications network expansion problems. *Operations Research*, vol. 43, 1995, pp. 58-76.
2. Chang S.-G., Gavish B.: Lower bounding procedures for multiperiod telecommunications network expansion problems. *Operations Research*, vol. 43, 1995, pp. 43-57.
3. Haurie A., Sawik T.: Systems analytic methods, industrial restructuring and the environment. In: Carraro C., Haurie A., and Zaccour G. (eds.): *Environmental Management in a Transition to Market Economy*, Editions Technip, Paris 1994, pp. 419-460.

Recenzent: Prof.dr hab.inż. Józef Grabowski

Abstract

W pracy przedstawiono model matematyczny zadania planowania rozbudowy i modernizacji miejskiej sieci ciepłowniczej. Zadanie polega na wyznaczeniu harmonogramu rozbudowy i modernizacji sieci rurociągów oraz instalacji nowych i modernizacji istniejących lokalnych źródeł energii cieplnej, który zapewni zaspokojenie zapotrzebowania na energię cieplną w rozważanym okresie przy minimalnych łącznych kosztach inwestycji i eksploatacji systemu. Problem sformułowano jako wieloetapowe zadanie programowania liniowego, mieszanego. Analiza specjalnej struktury zadania doprowadziła do wyznaczenia dodatkowych nierówności zastępczych (cięć), których wprowadzenie do modelu bazowego umożliwia wzmocnienie wyjściowego sformułowania poprzez redukcję obszaru rozwiązań dopuszczalnych bez eliminacji całkowitoliczbowego rozwiązania optymalnego. Model przeznaczony jest do wspomagania planowania rozbudowy krakowskiej sieci ciepłowniczej z uwzględnieniem ograniczeń dopuszczalnego poziomu szkodliwych emisji gazów i pyłów. Wstępne wyniki eksperymentów obliczeniowych przeprowadzonych przy użyciu pakietu AMPLPLUS/CPLEX, przedstawione na konferencji ADPP11, wskazują na możliwość zastosowania opracowanego modelu w praktyce.