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Vol. 16 No. 5 7400 SECOND BOULEVARD, DETROIT 2, MICHIGAN **April 1945**

Presidental Address to American Concrete Institute*

By R. W. CRUM

The special committee on postwar planning of the Institute recommended that the "Institute encourage and participate in the discussion of ways and means toward a better postwar world". In its own discussion which followed, the committee assumed that the postwar world would be a peaceful one.

It further recommended that the "American Concrete Institute rededicate itself to the task of increasing, correlating and disseminating the special engineering knowledge of its chosen field . . . " Such an endeavor can only flourish to the advancement of civilization in a peaceful world.

The Concrete Institute is one of many institutions in the fields of science and technology, the sum of whose achievements makes up the advancement in material well being of the peoples of the earth. Basically the activities of these institutions depend upon research, which is nothing but the search for new knowledge, for understanding of it, and for ways to use it for the benefit of man. The whole process was well summed up by Secretary Whipple, who said recently: "As a base of all our work we must, of course, have research to point the way. We do not, however, achieve a large audience except as a few pioneers apply research to practice, and evolve from that practice definite recommendations on what to do about it in design, construction and manufacture."

The material civilization the World enjoys has come about through the summation of such activities of all peoples and institutions in times of peace. In war also, it is true that research flourishes, but it is the research of destruction, and in spite of some war induced discoveries and developments that may have potential value to future civilization, if war conditions prevail long enough, these values may become of little account. For progress, learning must not be lost but enhanced, and for

*By the retiring president, at the Institute's Luncheon Meeting (41st Annual Convention) New York City, Feb. 16, 1945.

learning to increase, scientists and technologists must be trained and embued with the thirst for new knowledge. Even now, who is going to carry the torch after this war is over? Do we realize that already there is a four year gap in the training of scientific and engineering workers that cannot be closed for many years.

This picture is not theory; it is fact, demonstrated many times in the long history of the strivings of the human race toward better life. The civilization of the Roman empire disappeared utterly and was followed by centuries of darkness and distress.

There is no more important duty before the citizens of America today than the effort we must make to secure an enduring peace after this war. To this end the intellectual leadership represented in the membership of this and similar organizations must be exerted to the utmost.

In their essay on "The Problems of Lasting Peace", Herbert Hoover and Hugh Gibson list seven dynamic forces which collectively control international relations with respect to war and peace. Only one of these, "the will to peace", is a positive force in favor of peace as against war. Although this force has been at work throughout the recorded history of civilization it is somewhat discouraging to face the fact that for the most part it has been ineffective. If the "will to peace" were universal throughout the civilized world, the other forces that make for distrust, suspicion and hatred between men and nations could doubtless be overcome but regrettably there have always been men and nations, in whose sight war is good and who do not want lasting peace. Mistaken though they may be, their presence is a fact which renders solution of the problems of lasting peace immensely if not insurmountably difficult.

There have been many proposals for methods of keeping the peace, a few of which have been tried, with indifferent success. The one force at the disposal of government that has been successful in keeping the peace is that of law and law enforcement. It is a fact that in every community, no matter what its size, from a mere village to a group of nations, there is a lawless element that, if unrestrained would keep warfare perpetually active. Civilized nations have learned to maintain internal order through laws, courts and police power. Once, in historical times, international peace was kept by this method. For 300 years imperial Rome governed the civilized world and kept law and order. When its restraining hand fell, for other hundreds of years there was no peace anywhere.

The purpose of this brief essay is to arouse speculation upon the relation of the unescapable facts of war and peace to the prospects for lasting peace, in the years to follow the present worldwide conflict. The fact must be faced that to some nations, to be warlike is regarded as a virtue.

Numerous questions present themselves: Is there any hope that the "will to peace" can be aroused among these peoples? Is there any possibility that through international agreements or coalitions, conditions may be set up that in themselves are conducive to peace and not to war? Are there enough peace-loving nations that will associate themselves together to keep the peace in the civilized world, and if so, will they adopt thorough-going measures to that end? Is there enough wisdom in the human race to protect itself against recurrence of the desperate situation in which it is today?

Each of these questions could be made the basis for an extended essay.

To be more specific: Can enduring peace be assured without the use of police power and if the answer is "no", can enough nations with enough power to enforce the peace be found who will work steadfastly together through this means?

The brief backward look I have taken at history indicates to me that there is little hope for permanent or even long-lasting peace unless a body of just international law is developed, courts established to adjudicate disputes and adequate police power provided. As to the second part of the question, I should say that the record of the human race in developing the arts of peace in spite of wars and pestilences should give rise to well-founded hope that in the end concerted effective action by peace-loving peoples will keep peace between nations just as in any community of civilized individuals, the law-abiding elements work together to uphold the forces of law and order.

The United States of America is foremost among the nations that truly want peace. For many years through our fortunate geographical situation we were able to go our way without embroilment in the quarrels of the rest of the world and in that time we built up the best living conditions yet known to man. Now, through that very research, which under right conditions is our hope for continued upward realization of our aspirations, our safety through isolation has been destroyed. As evidenced by the happenings of this war, another world conflict might entirely destroy our civilization and set us back for centuries.

It is a vital necessity that this nation work wholeheartedly with the other peace-loving peoples to set up a regime of lasting peace, and to my mind that can only be assured through the establishment of international laws, courts and adequate enforcement machinery. I do not believe participation in such a plan for world order need be subversive of our national sovreignity, nor does it need to be incompatible with provision for defense against aggression.

No plan for lasting peace can succeed without the full support of the United States, nor can it succeed without the full support of the powers allied with us in this war. The problem is indeed difficult, but looking forward to durable world peace must not be dismissed as a mere utopian dream.

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Precast Concrete Pit Sheeting*

By JACOB FELD†
Member American Concrete Institute

SYNOPSIS

The development of light weight concrete slabs in place of wood planks for pit or box sheeting eliminates future settlements of underpinned structures when the buried sheeting rots. Practical use has demonstrated that concrete planks add little to the cost of such work. A summary of the history of box sheeting and of the various types used shows the possibilities for the use of concrete in this phase of construction work.

INTRODUCTION

Excavations for foundation support in restricted localities, such as occur in the digging of pits for the underpinning of buildings, must be performed within sheeted pits with horizontal or box sheeting. Lack of vertical clearance prevents the driving of vertical sheeting. In work where the vibration of the driving would injure adjacent structures or in soils which liquify during such vibration, box sheeting is also a necessary procedure. The underpinning of building foundations along the route of a city subway structure is a major operation, especially when the subway width covers practically the entire street width. In recent years, New York City provisions are that no wood sheeting may be left on the ground, since experience had shown a possible danger of settlements resulting from rotted timber sheeting. In 1935, the writer developed the use of precast concrete box sheeting, used it in the underpinning operations of many buildings and other structures and has found the cost reasonable and only slightly above the most economical wood sheeting.

*Received by the Institute, Dec. 20, 1945.
†Consulting Engineer, New York, N. Y.

PRACTICE AND THEORY

The introduction of horizontal box sheeting is usually attributed to the late James C. Meem, in his work as Chief Engineer for Cranford and McNamee, contractors of the original subway in Brooklyn, N. Y., about 1905. In Cranford and Meem's paper on "Resume of the Development of Underpinning," presented before the Brooklyn Engineers' Club in 1931, he states in connection with the underpinning of Elevated Railway Columns:

> The first important step in connection with this underpinning development was the adoption of a method of sheeting pits, usually not more than 4 x 4 ft. nor greater than 6 x 6 ft. in area, and of any required depth.
>
> The sheeting consisted of 2-in. planks cut so that the pieces on the north and south sides of the pit engaged and braced the square ends of the pieces on the east and west sides, successive courses alternating the bearings from the east and west to the north and south pieces. This was in effect an adaptation and improvement of the method long used by Long Island well diggers, who cut ordinary saplings to the proper length, and placed them in engagement on the opposite sides in the successive courses.
>
> The essential principle in this connection was the fact, well known to some engineers, that it is quite possible to sink separated pits of limited dimensions safely adjacent to a building foundation and to join them by intermediate pits when it is equally impossible to safely excavate there a long continuous trench. This method was, therefore, applied, not only to all the buildings, but to the foundation of the elevated railroad columns.

In a paper on "Pressure and Resistance of Soil," also presented before the Brooklyn Engineers' Club in 1920, Mr. Meem also states his experience that in pits not exceeding 5 ft. square, "the pressure is not cumulative because the horizontal arching component tends to maintain it uniformly, in the same principle, though differently applied, as in grain bins." He also describes the use of a fabricated steel bracing in the form of bolted channel frames placed under each other to act as pit sheeting. This type was quite costly and required considerable labor to remove the sheeting and backfill the pier which was poured within wood forms. Since the pit was 5 ft. square, working room restricted the concrete pier to an uneconomical size.

In 1923, the writer showed that no uncertain internal stress, such as Mr. Meem's horizontal arching component, need be assumed to prove that the lateral pressure of soil on the sides of a pit is not cumulative with depth. In the paper on "Lateral Earth Pressure Determination," published in the 1924 *Transactions*, Am. Soc. of C. E., V. 86, p. 1579, the writer developed a formula for the lateral pressure on the side wall of a pit, based on the modified wedge theory, as follows (see Fig. 1): Horizontal pressure on a unit area at depth x is

$$w\, x\, tan^2\, \tfrac{1}{2}(90^\circ - \phi) \qquad (1)$$

where w is the unit weight and ϕ is the angle of internal friction of the soil.

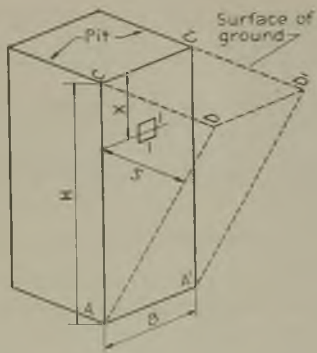


Fig. 1

For an unlimited length of wall, there being no differential motion in the fill, the total horizontal pressure per foot length of wall is

$E = \frac{1}{2}\, w\, H^2\, tan^2\, \frac{1}{2}(90^\circ - \phi)$ .... (2)

the usual Coulomb or wedge formula for earth pressure against a vertical wall of a horizontally bounded fill.

If there exists a definite length of wall or sheeting, B, cut out without disturbance, the total pressure on the sheeting equals the pressure on a width B as given by formula (2) less the resistance offered by the undisturbed soil along the vertical planes ACD and $A'C'D'$. It is accurate enough for this purpose to assume that AC is a straight line. The most recent researches indicate that the surface of rupture deviates only slightly from a plane. The tendency is for the wedge of rupture to move parallel to AD; therefore the frictional resistances along planes ACD and $A'C'D'$ are also parallel to AD. In accordance with the wedge theory of earth pressure, angle CAD is $\frac{1}{2}(90^\circ - \phi)$.

The total horizontal component of the lateral pressure on area ACD

is $\int_0^H w\, x\, S\, tan^2\, \frac{1}{2}(90^\circ - \phi)\, dx$ .... (3)

where S is the width of the triangle at depth x.

Since $S = (H - x)\, tan\, \frac{1}{2}\, (90^\circ - \phi)$ .... (4)

then substituting for S and integrating,

$$E \text{ (on } ACD) = \frac{1}{6}\, w\, H^3\, tan^3\, \frac{1}{2}(90^\circ - \phi) \quad \text{.................... (5)}$$

The resistances along the two planes, since $tan\ \phi$ is the coefficient of internal friction of the soil, are each equal to

E (on ACD) $tan\ \phi$, and the horizontal component of the resistance

is E (on ACD) $tan\ \phi\ sin\ \frac{1}{2}(90^\circ - \phi) =$

$\frac{1}{6}\, w\, H^3\, tan\ \phi\ tan^3\, \frac{1}{2}(90^\circ - \phi)\ sin\ \frac{1}{2}(90^\circ - \phi)$ .... (6)

For horizontal equilibrium, i.e., no increase in total horizontal pressure, the total pressure given by formula (2) over the width B, must be balanced by the two resistances, each given by formula (6). Such equilibrium will occur when

$$\frac{B}{H} = \frac{2}{3} \tan \phi \tan \tfrac{1}{2}(90° - \phi) \sin \tfrac{1}{2}(90° - \phi) \qquad (7)$$

From this formula, Table 1 has been computed, which gives for various values of ϕ,

Column (*c*) the ratio of B/H below which there is no increase in cumulative lateral pressure.

Column (*d*) the depth H_o beyond which there is no pressure in a pit 5 ft. wide.

It will be noted that H_o is a minimum for values of ϕ about 35°. The values in Table 1 indicate depths where the net total pressure is zero, i.e., the surplus resistance in the lower levels neutralizes the unbalanced part of the pressure at the upper levels. This state of affairs, most marked in soils of ϕ about 35° (the usual type encountered) explains the often reported observation that the pressure in pits and in rigidly

TABLE 1

ϕ Angle of Friction (deg.)	(a) Values of B/H for Zero Unit Pres.	(b) Values of B/H for Max. Unit Pres.	(c) Values of B/H for Zero Ttl. Pres.	(d) Depths in Pits 5 ft. Wide at which Total Pres. is Zero	(f) Depths in Pits 5 ft. Wide at which Unit Pres. is Max.	(g) Depths in Pits 5 ft. Wide at which Unit Pres. is Zero	(e) Max. Pres. on 5 ft. wide by 1 ft. deep (lb.) for W = 100 lb./c.ft.
0	.0	.0	.0	Infinite	Infinite	Infinite	Infinite
5	.054	.108	.036	139	46	93	19300
10	.098	.196	.065	77	26	51	9100
15	.126	.252	.084	60	20	40	6000
20	.147	.294	.098	51	17	34	4200
25	.159	.318	.106	47	16	31	3700
30	.167	.334	.111	45	15	30	2500
35	.168	.336	.112	45	15	30	2000
40	.165	.330	.110	45	15	30	1650
45	.159	.318	.106	47	16	31	1360
50	.149	.298	.099	51	17	34	1100
55	.135	.270	.090	56	19	37	950
60	.120	.240	.080	62	21	42	740
65	.104	.208	.069	72	24	48	600
70	.084	.168	.056	89	30	60	450
75	.065	.130	.043	116	39	77	390
80	.042	.084	.028	178	60	119	300
85	.024	.048	.016	435	104	208	260

sheeted excavations, seems to decrease with depth, is zero at the bottom of the excavation and maximum at the top or near the top.

For the design of horizontal sheeting, we are more interested in unit pressures at various depths than in total pressures, and especially in the value of maximum unit pressure.

The net pressure on a side of the pit, of width B, from equations (2) and (6) becomes

$$E(on\ ACC'A') = \tfrac{1}{2}\, w\, BH^2 \tan^2 \tfrac{1}{2}(90 - \phi) - \frac{1}{3}\, w\, H^3 \tan\phi \tan^3 \tfrac{1}{2}(90 - \phi) \sin \tfrac{1}{2}(90 - \phi)$$

$$= w\, H^2 \tan^2 \tfrac{1}{2}(90 - \phi) \left(\frac{B}{2} - \frac{HT}{3}\right) \ldots\ldots\ldots (8)$$

Where $T = \tan\phi \tan \tfrac{1}{2}(90 - \phi) \sin \tfrac{1}{2}(90 - \phi)$.

The net pressure on a strip one foot high, B wide and at depth H is

$$E_H = w \tan^2 \tfrac{1}{2}(90 - \phi)\,(BH - H^2T) \ldots\ldots\ldots (9)$$

E_H is maximum when $\dfrac{\partial E_H}{\partial H} = O = B - 2\,HT$

or when $B = 2\,HT \ldots\ldots\ldots (10)$

or $H = \dfrac{B}{2T}$

$$E_H \text{ maximum} = w \tan^2 \tfrac{1}{2}(90 - \phi)\, \frac{B^2}{4T} \ldots\ldots\ldots (11)$$

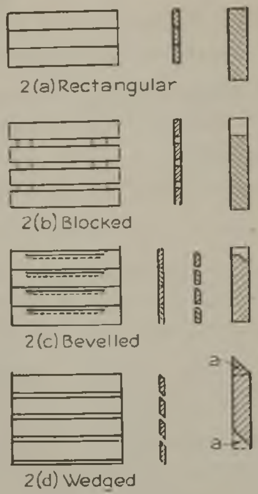
Table 1 gives values for maximum unit pressure (e) and depths at which maximum (f) and zero unit pressures (g) occur in pits 5 ft. wide.

It will be noticed that for the usual soils (values of ϕ between 15 and 60 degrees), the maximum unit pressure occurs within a depth of 20 ft. Pits sunk to any depth need sheeting to sustain only that maximum pressure. This explains such observations as for instance, the article by H. F. Peckworth on "Sheeting for Underpinning Pits" in *Civil Engineering* July 1934, page 367, that 6 ft. by 6 ft. pits were sunk through damp sand 65 ft. without the slightest difficulty, using 2 x 8 timber sheeting.

PRACTICAL CONSIDERATIONS

The simplest pit sheeting consists of square edged boards, successively placed one sheet at a time below the previously braced sheets (Fig. 2a). This necessitates careful excavation, by hand, cutting the soil to closely simulate the shape of the sheet to be placed. Where soil conditions do not permit such shaping, and where the soil will not remain with a vertical face until the next sheet is placed and wedged, any one of several "louvre" type sheets are used. The basic principal of louvre sheeting is

Fig. 2—Elevations, sections, details of types of box sheeting



the provision for packing in back of the sheets, to correct for irregularities of the excavation and to eliminate open voids in back of the sheeting.

Several types of louvre sheeting have been developed. The earliest (Fig. 2b) design was to interpose short blocks in gaps between rectangular sheets. Working space in a pit is limited and the fewer items to handle and place, the more work can be accomplished. As a result, the blocks are often omitted, to be placed after the pit is fully excavated.

To avoid the use of the small blocks, a patented beveled sheet has been developed (Fig. 2c) which provides continuous support of the pit sheeting with a gap for packing. Incidentally, these gaps are often packed with straw or hay, in fine water bearing soils, to stop the flow of the soil and to drain out free water. To reduce the cost of the sheeting and at the same time providing a sloping surface in the gaps between sheets, which makes it easier to pack and also reduces the possible flow of soil into the pit, the late Jacob Siebert, formerly Superintendent on numerous New York City subway contracts, developed the type shown in Fig. 2d. An 8-in. board covers 10 in. of vertical exposure. The sharpened lower edges of the boards permit digging into the bottom of the pit and rotating the board into position against the side of the excavation.

Bracing of the pit sides can be accomplished in several ways, all of which use the principle that the completed pit is in equilibrium and two opposite sides will support the pressure on the other two sides. All of the methods used in making wooden boxes are employed on a larger scale in pits. Several methods are shown in Fig. 3.

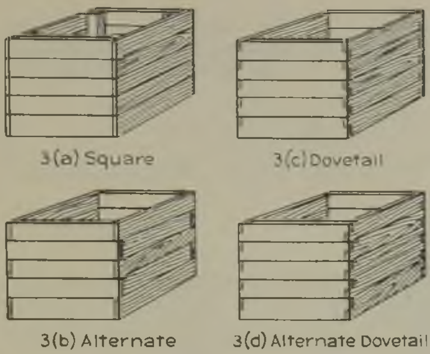


Fig. 3—Types of box sheeting

Where pits are sunk for underpinning and the sectional construction of a continuous wall, three sides of the pit sheeting are usually removed, after the pit has been concreted. The fourth side cannot be removed since it is under the building which is being underpinned. In the 1935 specifications of the N. Y. City Board of Transportation, a definite prohibition against the use of permanent wood sheeting was not included, but before construction started in 1936 on the Sixth Avenue Subway, the order was received by the Contractor that no wood sheeting could remain in the ground. The object was to eliminate future rotting of the timber and possible later consolidation and settlement of structures.

The writer, then Chief Engineer for the Brader Construction Corp., in charge of the subway section in 6th Avenue between 39 and 46th Street, the first section contracted, made a study of precast concrete slabs as a substitution for timber in underpinning pits. The usual and customary opposition to anything new, came from the field forces. However, by placing engineering assistants in direct charge of the work in the field, such inertia was soon overcome, and the underpinning foremen soon made public at every possible occasion, their experience in the use of "non-wood" pit sheeting. Some years earlier, the writer had designed, supervised the manufacture and installation and had tested thin precast concrete slabs used as the seat deck of the Rochester (N. Y.) Ball Park grandstand. It was natural to use such prior experience and the results of those tests were used to prove the feasibility of concrete slabs as sheeting.

Since there were about a hundred buildings to be underpinned in the Brader contract, pits expected from 5 to 40 ft. in depth, economy of construction was a controlling item. Several types of units were studied and designed and those which looked possible were tabulated and fabrication prices obtained from local concrete product manufacturers. Light weight concrete was preferred, since the work is entirely manual and in the cramped pit, handling planks is tiring labor. The result of

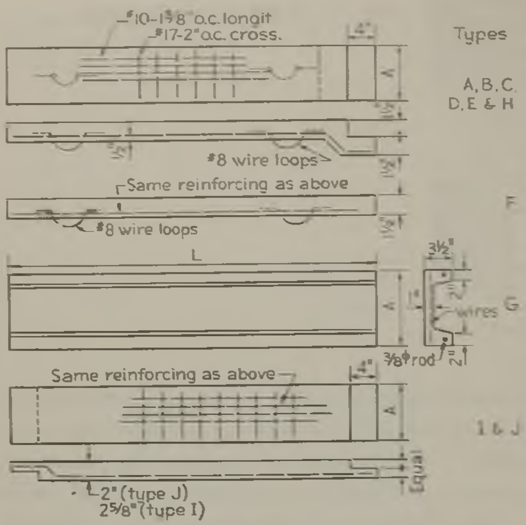


Fig. 4(a)—Precast concrete sheeting slabs—study for subway contract (see Table 2.)

TABLE 2—DETAILS AND COST OF PROPOSED TYPES

Type	A	L	Wt. lb. per Sq. ft.	Aggregate	Ultimate Load per Sq. Ft.	Cost per Sq. Ft.
A	8"	60"	18½	Sand	200	$0.20
B	12	60	"	"	"	.20
C	8	60	14	Haydite	"	.22
D	12	60	"	"	"	.22
* E	8	48	"	"	"	.22
* F	8	56	"	"	"	.22
G	24	96	12	Porete	240	.215
H	12	60	13	Haydite	200	.42
I	12	60	16	Porete	"	.36
J	12	60	17	Haydite	"	.32

Costs are bid prices F.O.B. job in truckload lots (7½ tons)
Add $0.01 per sq. ft. for Incor cement

*Types used.

the price canvas is reproduced in Fig. 4, which also shows the typical bracing method and the steps of the underpinning operation.

The simplest type was chosen and found entirely satisfactory. The planks were 8 in. wide and ordered in two lengths since flexibility was needed in pit widths. Each building required a separate underpinning to avoid property ownership encroachments. Pier widths were determined for each building in combinations of 48- and 56-in. lengths. It was felt that 8 in. was sufficient height, since the sheeting has to be undercut several inches more than the height of the sheet being placed. To avoid placing the sheets with reinforcing in the wrong face, two wire loops protruded from the front face. These loops served as hand grips

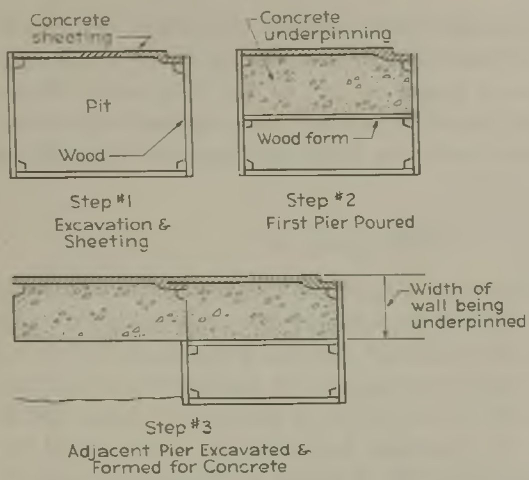


Fig. 4(b)—Use of sheeting

for lowering and placing the sheets and also a second purpose for hanging the sheet as it is placed and until it is braced in position.

The sheets weighed less than 10 lb. per foot of length and no objections were raised by the laborers. As a matter of fact, the units looked much heavier than their 40 lb. weight and men expressed surprise at how easy it was to pick up and place the planks.

TESTS OF SHEETING

Before installing the first planks, a load test was run on six slabs by two quarter point loadings with a 44-in. span. The slabs were loaded to failure in a Riehle testing machine, three at an age of seven days and three at 28 days after manufacture.

Tested slabs were 8 x 48 x 1½ in. (test span 44 in.) reinforced with No. 10 AS & W wires, 1⅜ in. o.c. longitudinal and No. 17 AS & W wires 2 in. o.c. crosswise, placed ½ in. from face of slab. Haydite aggregate and Incor portland cement were used by the Federal American Cement Tile Co. in manufacturing these first slabs.

In the 7-day age test, the first crack appeared at a minimum total load of 611 lb., with a deflection of 0.7 in., equivalent to a uniformly distributed load of 258 lb. per sq. ft. As additional load was added, more cracks opened up, but the steel did not fail in tension and at a load of 670 lb., the deflection was 2½ in.

In the 28-day age test, the first crack appeared at a minimum total load of 625 lb., equivalent to a load of 264 lb. per sq. ft. All specimens tested showed strengths within 5 percent of the values noted above, which convinced everyone that proper control was being exercised by the manufacturer. This was important, because the failure of a plank in a pit would almost always result in a serious personal injury, possibly a fatal one. No failures were found in any of the pits; some 20,000 planks were used. However, about 5 percent were rejected before use, most often because of cracks developing from improper handling during delivery.

COSTS

After using these units for several weeks, a careful check was made of labor costs compared to the previous experience with wood sheeting. The reports from several of the field staff, showed a maximum estimated time loss of 10 minutes per sheet for concrete as against wood sheeting. This corresponds to about $0.14 per sq. ft. of sheeting or about $0.40 per cu. yd. of pit volume. To this must be added the extra cost of the concrete sheeting over timber, about $0.18 per sq. ft. of sheeting, equivalent to about $0.50 per cu. yd. of pit volume. This adds about $1.00 total per cu. yd. to an average cost of $27.00 to $40.00 per cu. yd. for the completed underpinning (including the concrete filling of the pits and the wedging up of the loadings). The cost is based on labor prices of 1939 in the New York area.

Progress of excavation including placing of the sheeting averages about a foot per hour, so that the maximum loss in progress by using concrete sheeting would not exceed 15 percent and probably would be practically zero.

The field studies indicated that the extra time resulted from the greater flexibility of wooden pit boards. When the excavation is not "full" and the fit is tight, a wood plank can be driven into place by means of a mall, the concrete plank cannot be treated in this manner, for fear of shattering or cracking it. It was therefore seen to it that the excavation was carefully done to proper fit. Also when boulders were encountered, the wood plank was cut out with an axe to fit around the boulder, but with concrete plank, the boulder had to be drilled and cut off to make room for the boards. However, the concrete pit board is a permanent material which will not be affected by dry rot or water level fluctuations.

More recently, when the problem arose of building a retaining wall to safeguard an important watermain during the adjacent excavation for a reservoir, it was considered essential to use a permanent pit sheeting to guarantee against possible future loss of lateral support. Fig. 5 shows the plan and section of the bracing piers installed. By inserting sheeting in the gaps between piers, full protection was provided by building only a 50 percent continuous wall. Protection over a length of 160 ft. (16 piers) cost approximately $6400. The work was done in 1942 in a mid-eastern city, with almost ideal digging materials.

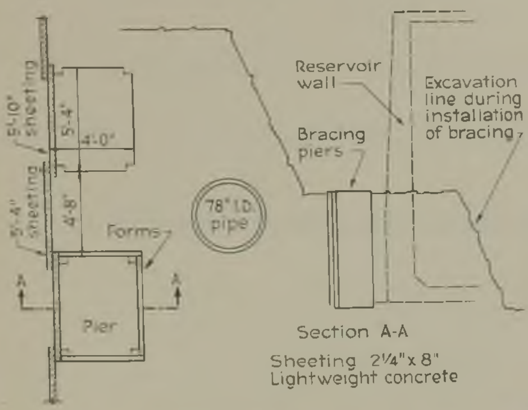


Fig. 5—Precast concrete sheeting in permanent protection wall

Fig. 6 (below) — Sheeting in the pit

Photographs of underground work are seldom available to illustrate the technique of new operations. However, Fig. 6 (taken by the author) looking down into a pit under the foundation of a building being underpinned shows a succession of pit boards tied up by wire through the wire loops. The planks are 56 in. long and 8 in. high.

To facilitate selective distribution, **separate prints** of this title (41-19) are currently available from ACI at 25 cents each—quantity quotations on request. **Discussion** of this paper (copies in triplicate) should reach the Institute not later than July 1, 1945

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A Practical Procedure for Rigid Frame Design*

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Member American Concrete Institute

SYNOPSIS

Moment distribution has appreciably simplified theoretical studies of rigid building frames but in itself is not always a practical office tool. Two-cycle moment distribution coupled with short-cuts in loading for maxima has reduced the time element in design procedure considerably but still leaves something to be desired for actual office usage. This paper attempts to carry the two-cycle method one step further, illustrating a procedure whereby any rigid frame can be completely designed within a time period that is economically feasible for average office usage.

SYMBOLS AND NOTATIONS

a : coefficient used in $A_s = \frac{M}{ad}$
A : gross area of columns
A_s : area of tensile reinforcement; also area of web reinforcement
b : width of beams or columns
c : depth of column in direction of bending
C : $\frac{f_a}{0.45f'_c}$
d : effective depth of beams
D : $\frac{t^2}{2R^2}$; used in design of columns having bending moments
DF_{bm} : distribution factor for beams
DF_{col} : distribution factor for columns
DL : dead load
E_c : modulus of elasticity of concrete
E_s : modulus of elasticity of steel
f_a : average allowable stress on axially loaded column
f_c : compressive stress in extreme fiber
f'_c : ultimate compressive strength of concrete
f_s : stress in tensile reinforcement
f_v : stress in web reinforcement
FEM_D : fixed end moment due to dead load
FEM_T : fixed end moment due to total load
FM : final moment
g : distance (gc) between bars at opposite faces of column to over-all dimension (c)
h : length of columns (floor to floor)
j : ratio of distance (jd) between resultants of compressive and tensile stresses to effective depth
k : ratio of distance (kd) between extreme fiber and neutral axis to effective depth
K : $\frac{1}{2}f_cjk$; also relative stiffness of beams and columns

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l : centerline distance between columns; also span supported by columns
LL : live load
M : external moment
Mid_D : moment at mid-span for dead load due to fixity at ends
Mid_T : moment at mid-span for total load due to fixity at ends
n : ratio of $\frac{E_c}{E_s}$
N : external concentric load on columns
p : ratio of longitudinal reinforcement in columns
P : equivalent total concentric load on columns
s : spacing of concrete frame
S : base length of shear diagram
TL : total load
v : shearing stress
v' : shearing stress taken by web reinforcement
V : total shear
w_D : uniformly distributed dead load
w_T : uniformly distributed total load

1. PROBLEM

The introduction of moment distribution* to the engineering world during the past decade was hailed by many as marking a new era in rigid frame design and analysis. No one can deny that this has taken place. The mechanical procedure of moment distribution is so easy to learn and to apply that many problems bordering on "higher engineering" in complexity are now within the capabilities of average, practicing engineers. Surely in the field of theoretical studies and analyses, great masses of engineers are now capable of fully understanding and intelligently discussing problems which two decades ago were the chief property of professors and advanced graduate students of engineering.

All fields of engineering, however, have not benefited equally. Thus, in the design of reinforced concrete building frames where loading for maxima are required, the use of moment distribution makes the problem theoretically practical when compared to the older classical methods; but so much time is required to determine moments for the some twenty loading cases necessary for an average small rigid frame as to render this method economically impractical for general office usage. Instances can be cited where some offices have vastly reduced this voluminous amount of work by employing the simple expedient of simultaneously loading all spans with full live load and designing the frame for the resulting moments. This is, of course, not only contrary to the requirements of any recognized building code, but is also contrary to the concepts of any thinking engineer.

The use of two-cycle moment distribution combined with short-cuts in application and loading arrangements for finding maximum moments† has certainly placed the problem of rigid building frame design within the grasp of average engineers. Yet a step by step procedure of a frame three stories high and four bays wide requires too much time to be acceptable to some offices. Certainly many offices have accepted it; but for each office or engineer that uses this method, one can point to

*See "Continuity in Building Frames" by Cross and Morgan. Also PCA ST-40, 41, 42 and 43.
†See PCA "Continuity in Concrete Building Frames", Third Edition.

another group that has rejected it and returned to archaic, uneconomical, and sometimes "code-contrary" arbitrary moment coefficients.

To that group of engineers which still believes that an "exact" method of rigid frame design is too time-consuming for economical office usage, the following arrangement of the two-cycle method is offered.

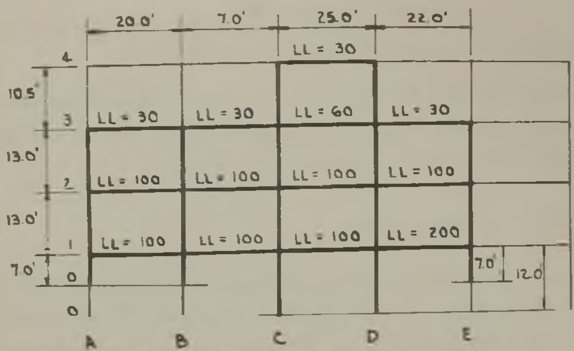
2. SOLUTION

Most concrete engineers were originally taught to design beams by use of arbitrary moment coefficients. This method requires a minimum number of sketches, most of the time being devoted to routine slide-rule computations. When trying moment distribution, once its novelty wears off the designer cannot keep from noting the vast amount of time necessary to make the numerous sketches required to prepare a pre-design of the frame from which relative stiffnesses and distribution factors for beams and columns can be computed, and to calculate *FEM*s. Until the time required to perform these operations is shortened, many designers are prone to stay away from any form of moment distribution for practical office usage.

A partial solution immediately suggests itself. The preparation of blank forms applicable to any reasonably sized frame is a step in the right direction. These forms can be penciled on tracing paper, inexpensively reproduced, and then filed away ready for instant use.

Again referring to the normal design of beams by arbitrary moment coefficients, the usual procedure is to completely design one member at a time, starting with dimensions and loads and winding up with size and spacing of stirrups. Many more calculations are necessary when moment distribution is used and a more efficient layout procedure must be adopted. It is apparent that if similar operations on all beams and columns are performed successively, rather than each member designed completely as a unit, a more efficient use of time will result. Another important advantage accrues from this procedure. The mass repetition of a single operation lends itself well to columnar layout on standard form sheets. It also has the further advantage of permitting easier checking.

These two time-savers, prepared blank forms and immediate repetition of similar operations, are the basis of this paper. Admittedly this procedure still takes more time than that required by arbitrary moment coefficients, but after one or two complete designs, the time necessary is well within any reasonable office allowance. Broken down to its elements, this method is so simple that it can be used successfully by anyone who has just a fair understanding of moment distribution; and



SPECIFICATIONS

f_s = 24000 psi
f'_c = 3000 psi
f_c = 1200 psi
n = 10
f_v = 60 psi

For 24000/10/1200

K = 178
k = 0.333
j = 0.889
a = 1.76

Beam	Slb & Jst	Ceil	Roof	Girder	Partition	FL Fin.				Total DL	LL	TL
4 CD	50	5	10	20	-	-				85	30	115
3 AB	50	10	10	20	-	-				90	30	120
3 BC	40	10	10	20	-	-				80	30	110
3 CD	60	10	-	20	20	5				115	60	175
3 DE	50	10	10	20	-	-				90	30	120
2 AB	70	10	-	25	20	5				130	100	230
2 BC	40	10	-	20	-	10				80	100	180
2 CD	70	10	-	25	20	5				130	100	230
2 DE	90	10	-	35	-	5				140	100	240
1 AB	70	-	-	25	20	5				120	100	220
1 BC	40	-	-	20	-	10				70	100	170
1 CD	70	-	-	25	20	5				120	100	220
1 DE	90	-	-	35	-	-				125	200	325

Sheet 1—Frame layout and summary of loads

it is particularly suited to larger offices where definite design standards must be adhered to.

3. EXAMPLE

The procedure is best illustrated by carrying through the complete design of an actual building frame. Eleven sheets are necessary. The first nine comprise the design calculations, while the last two are to be added to the contract drawings. Comment and description of these sheets follow.

Sheet 1. Frame layout, and summary of loads

The frame selected has considerable variation in both span and liveloading. Since the second span is flanked by spans of considerably greater length, large negative moments at midspan can be expected. Note that all the data on this sheet, as well as on all other sheets, are placed on prepared blank forms.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	Uniform (8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Beam	DL	TL	s (width)	w_D (2)(4) (kips)	w_T (3)(4) (kips)	ℓ (length)	FEM_D Table II	FEM_T Table II	Mid_D $\frac{1}{2}$ (8)	Mid_T $\frac{1}{2}$ (9)	b (beam width)	d $\sqrt{\frac{(9)\,12}{K\,(12)}}$	d Adjusted	2K Table III
4CD	85	115	19	1.6	2.2	25	83	114	42	57	12	25	25	125
3AB	90	120	19	1.7	2.3	20	57	77	28	38	12	21	31	298
3BC	80	110	19	1.5	2.1	7	6	9	3	4	12	7	31	853
3CD	115	175	19	2.2	3.3	25	114	172	57	86	12	31	31	238
3DE	90	120	19	1.7	2.3	22	69	93	34	46	12	23	31	271
2AB	130	230	19	2.5	4.4	20	83	146	42	73	14	27	33	420
2BC	80	180	19	1.5	3.4	7	6	14	3	7	14	8	33	1200
2CD	130	230	19	2.5	4.4	25	130	229	65	114	14	33	33	336
2DE	140	240	19	2.7	4.6	22	109	185	55	93	14	30	33	382
1AB	120	220	19	2.3	4.2	20	77	140	38	70	16	24	24	184
1BC	70	170	19	1.3	3.2	7	5	13	3	7	16	7	24	527
1CD	120	220	19	2.3	4.2	25	120	218	60	109	16	30	33	384
1DE	125	325	19	2.4	6.2	22	97	250	48	125	16	33	33	436

Sheet 2—FEM, Mid Mom, and K-values for beams

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Col	h (height)	w_T	ℓ (length)	TL 3·4	Outside Wall	Conc Load	Weight Col.	N Σ 5+6+7+8	ΣN	A $\frac{\Sigma N}{0.9}$	b	$C = \frac{A}{b}$ min = 10"	K Table IV
A 32	13.0	2.3	10.0	23	8		2	33	33	36	12	10	8
A 21	13.0	4.4	10.0	44	25		3	72	105	117	14	10	9
A 10	7.0	4.2	10.0	42	25		3	70	175	194	16	12	33
B 32	13.0	2.2	13.5	30	-		2	32	32	36	12	10	8
B 21	13.0	3.9	13.5	53	-		3	56	88	97	14	10	9
B 10	7.0	3.7	13.5	50	-		3	53	141	158	16	10	19
C 43	10.5	2.2	12.5	28	8		2	38	38	42	12	10	10
C 32	13.0	2.7	16.0	43	20		2	65	103	115	12	10	8
C 21	13.0	3.9	16.0	62	-		3	65	168	187	14	14	25
C 10	12.0	3.7	16.0	59	-		3	62	230	256	16	16	45
D 43	10.5	2.2	12.5	28	8		2	38	38	42	12	10	10
D 32	13.0	2.8	23.5	66	20		2	88	126	139	12	12	13
D 21	13.0	4.5	23.5	106	-		3	109	235	260	14	20	73
D 10	12.0	5.2	23.5	122	-		3	125	359	400	16	24	155
E 32	13.0	2.3	11.0	25	8		2	35	35	39	12	10	8
E 21	13.0	4.6	11.0	51	25		3	79	114	127	14	10	9
E 10	7.0	6.2	11.0	68	25		3	96	210	234	16	16	77

Sheet 3—Axial load and K-values for columns

Sheet 2. FEM, mid moment and K-values for beams

The *FEM*'s and midmoments shown in cols. 8, 9, 10, and 11 are taken from Table 2 in "Continuity in Concrete Building Frames". Those who object to using tables can compute moments by employing conventional slide rule procedure. In computing relative stiffnesses, note that $2K$ is used instead of K, the doubled value being used to more nearly approximate the T-action to which these beams are subjected.

In sizing the structure for computing distribution coefficients, the designer must decide whether a uniform depth or variable (for balanced design) depth of beam for any one floor should be adopted. The former practice will simplify form construction and erection at the cost of some extra concrete. For either method, a trial depth is calculated (col. 12) based on the *FEM* (col. 9) and that beam width which will produce a depth not interfering with any headroom requirements. If a uniform beam depth per floor is adopted, the greatest trial depth (col. 13) should be used. Many test cases have verified the reasonable accuracy of this procedure. When a variable depth is to be used, the problem is considerably complicated by the many combinations of lengths of spans and loadings. As a general rule, the maximum trial depth will change negligibly, while the lesser trial depths increase in size. In this frame, constant depths are to be used for floors 2 and 3, while floor 1 is divided into two groups of two beams each, the depth of each group being determined by the maximum trial depth in that group. Final computed depths (Sheet 7) indicate that sizing of the members is correct for all practical purposes.

The design illustrated considers uniform loads only. If concentrated loads exist, additional columns for concentrated *FEM*s must be inserted between cols. 11 and 12.

Sheet 3. Axial load and K-values for columns

To shorten the number of calculations, the running load per foot of supported floor (col. 3) is the average of the two supported beams (Sheet 2, col. 6). In sizing the columns, an average compressive strength of 900 psi is used to compute the required area. The relative stiffnesses of the columns K, can be either taken from Table IV or computed in conventional slide rule manner.

Sheet 4. K-values and distribution factors at joints

No comment.

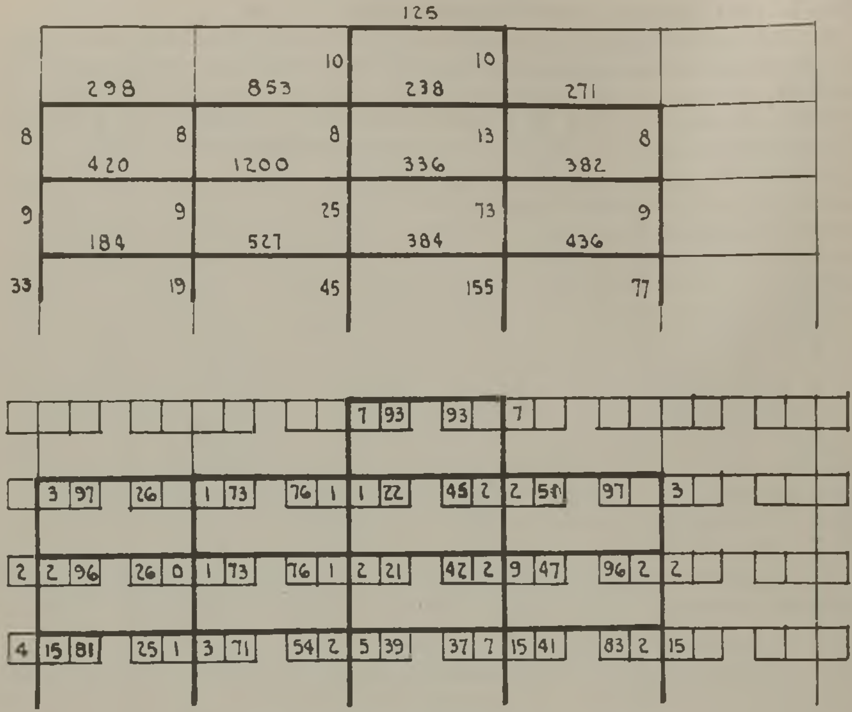
Sheets 5 and 6. Maximum positive, negative, and exterior column moments. Minimum positive and maximum interior column moments

The distribution process follows exactly the two-cycle method outlined in "Continuity in Concrete Building Frames." It is a safe conjecture to make that if the normal process of moment distribution were applied to this frame, coupled with the multiplicity of loadings necessary to produce maximums, the work entailed would have been increased four- or five-fold.

Sheet 7. Design of beams

Since both the Joint Committee report (1940) and ACI Code* require that moments be computed by centerline distances but permit beams to be designed by moments at faces of supports, a correction of the moments already computed may be made. This correction is made by deducting $\frac{1}{3}Vc$ at the supports and $\frac{1}{6}Vc$ at the centerline, where V equals simple shear and c equals the depth of the column. For minimum moments, the correction is added rather than deducted. A complete discussion of this point can be found in "Continuity in Concrete Building Frames."

*Building Regulations for Reinforced Concrete" (ACI 318-41)

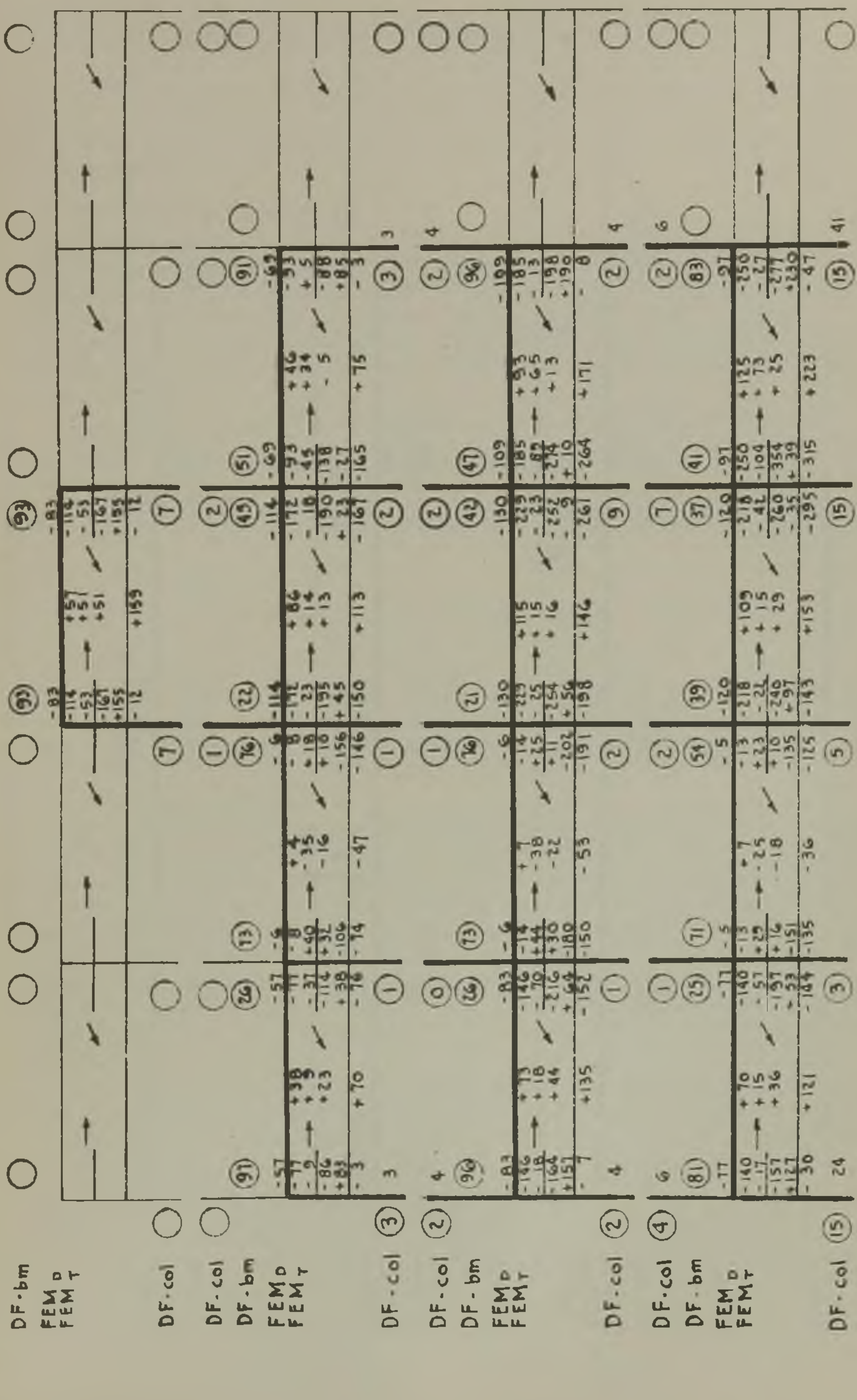


Sheet 4—K-values and distribution factors at joints

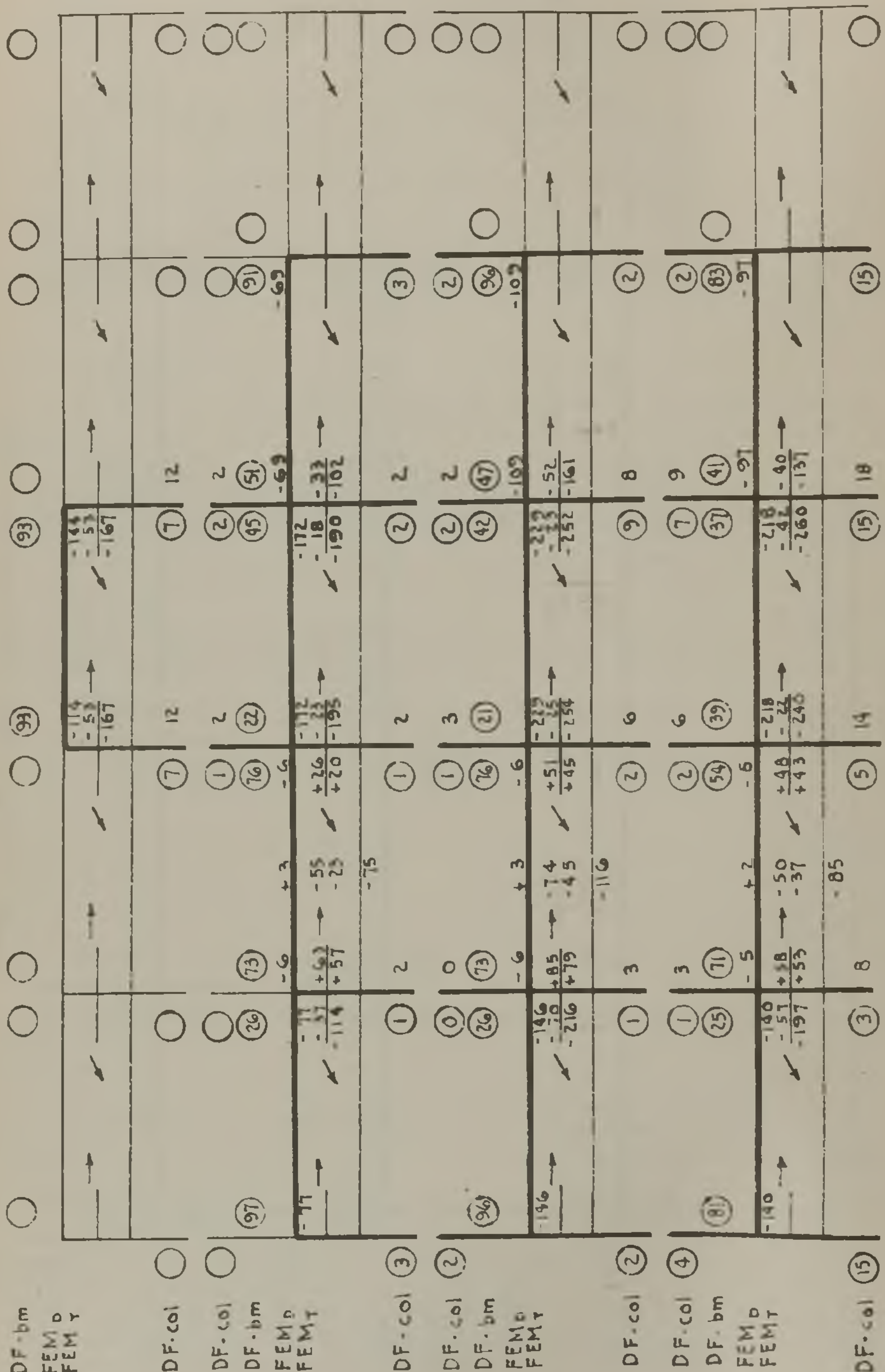
A comparison of the final required beam depths with the preliminary depths (Sheet 2) is very favorable for floors 2 and 3. For floor 4 (upper Roof level) the estimated depth of 25 inches is 4 inches less than the required depth of 29 inches. This increased depth will also increase the stiffness of the beam, but since it is already taking 93 per cent of the unbalanced moment at the corner joints, any change will be relatively small. However, the head room of this already low ceiling room has been reduced another four inches. It is therefore necessary to increase the 10.5 foot distance between levels 3 and 4 to 11.0 feet, or design the beam with compressive reinforcement.

At floor 1, beams 1-AB and 1-BC check out exactly, but the estimated 33-inch depth for 1-CD and 1-DE must be increased to 36 in. Revised moments computed on the increased stiffness of this deeper beam result in no moment increase greater than several per cent. This is shown on Check Sheet 1 for which beam stiffnesses based on a depth of 36-in. are used. This demonstration is included to show that final depths can vary appreciably from preliminary depths with only negligible changes in the moments. Such a check may usually be omitted.

Inflection points are computed to the nearest half-foot by determining the ratio of final moments to *FEM*s and entering this figure on the diagram shown. This diagram assumes symmetrical end moments but is reasonably accurate for unsymmetrical conditions. However, where one end moment is close to zero, appreciable errors may result from the use of this diagram. A study of inflection points for beams having zero moments at one end reveals that coefficients for finding the inflection points at the opposite ends are exactly equal to one-sixth of the ratio of final moments to *FEM*s.



Sheet 5—Maximum positive, negative and exterior column



Sheet 6—Minimum positive and maximum interior column

All end spans for this frame have negligible moments at the wall end, and this rule for locating inflection points at the opposite end is used.

Note that in designing the steel for beams, the coefficient a is used in lieu of $\frac{fs}{12000}$ x (average j value). This small time saver per beam represents a substantial saving in time for all 13 beams.

Sheet 8. Design of columns

Accurate column design involving both bending and axial load is usually a fairly time-consuming problem. The method illustrated on sheet 8 is accurate within a very small per cent and requires little more time than for columns having direct load only. In column 5, the value of CD is assumed equal to 3 for tied columns, 4 for square spiral columns, and 6 for round spiral columns. Thus the equivalent axial load can be immediately found and the column is designed by Table 18 in the ACI Reinforced" Concrete Design Handbook".*

Some question might be raised regarding the accuracy of the above assumed values of CD. To prove its accuracy, consider a redesign of Column C-21 by the method outlined in "Continuity in Concrete Building Frames."

Refer to Table VII.

For $g = 0.71$, $(n-1)p = 0.20$ (estimated) and rectangular section with ties: $D = 5.6$

Refer to Table VI.

For $f'_c = 3000$, $f_s = 20{,}000$, $p = 0.020$ (estimated) and tied columns: $C = 0.54$.

Compute $CD\dfrac{M}{c} = 0.54 \times 5.6 \times \dfrac{6}{14} \times 12 = 16$

Add	$N = 168$	
Design section for total load	$P = 184$	About equal
By short-cut above	$P = 183$	

Sheet 9. Design of stirrups

Since continuity is an essential factor in this design, it will be necessary to correct shears due to unequal end moments. The shear correction for end spans can be computed directly from end moments shown on Sheet 5. For interior spans, however, the smaller of the two end moments for any beam can always be reduced by a proper reloading of the frame, thereby increasing the maximum end shear. A study of many interior beams reveals that the shear correction is rarely more than double the value obtained by using the end moments shown on Sheet 5. Since the shear corrections due to continuity is usually only a small per cent of the simple shear, the doubling method just outlined is a rapid, safe, and reasonably economical way to achieve an accurate result.

In computing shears, note that the simple shear (col. 5) has already been computed on Sheet 7. Once the corrected shears have been calculated, the method of stirrup design is modeled directly after the procedure outlined in "Reinforced Concrete Design Handbook."

Sheets 10 and 11. Beam and column schedules

These two sheets compile the actual design requirements into the usual tables which appear on most contract drawings. Their arrangement is such that the detailer can trace them directly on the final plans.

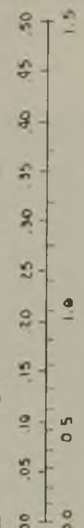
*Report of ACI Committee 317, 1939.

							0	
Theor. Max M							-12	+159
V Max end shear							28	
Less $\frac{1}{4}V_c$ or $\frac{1}{8}V_c$							8	4
Design M							-4	+155
$d=\sqrt{\frac{M\cdot 12}{Kb}}$								29
$A_s=\frac{M}{ad}$								3.00
Top at support							2·1ϕ	
Trussed bars								2·1ϕ
Straight, bottom								2-1ϕ
Steel provided								3.16
	0		3				4½	
Theor Max M	-3	+70	-76	-74	-75	-146	-150	+113
V Max end shear	23		23	7		7	41	
Less $\frac{1}{4}V_c$ or $\frac{1}{8}V_c$	6	3	6	2	-1	2	11	6
Design M	+3	+67	-70	-72	-76	-144	-139	+107
$d=\sqrt{\frac{M\cdot 12}{Kb}}$								
$A_s=\frac{M}{ad}$		1.27	1.31	1.37	1.49	3.00	2.74	2.04
Top at support	2·$\frac{5}{8}\phi$			2-1ϕ ←	cont	→ 2·1ϕ	2·1ϕ	
Trussed bars		2·$\frac{5}{8}\phi$ (L only)			-			2·1ϕ
Straight, bottom		2·$\frac{5}{8}\phi$			2·$\frac{5}{8}\phi$ (nominal)			2·$\frac{5}{8}\phi$
Steel provided		1.24	1.58	1.58	1.58	3.16	3.16	2.20
	0		3½				4½	
Theor Max M	-7	+135	-152	-150	-116	-191	-198	+146
V Max end shear	44		44	12		12	55	
Less $\frac{1}{4}V_c$ or $\frac{1}{8}V_c$	12	6	12	3	-2	5	21	11
Design M	+5	+129	-140	-147	-118	-186	-177	+135
$d=\sqrt{\frac{M\cdot 12}{Kb}}$								
$A_s=\frac{M}{ad}$		2.16	2.34	2.46	1.62	3.14	2.96	2.26
Top at support	2·$\frac{3}{4}\phi$		2·$\frac{3}{4}\phi$	2·1ϕ ←	cont	→ 2·1ϕ	2·1□	
Trussed bars		2·$\frac{3}{4}\phi$			-			2·1□
Straight, bottom		2·$\frac{7}{8}\phi$			2·$\frac{5}{8}\phi$ (nominal)			2·$\frac{5}{8}\phi$
Steel provided		2.08	2.46	2.46	1.58	3.58	3.58	2.62
	0		3½				3½	
Theor Max M	-30	+121	-144	-135	-85	-125	-143	+153
V Max end shear	42		42	11		11	52	
Less $\frac{1}{4}V_c$ or $\frac{1}{8}V_c$	14	6	12	3	-2	5	23	12
Design M	-16	+115	-134	-132	-87	-120	-120	+141
$d=\sqrt{\frac{M\cdot 12}{Kb}}$			24					
$A_s=\frac{M}{ad}$		3.10	3.18	3.14	2.06	3.84	1.90	2.22
Top at support	2·1ϕ		2·$\frac{5}{8}\phi$	2·1$\frac{1}{8}$□ ←	cont	→ 2·1$\frac{1}{8}$□	2·1ϕ	
Trussed bars		2·1ϕ (L only)			-			2·1ϕ (L only)
Straight, bottom		2·1ϕ			2·$\frac{5}{8}\phi$ (nominal)			2·$\frac{5}{8}\phi$
Steel provided		3.16	3.16	3.16	2.54	4.12	4.12	2.20

Sheet 7—Design of Beams (continued opposite page)

← Inflection Points *

0			
-12			
28			
8			
-4			
2-1φ			
5	6½		0
-167	-165	+75	-3
41	25		25
14	8	3	7
-153	-157	+72	+4
	29		
2.96	3.04	1.57	
2-1φ	2-1φ		2-⅝φ
		2-⅝φ (R. only)	
		2-¾φ	
3.16	3.16	1.50	
6½	5		0
-261	-264	+171	-8
55	51		51
31	28	7	14
-230	-236	+169	+6
	34		
3.85	3.95	2.74	
2-1□	2-1□		2-1□
		2-1□	
		2-¾φ	
4.00	4.00	2.88	
8½	4		0
-295	-315	+223	-47
52	68		68
35	45	15	30
-260	-270	+208	-17
	36		
4.10	4.26	3.80	
1-1⅛□, 1-1□	2-1□		2-1□
		2-1□ (R. only)	
		2-1□	
4.27	4.27	4.00	



* Inflection points: a) Approx. symmetry: Ratio of $\frac{FM}{FEM}$ plus diagram

b) Zero moment one end: Coeff. of L = $\frac{1}{6} \cdot \frac{FM}{FEM}$

Sheet 7—(continued from opposite page)—**Design of beams**

1	2	3	4	5	6	7	8	9
Col.	Est. Size b x e	N Axial	M	$\frac{CD^* M}{e}$ $\frac{CD^* (4) 12}{(2)}$	P (5)+(3)	Load on Conc. Table 18	Load on Steel (6)-(7)	Steel Table 18
A 32	12 x 10	33	4	14	47	65	-	4-$\frac{5}{8}\phi$
A 21	14 x 10	105	6	22	127	76	51	6-$\frac{3}{4}\phi$
A 10	16 x 12	175	20	60	235	104	131	8-1ϕ
B 32	12 x 10	32	2	7	39	65	-	4-$\frac{5}{8}\phi$
B 21	14 x 10	88	3	11	99	76	23	4-$\frac{5}{8}\phi$
B 10	16 x 10	141	8	30	171	86	85	6-1ϕ
C 43	12 x 10	38	12	36	74	65	9	4-$\frac{5}{8}\phi$
C 32	12 x 10	103	3	11	114	65	49	4-$\frac{7}{8}\phi$
C 21	14 x 14	168	6	15	183	106	77	4-1$\square$
C 10	16 x 16	230	14	31	261	138	123	4-1$\frac{1}{4}\square$
D 43	12 x 10	38	12	43	81	65	16	4-$\frac{5}{8}\phi$
D 32	12 x 12	126	2	6	132	78	54	4-1ϕ
D 21	14 x 20	235	9	16	251	151	101	4-1$\frac{1}{8}\square$
D 10	16 x 24	359	18	27	386	207	179	6-1$\frac{1}{4}\square$
E 32	12 x 10	35	4	14	49	65	-	4-$\frac{5}{8}\phi$
E 21	14 x 10	114	6	22	136	76	60	4-1ϕ
E 10	16 x 16	210	41	92	302	138	164	6-1$\frac{1}{4}\square$

* CD = 3 for Tied Columns
4 for Square Spiral Column
6 for Round Spiral Column
} Assumed tied this problem

Sheet 8—Design of columns

Sheets 10 and 11 cont'd

Note that a minimum distance of two feet is used for all inflection points. Also note that in the short seven-foot spans, inflection points are shown to exist at a distance of two feet from the column centerline. Of course, these beams have no inflection points but this arbitrary distance is used to establish cut-off points for negative steel from adjacent beams.

4. CONCLUSIONS

The total time necessary to fill out the eleven blank forms required about twenty hours, or two and one-half working days. To this must be added the time necessary to check the work and make corrections.

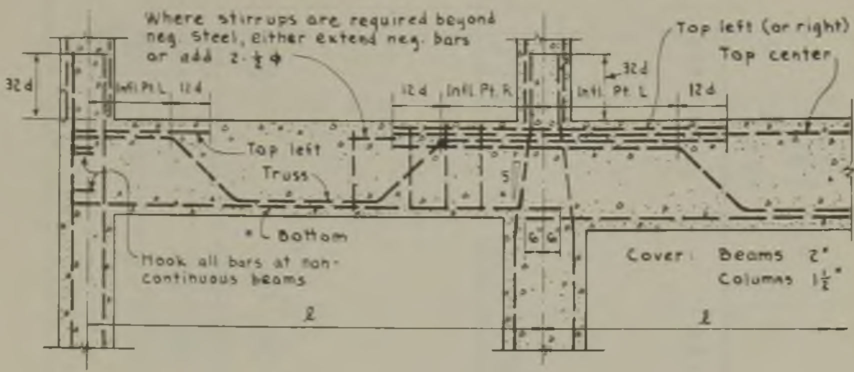
(1) Beam	(2) b	(3) d	(4) ℓ	(5) V_{sim}	(6) V_{con}*	(7) V $(5)+(6)$	(8) v $\frac{(7)}{(2)\frac{7}{8}(3)}$	(9) v' $(8)-.02f'_c$	(10) S $\frac{(9)}{(8)}\frac{(4)}{2}$	(11) Stirrup Size	(12) $A_s f_v$ Diagram 17	(13) Max $\frac{1}{S}$ $\frac{(9)(2)}{(12)}$	(14) Index $1.5\frac{(10)}{(13)}$	(15) Stirrups Required Diagram 17
4 CD	12	29	25	28	0	28	92	32	4.4	$\frac{3}{8}\phi$	5280	0.073	130	1/6 4/12
3 AB	12	29	20	23	4	27	89	29	3.3	$\frac{3}{8}\phi$	5280	0.066	75	1/6 3/12
3 BC	12	29	7	7	20	27	88	28	1.1	$\frac{3}{8}\phi$	5280	0.066	25	1/6 1/12
3 CD	12	29	25	41	2	43	142	82	7.2	$\frac{3}{8}\phi$	5280	0.186	58	1/3 3/6 3/8 3/12
3 DE	12	29	22	25	7	32	105	45	4.7	$\frac{3}{8}\phi$	5280	0.102	69	1/6 4/12
2 AB	14	34	20	44	7	51	122	62	5.1	$\frac{3}{8}\phi$	5280	0.165	46	1/3 1/6 2/8 3/12
2 BC	14	34	7	12	12	24	58	-	-	-	-	-	-	-
2 CD	14	34	25	55	5	60	144	84	7.3	$\frac{3}{8}\phi$	5280	0.233	49	1/2 2/4 4/6 2/8 3/12
2 DE	14	34	22	51	12	63	151	91	6.6	$\frac{1}{2}\phi$	9600	0.132	75	1/4 2/8 5/12
1 AB	16	24	20	42	6	48	143	83	5.8	$\frac{3}{8}\phi$	5280	0.252	35	1/2 4/3 2/6 2/8 2/12
1 BC	16	24	7	11	4	15	45	-	-	-	-	-	-	-
1 CD	16	36	25	52	12	64	127	67	6.6	$\frac{3}{8}\phi$	5280	0.203	49	1/3 3/6 2/8 3/12
1 DE	16	36	22	68	12	80	159	99	6.9	$\frac{1}{2}\phi$	9600	0.164	63	1/3 1/6 3/8 4/12

* Correct for end beams. Values doubled for interior beams. [End moments taken from sheet 5]

Sheet 9—Design of stirrups

Beam	ℓ	w	d	Truss	Bottom	Top Left	Top Center	Top Right	Infl. Points Left	Infl. Points Right	Stirrups Size	Stirrups Spacing	Remarks
4CD	25	12	29	2-1φ	2-1φ				2'	2'	$\frac{3}{8}$φ	1/6 4/12	
3AB	20	12	29	2-$\frac{5}{8}$φ	2-$\frac{5}{8}$φ				2'	3'	$\frac{3}{8}$φ	1/6 3/12	Truss left side only
3BC	7	12	29		2-$\frac{5}{8}$φ		2-1φ		2'	2'	$\frac{3}{8}$φ	1/6 1/12	
3CD	25	12	29	2-1φ	2-$\frac{5}{8}$φ				4$\frac{1}{2}$'	5'	$\frac{3}{8}$φ	1/3 3/6 3/8 3/12	
3DE	22	12	29	2-$\frac{5}{8}$φ		2-1φ			6$\frac{1}{2}$'	2'	$\frac{3}{8}$φ	1/6 4/12	Truss right side only
2AB	20	14	34	2-$\frac{3}{4}$φ	2-$\frac{7}{8}$φ				2'	3$\frac{1}{2}$'	$\frac{3}{8}$φ	1/3 1/6 2/8 3/12	
2BC	7	14	34		2-$\frac{5}{8}$φ		2-1φ		2'	2'	-		
2CD	25	14	34	2-1□	2-$\frac{5}{8}$φ				4$\frac{1}{2}$'	6$\frac{1}{2}$'	$\frac{3}{8}$φ	1/2 2/4 4/6 2/8 3/12	
2DE	22	14	34	2-1□	2-$\frac{3}{4}$φ				5'	2'	$\frac{1}{2}$φ	1/4 2/8 5/12	
1AB	20	16	24	2-1φ	2-1φ			2-$\frac{5}{8}$φ	2'	3$\frac{1}{2}$'	$\frac{3}{8}$φ	1/2 4/3 2/6 2/8 2/12	Truss left side only
1BC	7	16	24		2-$\frac{5}{8}$φ		2-1$\frac{1}{8}$□		2'	2'	-		
1CD	25	16	36	2-1φ	2-$\frac{5}{8}$φ			1-1$\frac{1}{8}$□ 1-1□	3$\frac{1}{2}$'	8$\frac{1}{2}$'	$\frac{3}{8}$φ	1/3 3/6 2/8 3/12	Truss left side only
1DE	22	16	36	2-1□	2-1□	2-1□			4'	2'	$\frac{1}{2}$φ	1/3 1/6 3/8 4/12	Truss right side only

Sheet 10—Beam schedule



4						
			12 × 10 4 · $\frac{5}{8}\phi$	12 × 10 4 · $\frac{5}{8}\phi$		
3						
	12 × 10 4 · $\frac{5}{8}\phi$	12 × 10 4 · $\frac{5}{8}\phi$	12 × 14 4 · $\frac{7}{8}\phi$	12 × 12 4 · 1ϕ	12 × 10 4 · $\frac{5}{8}\phi$	
2						
	14 × 10 6 · $\frac{3}{4}\phi$	14 × 10 4 · $\frac{5}{8}\phi$	14 × 14 4 · 1□	14 × 20 4 · $1\frac{1}{8}$□	14 × 10 4 · 1ϕ	
1						
	16 × 12 8 · 1ϕ	16 × 10 6 · 1ϕ	16 × 16 4 · $1\frac{1}{4}$□	16 × 24 6 · $1\frac{1}{4}$□	16 × 16 6 · $1\frac{1}{4}$□	
0						
	A	B	C	D	E	

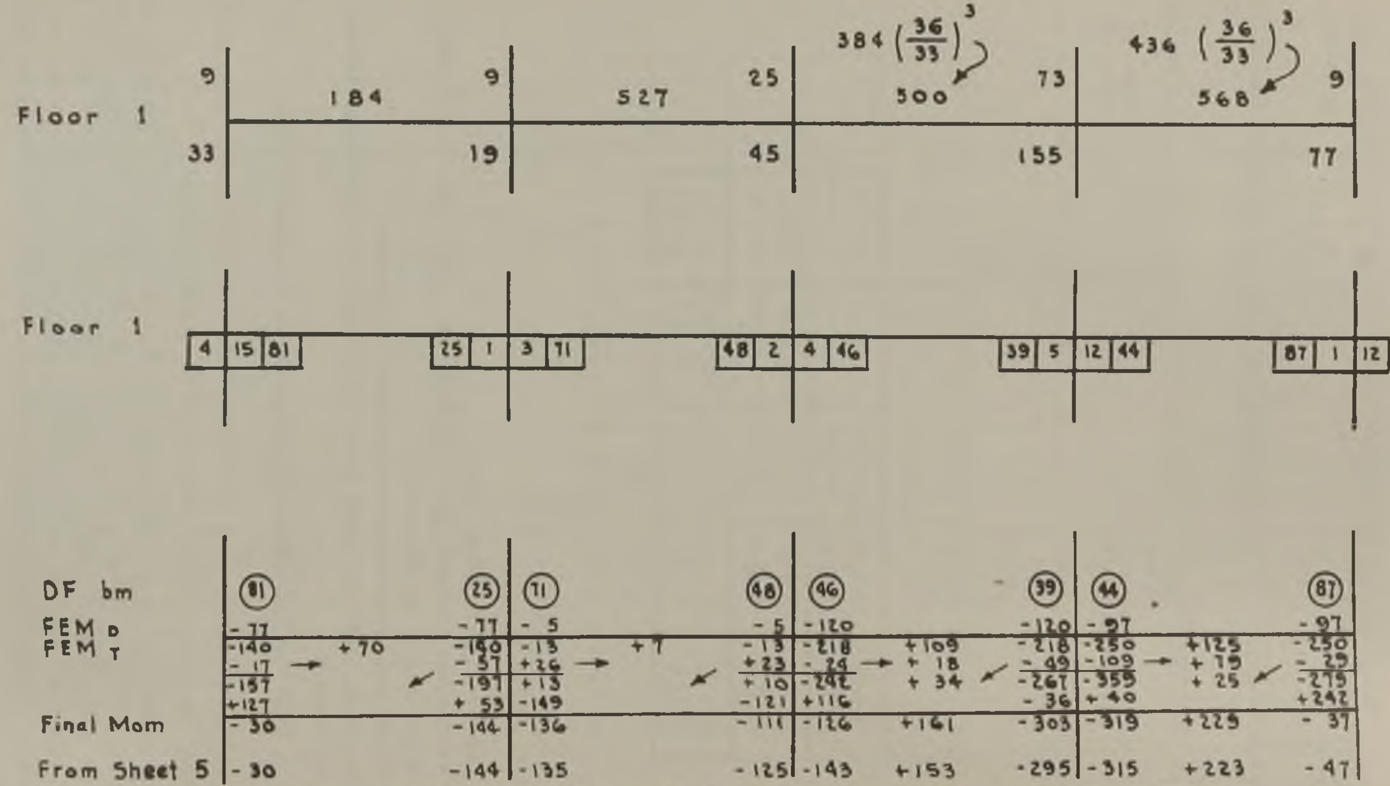
Column Notes

1. Lap bars 32 d above floor
2. All columns to have $\frac{3}{8}\phi$ ties @ 12" oc. Use 1 tie for 4 bars, 2 for 6, 3 for 8, etc.
3. First dimension is normal to plane of frame.
4. Arrange half of steel along each face of dimension normal to frame.

Sheet 11—Typical beam and column details and column schedule

Since thirteen beams and seventeen columns have been completely designed, the time required does not seem unreasonable.

Some concern may be felt regarding the necessity of completely redesigning a frame because the calculated dimensions are appreciably different from the preliminary sizing. Reference to Check Sheet 1 should minimize concern on this point, provided the predesign is based on *FEMs*. Admittedly, there is room for considerable study on this



Check Sheet 1—K-values, distribution factors at joints, maximum positive and negative moments—Moment comparison for revised beam stiffnesses feature of the procedure, particularly where variable beam depths are to be used.

A casual inspection of the work reveals that few new ideas or thoughts have been introduced in this procedure. It is essentially a careful arrangement of existing methods. As a result, it can be readily understood and applied by inexperienced engineers.

[To facilitate selective distribution, **separate prints** of this title (41-20) are currently available from ACI at 25 cents each—quantity quotations on request. **Discussion** of this paper (copies in triplicate) should reach the Institute not later than July 1, 1945]

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Dynamic Testing of Pavements*

By GERALD PICKETT†

SYNOPSIS

A theoretical analysis is made of the problem of the vibration of a pavement in contact with an elastic subgrade. The analysis shows that for each frequency of sustained vibration, waves may be propagated horizontally with three different velocities. The properties of the subgrade have very little effect on the highest or the lowest of these three velocities but have considerable effect on the intermediate velocity. Near the origin the velocity of each wave may be greater or less than the velocity farther from the origin. The analysis indicates that the properties of both pavement and subgrade may be determined for any small region of the pavement by placing the source of sustained vibrations in that region.

INTRODUCTION

The dynamic method of determining Young's modulus of elasticity of concrete has been used for several years as a laboratory test.[1-4] A specimen under test is caused to vibrate at one or more of its resonant frequencies while being supported either on rubber or in such a way that the support has a negligible effect upon the results. Equations giving the approximate value of these resonant frequencies in terms of the weight, elastic constants, and dimensions of the specimen are available for the usual shapes of laboratory specimens, and thus the modulus of elasticity can be readily computed after the resonant frequency has been determined experimentally.

The application of dynamic testing to concrete pavements and structures in place has probably been retarded both by lack of suitable measuring instruments and by the lack of the necessary mathematical expressions for interpreting the results of measurements. Bernhard[5] compared the vibration properties of pavement slabs of different thicknesses. He used a mechanical oscillator with a frequency range of from ½ to

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[1-4] See references at end of paper.

40 cycles per second. Recently, Long, Kurtz, and Sandenaw[6,7] have developed instruments that give reasonable accuracy for the time of travel of waves between two points on the surface of a pavement slab, even when the points are no more than a foot apart. They measured velocity of waves produced by impact and also those produced by sustained vibration at frequencies ranging from 1000 to 2000 cycles per second. They also made some progress in interpreting the results in terms of Young's modulus of elasticity of the pavements by the use of equations developed by Lamb[8] and Timoshenko.[9]

The accomplishments noted above and the fact that the use of the equations of Lamb and Timoshenko for this purpose may be questionable stimulated the author to prepare the present paper.

LIMITATIONS OF PREVIOUS EQUATIONS

The equations of Lamb and Timoshenko are not strictly applicable to pavements because:

(1) They are for freely vibrating slabs, whereas the pavement is in contact with the subgrade.

(2) They are for a two-dimensional problem (plane waves), whereas the vibration of a pavement is a three-dimensional problem.

In general both the shape and the support of field structures are such as to make analysis of vibration more complicated than that for small laboratory specimens.

So far as is known, the nearest analytical approach to the vibration of a pavement was a study made by Love[10] in which he investigated the vibration of the earth's crust, assuming that the elastic properties of the crust differ from those of the interior. However, Love's studies were also confined to two-dimensional problems and only dealt with cases in which the crust was less rigid than the interior, whereas a concrete pavement will usually be more rigid than its subgrade.

SCOPE OF PRESENT DISCUSSION

This paper will discuss certain possible modes of vibration of a pavement in contact with its subgrade that are likely to occur in the dynamic testing of pavements. An equation giving the relation between driving frequency, thickness of slab, elastic constants of both pavement and subgrade, and the velocity of wave is derived on the assumption of continuity of motion between pavement and subgrade. The equations of elasticity applicable to a homogeneous, isotropic, elastic solid are used for both the slab and the subgrade.

If experiments prove that the equations are generally applicable to pavements in place, then from a few dynamic measurements it should be possible to determine not only Young's modulus for the concrete but also the thickness of the pavement and a modulus for the subgrade.

PARTICULAR SOLUTIONS OF THE DIFFERENTIAL EQUATIONS OF VIBRATION

When expressed in the cylindrical coordinates r, Θ, z the differential equations of vibration for a homogeneous, isotropic, elastic solid are:[11]

$$(\lambda + 2G)\frac{\partial \Delta}{\partial r} - \frac{2G}{r}\frac{\partial W}{\partial \Theta} + 2G\frac{\partial V}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2}$$

$$(\lambda + 2G)\frac{1}{r}\frac{\partial \Delta}{\partial \Theta} - 2G\frac{\partial U}{\partial z} + 2G\frac{\partial W}{\partial r} = \rho\frac{\partial^2 v}{\partial t^2}$$

$$(\lambda + 2G)\frac{\partial \Delta}{\partial z} - \frac{2G}{r}\frac{\partial}{\partial r}(rV) + \frac{2G}{r}\frac{\partial U}{\partial \Theta} = \rho\frac{\partial^2 w}{\partial t^2}$$

where u, v, w are displacements in the r, Θ, z directions, respectively;
t is time;

$$\Delta = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \Theta} + \frac{\partial w}{\partial z};$$

$$2U = \frac{1}{r}\frac{\partial w}{\partial \Theta} - \frac{\partial v}{\partial z};$$

$$2V = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r};$$

$$2W = \frac{\partial v}{\partial r} + \frac{v}{r} - \frac{1}{r}\frac{\partial u}{\partial \Theta};$$

ρ = mass density;

$\lambda = \dfrac{\mu E}{(1 + \mu)(1 - 2\mu)}$, Lamé's constant;

$G = \dfrac{E}{2(1 + \mu)}$, modulus of elasticity in shear;

E = Young's modulus; and

μ = Poisson's ratio.

Four particular solutions of the differential equations will be used in the following discussions. Two of these, designated H and V, apply to the pavement slab, and two, designated H_1 and V_1 apply to the subgrade. The solutions will be used in pairs, the H solution for the pavement with the H_1 solution for the subgrade, etc. In the H and H_1 solutions points on the line $r = 0$ move in a horizontal direction and in the V and V_1 solutions these points move in a vertical direction. In the following all subscripts of unity except that designating a Bessel function of the first order refer to the subgrade. (J_o and J_1 are Bessel functions.) The solutions are as follows:

$$\begin{aligned} u &= Z \sin \Theta \cos pt \left[J_o (a\, r) - \frac{J_1 (ar)}{ar} \right] \\ v &= Z \cos \Theta \cos pt \frac{J_1 (ar)}{ar} \\ w &= Z' \sin \Theta \cos pt\, J_1 (ar) \end{aligned} \tag{H}$$

$$\begin{aligned} u_1 &= Z_1 \sin \Theta \cos pt \left[J_o (ar) - \frac{J_1 (ar)}{ar} \right] \\ v_1 &= Z_1 \cos \Theta \cos pt \frac{J_1 (ar)}{ar} \\ w_1 &= -Z_1' \sin \Theta \cos pt\, J_1 (ar) \end{aligned} \tag{H$_1$}$$

$$\begin{aligned} u &= Z \cos pt\, J_1 (ar) \\ v &= 0 \\ w &= -Z' \cos pt\, J_o (ar) \end{aligned} \tag{V}$$

$$\begin{aligned} u_1 &= Z_1 \cos pt\, J_1 (ar) \\ v_1 &= 0 \\ w_1 &= Z_1' \cos pt\, J_o (ar) \end{aligned} \tag{V$_1$}$$

where

$$\begin{aligned} Z &= M \cosh mz + M' \sinh mz + N \cosh nz + N' \sinh nz; \\ Z' &= \frac{Mm}{a} \sinh mz + \frac{M'm}{a} \cosh mz + \frac{Na}{n} \sinh nz \\ &\quad + \frac{N'a}{n} \cosh nz; \\ Z_1 &= Re[M_1 e^{-m_1 z} + N_1 e^{-n_1 z}]\ ; \\ Z_1' &= Re\left[\frac{M_1 m_1}{a} e^{-m_1 z} + \frac{N_1 a}{n_1} e^{-n_1 z}\right]; \\ m &= a\sqrt{1 - a^2}\ , \qquad m_1 = a\sqrt{1 - a_1^2}\ ; \\ n &= a\sqrt{1 - b^2}\ , \qquad n_1 = a\sqrt{1 - b_1^2}\ ; \\ a &= \frac{p}{a}\sqrt{\frac{\rho}{\lambda + 2G}}\ , \qquad a_1 = \frac{p}{a}\sqrt{\frac{\rho_1}{\lambda_1 + 2G_1}}\ ; \\ b &= \frac{p}{a}\sqrt{\frac{\rho}{G}}\ , \qquad b_1 = \frac{p}{a}\sqrt{\frac{\rho_1}{G_1}}\ ; \end{aligned}$$

p = 2π times the frequency of vibration;

α = $p/V = 2\pi/l$ where V is the radial velocity which the wave approaches asymptotically as it gets farther from its source and l is the corresponding wave length; and

M, N, M', N', M_1, and N_1 are constants proportional to the amplitude of vibration.

Re [ ] signifies the real part of the expression in the bracket if either m_1 or n_1 is imaginary in a mathematical sense. If both m_1 and n_1 are real, then the Re in front of the brackets may be disregarded.

In each pair of solutions, for example, H and H_1, the frequency $p/2\pi$, the velocity p/α, the six amplitude constants M, N, M', N', M_1, and N_1, and the physical properties are arbitrary. That is, the differential equations are satisfied for any arbitrary values of these parameters. However, the boundary requirements at the top of the pavement and at the common boundary between pavement and subgrade permit the elimination of the six amplitude constants. The result is an equation, called the frequency equation, giving the relation between frequency, velocity of wave propagation and the physical properties of pavement and subgrade. It is of interest that the frequency equation is the same whichever pair of solutions is used and depends only on the assumptions made in regard to boundary conditions. In the derivation which follows the assumption is made that the top of the pavement is free of force and that the boundary stresses and displacements of the pavement are equal to those of the subgrade at their common boundary.

DERIVATION OF THE FREQUENCY EQUATION

The plane $z = 0$ is taken in the middle of a slab of thickness $2c$ and the direction $\Theta = 0$ is taken as due east as shown in Fig. 1.

The assumed boundary requirements result in the following relations:*

1. The top of the pavement is free of vertical force. $\sigma_z = 0$ at $z = -c$.
2. The top of the pavement is free of radial and tangential forces. $\tau_{rz} = \tau_{\Theta z} = 0$ at $z = -c$.
3. At the common boundary the vertical displacement of the pavement equals the vertical displacement of the subgrade. $w = w_1$ at $z = c$.
4. At the common boundary the radial and tangential displacements of the pavement equal the radial and tangential displacements, respectively, of the subgrade. $u = u_1$ and $v = v_1$ at $z = c$.
5. At the common boundary the vertical normal stresses are equal. $\sigma_z = \sigma_{1z}$ at $z = c$.

*Two relations are given in each of requirements 2, 4, and 6; but in each case the second relation is satisfied if the first is satisfied. Therefore the two relations give only one independent equation.

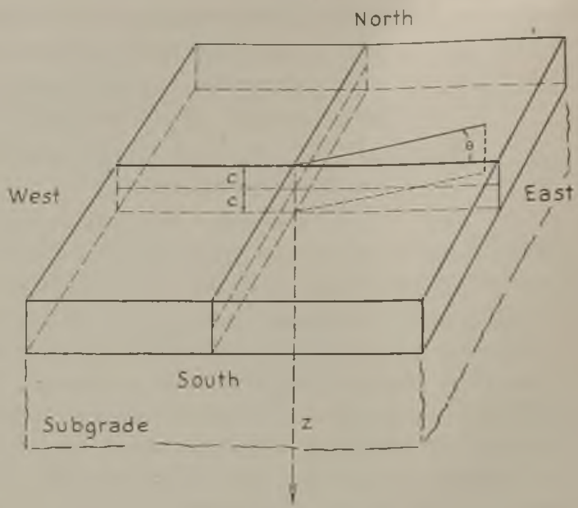

Fig. 1—Element of pavement slab and subgrade

6. At the common boundary the boundary shear stresses are equal. $\tau_{rz} = \tau_{1rz}$ and $\tau_{\Theta z} = \tau_{1\Theta z}$ at $z = c$.

When either pair of solutions for displacements is used and use is made of the relations previously given between m, n, λ, G, a, b, m_1, n_1, λ_1, G_1, a_1, and b_1 and of the following relations between stresses and displacements,

$$\sigma_z = \lambda \left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \Theta}\right) + 2G\frac{\partial w}{\partial z}$$

$$\tau_{rz} = G\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)$$

$$\tau_{\Theta z} = G\left(\frac{1}{r}\frac{\partial w}{\partial \Theta} + \frac{\partial v}{\partial z}\right) \quad ,$$

the six boundary relations listed above result in the following six equations, respectively:

$$(2 - b^2)[M \cosh mc - M' \sinh mc] + 2\,[N \cosh nc - N' \sinh nc] = 0 \;.\;.\; (1)$$

$$\frac{2m}{\alpha}[\,-M \sinh mc + M' \cosh mc]$$

$$+\;(2 - b^2)\frac{\alpha}{n}[\,-N \sinh nc + N' \cosh nc] = 0 \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;(2)$$

$$\frac{m}{\alpha}[M \sinh mc + M' \cosh mc] + \frac{\alpha}{n}[N \sinh nc + N' \cosh nc]$$

$$= -\,Re\left[\frac{M_1 m_1}{a} e^{-m_1 c} + \frac{N_1 \alpha}{n_1} e^{-n_1 c}\right] \;.\; (3)$$

$$M \cosh mc + M' \sinh mc + N \cosh nc + N' \sinh nc = Re\left[M_1 e^{-m_1 c} + N_1 e^{-n_1 c} \right] \qquad (4)$$

$$(2 - b^2)\,[M \cosh mc + M' \sinh mc] + 2[N \cosh nc + N' \sinh nc] = \frac{G_1}{G} Re\left[(2 - b_1^2)\, M_1 e^{-m_1 c} + 2N_1 e^{-n_1 c} \right] \qquad (5)$$

$$\frac{2m}{a}[M \sinh mc + M' \cosh mc] + (2 - b^2)\frac{a}{n}[N \sinh nc + N' \cosh nc] = -\frac{G_1}{G} Re\left[2\frac{M_1 m_1}{a} e^{-m_1 c} + (2 - b_1^2)\frac{a}{n_1} N_1 e^{-n_1 c} \right] \qquad (6)$$

The elimination of the six amplitude constants M, M', N, N', M_1, and N_1 from these six equations gives the following frequency equation:

$$\frac{h_1 f_1 (1 - A_1 - A_1')}{1 - A_2 - A_2'} - \frac{1 - A_3 - A_3'}{1 - A_4 - A_4'} = 0 \qquad (A)$$

where $A_1 = \phi\left[\frac{2 - b^2}{2}\coth mc - k \coth nc + \frac{2 - b_1^2}{2}\frac{b^2}{2}\frac{\sqrt{1 - a^2}}{f_1}\right]$;

$$A_2 = \phi\left[\frac{2 - b_1^2}{2}\,\frac{2 - b^2}{2}\coth mc - \frac{2 - b_1^2}{2}k \coth nc + h_1\frac{b^2}{2}\sqrt{1 - a^2}\right];$$

$$A_3 = \phi\left[\frac{2 - b_1^2}{2}\,\frac{2 - b^2}{2}\coth mc - \frac{2 - b_1^2}{2}k \coth nc + f_1\frac{b^2}{2}\sqrt{1 - b^2}\coth mc \coth nc\right];$$

$$A_4 = \phi\left[\frac{2 - b^2}{2}\coth mc - k \coth nc + \frac{2 - b_1^2}{2}\,\frac{b^2}{2}\,\frac{\sqrt{1 - b^2}}{h_1}\coth mc \coth nc\right];$$

$$\phi = \frac{G_1}{2G\left[\left(\frac{2 - b^2}{2}\right)^2 \coth mc - k \coth nc\right]};$$

$k = \sqrt{(1 - a^2)(1 - b^2)}$;

$h_1 = \sqrt{1 - b_1^2}$ if $b_1^2 < 1$, i.e., if n_1 is real

$h_1 = -\sqrt{b_1^2 - 1}\cot(ac\sqrt{b_1^2 - 1})$ if $b_1^2 > 1$, i.e., if n_1 is imaginary

$f_1 = \sqrt{1 - a_1^2}$ if $a_1^2 < 1$, i.e., if m_1 is real;

$f_1 = \sqrt{a_1^2 - 1}\tan(ac\sqrt{a_1^2 - 1})$ if $a_1^2 > 1$, i.e., if m_1 is imaginary.

The expressions for A_1', A_2', A_3', and A_4' used in the frequency equation are the same as for the expressions for A_1, A_2, A_3, and A_4, respectively, except that in every instance coth is replaced by tanh.

THE MEANING OF IMAGINARY AND COMPLEX VALUES FOR m, n, m₁, AND n₁

The question of real and imaginary quantities entered into the solutions for the subgrade. The reason for this was that certain of the expressions became complex (contained both real and imaginary parts) under some conditions.

If the velocity of propagation of the waves horizontally is greater than the normal travel of disturbances within the subgrade, then either n_1 or both n_1 and m_1 become either imaginary or complex quantities in a mathematical sense. If internal friction is neglected as in the present study, then they become imaginary, but if internal friction is taken into account they become complex quantities. That is, if in the foregoing development p/α is greater than $\sqrt{G_1/\rho_1}$, then b_1 will be greater than unity and n_1 will be imaginary, and if p/α is also greater than $\sqrt{(\lambda_1 + 2G_1)/\rho_1}$, then m_1 will also be imaginary. When n_1 and m_1 are imaginary, the expressions e^{-n_1z} and e^{-m_1z} become trigonometric functions of the depth with both real and imaginary parts. For example, the real part of the expression for Z_1 becomes

$$Z_1 = M_1 \cos (\alpha z \sqrt{a_1^2 - 1}) + N_1 \cos (\alpha z \sqrt{b_1^2 - 1})$$

and for Z_1' becomes

$$Z_1' = M_1 \sqrt{a_1^2 - 1} \sin (\alpha z \sqrt{a_1^2 - 1}) - N_1 \frac{\sin (\alpha z \sqrt{b_1^2 - 1})}{\sqrt{b_1^2 - 1}}$$

The factor n may also be imaginary, but such a possibility does not introduce any ambiguity into the solutions since cosh nz and (sinh $nz)/n$ are both real whether n is real or imaginary. The factor m will ordinarily not be imaginary because the shear modulus of the pavement is greater than the shear modulus of the subgrade.

If either m_1 or n_1 is imaginary, then as just indicated, a part of the expression for a displacement within the subgrade will be a trigonometric function of the depth z. Consequently, this part of the expression will indicate no diminution of maximum amplitude of vibration with increase of depth in the subgrade. Had internal friction been taken into account, the solutions would indicate a decrease in amplitude with increase of depth. For example, if the assumption is made that internal friction is adequately considered by replacing the usual Hooke's law by equations of the type*

*The term with f has been added to the usual expression for Hooke's law such as given by Timoshenko in *Theory of Elasticity* (New York: McGraw-Hill, 1934), pp. 7-10.

$$e_x + f\frac{\partial e_x}{\partial t} = \frac{1}{E}\left[\sigma_x - \mu\sigma_y - \mu\sigma_z\right]$$

$$\gamma_{xy} + f\frac{\partial \gamma_{xy}}{\partial t} = \frac{\tau_{xy}}{G}$$

then solutions corresponding to those given in V_1 above for the subgrade can be written

$$u_1 = Z_1 \sin\Theta \, Re\left[e^{ipt}\left\{ H_o^{(1)}(ar) - \frac{H_1^{(1)}(ar)}{ar}\right\}\right]$$

with similar expressions for v_1 and w_1, where the "real" Bessel functions of the first kind (J_o and J_1) have been replaced by "complex" Bessel functions of the third kind ($H_o^{(1)}$ and $H_1^{(1)}$)*; the "real" function $\cos pt$ has been replaced by the "complex" function e^{ipt}; and m_1 and n_1 have the values

$$m_1 = a\sqrt{1 - \frac{a_1^2}{1 + ifp}}, \quad n_1 = a\sqrt{1 - \frac{b_1^2}{1 + ifp}}$$

in place of the values given previously.

A separation of m_1 and n_1 into their real and imaginary parts shows that each has a positive real part for all values of a_1 and b_1. Therefore, when internal friction of the subgrade is taken into account, the amplitude of vibration decreases with increase of depth.

In the above type of solution for vibration with internal friction the displacements and stresses are discontinuous at the line $r = 0$. It is on this line that the energy necessary to maintain sustained vibration is assumed to be supplied. To restrict the driving force to points on the pavement rather than at the line $r = 0$ would necessitate a still more complicated solution.

Since it is believed that internal friction has only a small effect on the velocity of propagation of the waves and since it was desired to keep the analysis relatively simple, friction was neglected in the derivation of the frequency equation.

NUMERICAL SOLUTION OF THE FREQUENCY EQUATION

The frequency equation derived above expresses the relation between the velocity† of wave propagation p/a, the frequency of vibration $p/2\pi$, and the physical properties of the pavement and subgrade. Unfortunately, this relation (Equation A) is rather involved, and numerical solution is not made readily. The method of solution found to be best in general was as follows:

*Bessel functions of first, second, and third kinds are treated in *Tables of Functions*, by Jahnke and Emde, 3d Ed. (New York: G. E. Stechert & Co., 1938), pp. 126-268.

†It must be remembered that the velocity of waves traveling radially depends on distance from the source. p/a is the velocity that they approach with increase of distance.

First, assume values for G_1/G, λ/G, λ_1/G_1, ρ_1/ρ.

Second, select a value of b, the ratio of velocity of propagation to $\sqrt{G/\rho}$.

Third, determine a, a_1, and b_1 and the functions of these quantities and of b that will be needed later such as $\sqrt{1 - b^2}$, $\sqrt{1 - b_1^2}$, $(2 - b^2)/2$, etc.

Fourth, select a value of $\alpha c/\pi$, the ratio of frequency times pavement thickness to the velocity of wave propagation.

Fifth, perform all indicated substitutions into the frequency equation, Equation A.

Sixth, select a second value of $\alpha c/\pi$ and make the indicated substitutions in Equation A.

Seventh, continue the process of selecting values of $\alpha c/\pi$ until Equation A is satisfied to the desired accuracy.

Eighth, select other values of b and repeat the above procedure as many times as seems desirable.

Results of such a procedure are shown in Fig. 2 where the velocity of wave propagation is plotted against frequency of vibration times thickness of subgrade.

Fig. 2 shows that there are three possible wave velocities for each value of frequency times thickness. The highest of these velocities is almost the same as the velocity of longitudinal waves in the pavement, if it were free of the subgrade. The lowest of these velocities is almost the same as the velocity of transverse (flexural) waves in the pavement if it were free of the subgrade. The intermediate velocity is between that of "Rayleigh waves" in the subgrade alone and that of Rayleigh waves in the pavement. At high frequencies all three velocities approach the velocity of Rayleigh waves in the pavement.

The fact that the highest and lowest velocities are practically independent of the properties of the subgrade is of considerable importance. For practical purposes it may therefore be unnecessary to obtain numerical solutions to the foregoing frequency equation. Instead, the following frequency equations derived by Lamb and Timoshenko will be adequate:

For the higher velocity (longitudinal vibration)

$$\left(\frac{2 - b^2}{2}\right)^2 \coth mc = k \coth nc \quad \text{(L)}$$

and for the lower velocity (transverse vibration)

$$\left(\frac{2 - b^2}{2}\right)^2 \tanh mc = k \tanh nc \quad \text{(T)}$$

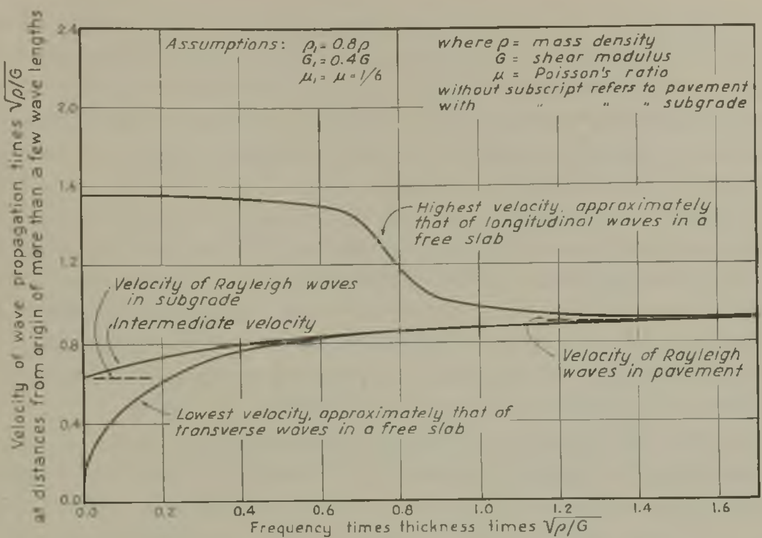

Fig. 2—Effect of frequency of vibration, thickness of pavement, and properties of subgrade on velocity of propagation of waves

These two frequency equations are readily obtained from Equation A by setting G_1 equal to zero. When this is done, either Equation L or T must be satisfied in order that a solution exist.

No correspondingly simple equation can be written for the approximate determination of the intermediate velocity. However, at low frequencies it is approximately the velocity of Rayleigh waves in the subgrade; at high frequencies it is approximately the velocity of Rayleigh waves in the pavement; and at intermediate frequencies it lies between these two limiting velocities. Therefore, a knowledge of these two limiting velocities may be all that is necessary for practical purposes. The relations are:

For Rayleigh waves in the subgrade

$$\left(\frac{2 - b_1^2}{2}\right)^2 = \sqrt{(1 - a_1^2)(1 - b_1^2)} \quad \text{(R}_1\text{)}$$

and for Rayleigh waves in the pavement

$$\left(\frac{2 - b^2}{2}\right)^2 = k \quad \text{(R)}$$

The foregoing analysis is based upon the assumption that the properties of the subgrade do not vary with depth. The properties of the subgrade, of course, do vary with depth; but, because of dampening, the properties at greater depths probably have no appreciable effect on the wave. It was shown by Love[10] that the velocity of Rayleigh waves decreased with frequency if the rigidity of the earth increased with depth. Therefore, one might expect in an actual test that the intermediate velocity would first decrease and later increase with increase of frequency rather than exhibit a continual increase as shown by Fig. 2.

Since for a given pavement on a given subgrade there are three possible wave velocities for each frequency, there may be some question as to the situation in a given test. It should also be kept in mind that for each of these wave velocities the motion may be that corresponding to either the H-solution or the V-solution or a combination of them. Final conclusions will probably have to await a study of test data, but it is believed that the type of vibration can be largely controlled by the manner of driving. This belief is based upon a study of the theoretical equations. This study indicates the following:

(1) If the driving force is directed horizontally, waves of the H-solution with the highest of the three velocities should predominate. This conclusion is based upon the fact that in this case points on the line $r = 0$ have only a horizontal motion and of relatively high amplitude. Relatively large motion of the point of application of the driving force and in the direction of the driving force is necessary for large energy input.

(2) If the horizontally directed force is not applied near the central plane of the slab, waves of the lowest of the three velocities will also be produced. These waves will also be of the H-solution. These lower-velocity waves should be produced to the exclusion of the waves of the higher velocity if the driver produces a couple rather than a resultant force. It is believed that with the driving force applied horizontally at the top of the pavement, waves of the highest rather than the lowest velocity will predominate.

(3) If the driving force is directed vertically, waves of the V-solution with the lowest of the three velocities should predominate. This type gives the largest relative vertical motion for points on the line $r = 0$.

(4) Waves of the intermediate velocity should be produced in all cases. Since their velocity is largely controlled by the properties of the subgrade, it is believed that their relative amplitude will be largely controlled by the dampening properties of the subgrade. Perhaps these waves will be most pronounced at low frequencies. It is believed that a rotary driving force such as would be produced by an unbalanced rotating mass would be most favorable for the production of these waves. That is, forces which produced motions for both H- and V-solutions out of phase with each other are believed to be best.

(5) Waves of the V-solution and of the highest of the three velocities could be produced by a pressure bulb inserted at the center of the pavement if the pressure fluctuated periodically.

It should be remembered that the velocities mentioned above are the limiting velocities which are approached asymptotically as the waves get farther from the source. Since velocity varies with distance from the source, the nodes are non-uniformly spaced, especially near the origin, and spaced differently for the different types of vibration. Moreover, in vibration according to the H-solution the character of the motion and the spacing of the nodes are different in the direction of the driving force, north-south directions in Fig. 1, from what they are in a direction at right angles to the driving force, east-west in Fig. 1. The location of the nodes will also depend on which displacements, horizontal or vertical, are being detected.

The table below giving $1/\pi$ times the roots of the Bessel functions involved should be helpful because the relative distances of the nodes from the origin should correspond to the tabular values given. For example, as is evident from the equations for displacement, the vertical displacement in the V-solution is zero at every distance r from the origin for which $J_o\ (ar)$ is zero.

$1/\pi$ Times the Roots of Bessel Functions

Order of the Root	$J_o(x)$	$J_1(x)$	$\left[J_o(x) - \frac{J_1(x)}{x} \right]$
0	—	0	—
1	0.7655	1.220	0.5860
2	1.757	2.233	1.696
3	2.755	3.238	2.717
4	3.754	4.241	3.726
5	4.753	5.243	4.731
n*	$n - 0.25$	$n + 0.25$	$n - 0.25$

*Where n is a large number.

As is evident from an examination of the equations for displacements, the amplitude of motion of the antinode decreases with distance from the origin. The rate of decrease depends on the angle Θ for waves of the H-solution: for example, *in the direction* of the driving force the amplitude is approximately inversely proportional to the square root of the distance from the origin; *at right angles* to the driving force the amplitude is approximately inversely proportional to the three-halves power of the distance. In the V-solution the motion of the antinodes is approximately inversely proportional to the square root of the distance away from the origin *in any direction*. These facts should be helpful in interpreting results.

Based upon the foregoing theoretical analysis the following experimental procedure with equipment such as that described by Long et al. [6] is recommended.

1. Place driver so as to produce a vertical driving force to the top of the pavement slab.

2. With input to driver set at a given frequency determine location of nodes by moving "pick-up" on radial lines from the driver and noting phase shift and magnitude of response on oscilloscope. (One pair of plates of the cathode ray oscilloscope will be connected to the pick-up circuit and the other pair will be connected to the driver circuit.)

3. Repeat the test at various frequencies.

4. Either repeat the above with driver set so as to produce horizontal motion or use the Long et al. method of determining the velocity of longitudinal waves.

5. Plot velocity of waves versus frequency and compare with curves of Fig. 2.

6. Determine proper values of G/ρ and thickness of slab that will give best agreement with theoretical curves.

7. If data on the properties of the subgrade are desired and the waves of intermediate velocity have not been detected and their velocity determined, use a mechanical driver of relatively low frequency. This should produce the waves of intermediate velocity and of sufficient relative amplitude for detection.

Example

Suppose that at 1000 cycles per second the distances between two nodes (other than the first two nodes) with the driving force acting first vertically and then horizontally are 2.25 ft. and 7.00 ft., respectively. The corresponding velocities will be 4,500 ft. per sec. and 14,000 ft. per sec. ($V = 2l$ times frequency) and the ratio of the velocities will be $4500/14{,}000 = 0.321$. Assume the lower velocity to be that of transverse waves and the higher velocity to be that of longitudinal waves. An examination of Fig. 2 shows that the velocities of these waves have a ratio of 0.321 at a value of about 0.115 for the abscissas and that the corresponding values of the ordinates are approximately 1.55 and 0.50. This means that

$$\sqrt{G/\rho} = \frac{14{,}000}{1.55} = 9{,}030 \text{ ft/sec}$$

and

$$\text{thickness} = \frac{9{,}030 \times 0.115}{1{,}000} = 1.04 \text{ ft}$$

If density of pavement is 150 lb. per ft^3, then $\rho = 4.66 \dfrac{\text{lb sec}^2}{\text{ft}}$

and $G = 4.66(9030)^2 = 380 \times 10^6$ lb. per ft^2 $= 2.64 \times 10^6$ psi

If $\mu = 1/6$, then $E = 14/6 \times 2.64 \times 10^6 = 6.15 \times 10^6$ psi

A test at another frequency, say 2000 cycles per second, should result in approximately the same values for thickness and moduli for the pavement. If radically different values are obtained, then the waves have not been correctly identified. One velocity in at least one of the tests might have been the intermediate velocity.

If the intermediate velocities are not in accord with the middle curve of Fig. 2, then other curves based on other assumptions should be prepared for these velocities.

All of the above discussion has pertained to sustained vibration (stationary waves). Sustained vibration can be considered as the result of two equal continuous wave-trains (progressive waves) traveling in opposite directions. An adequate treatment of a single, impact-generated wave-train of finite length traveling away from a source is beyond the scope of this paper. Because of the finite length of the train and because many different frequencies are usually represented, not all parts travel at the same velocity and the wave-form changes as the wave proceeds. However, of practical importance is the fact that the higher velocity (upper curve, Fig. 2) is almost independent of frequency for low frequencies. Therefore, a wave-train of longitudinal waves of low frequency can be propagated with relatively little change of wave-form. It is probably because of this fact that the velocities of "longitudinal" waves in pavements have been determined successfully by measuring the time required for a short wave-train of longitudinal waves to travel between two points, the short wave-train being produced by an impact.[6]

The effects of variations in physical properties of the subgrade at different depths below the pavement, and the significance of the fact that the pavement does not extend indefinitely in a horizontal direction have not been considered. However, vibration of appreciable amplitude will probably not extend very far from the source either down into the subgrade or horizontally in the pavement owing to the effect of internal friction, especially at high frequencies. It is therefore believed that only the pavement within a few feet of the source and only the material immediately below the pavement will have an appreciable effect upon the wave velocity near the source when the frequency is relatively high.

If the character of the subgrade at considerable depth is desired, relatively low frequencies would be required. The above analysis may be inadequate for a study of the variation in properties of the subgrade with depth since no provision was made for such variation in the equation.

SUMMARY

Equations are derived for the combined vibration of pavement and subgrade. Numerical solution of the frequency equation shows that for a given slab on a given subgrade three wave velocities are possible for each frequency of vibration. The highest of these velocities is almost the same as that of longitudinal vibration of a free slab; the lowest, almost the same as that of transverse (flexural) vibration of a free slab; and the intermediate, somewhere between the velocity of Rayleigh waves in the uncovered subgrade and the velocity of Rayleigh waves in the pavement.

Suggestions are given for the production of any one of the three possible velocities to the virtual exclusion of the other two.

NOTATION (PARTIAL LIST)

r, Θ, z = cylindrical coordinates

u, v, w = displacements in r, Θ, z directions, respectively, of the point (r, Θ, z) in the pavement

u_1, v_1, w_1 = displacements in r, Θ, z directions, respectively, of the point (r, Θ, z) in the subgrade

t = time

ρ, ρ_1 = mass densities of pavement and subgrade, respectively

$\lambda = \dfrac{E}{(1+\mu)(1-2\mu)}$

$\lambda_1 = \dfrac{E_1}{(1+\mu_1)(1-2\mu_1)}$

E, E_1 = Young's modulus for pavement and for subgrade, respectively

μ, μ_1 = Poisson's ratio for pavement and subgrade, respectively

G, G_1 = Shear modulus for pavement and subgrade, respectively

$\sigma_z, \sigma_y, \sigma_x$ = normal stresses

$\tau_{rz}, \tau_{\Theta z}$, etc. = shear stresses

e_x = normal strain in x-direction

$\gamma_{rz}, \gamma_{\Theta z}, \gamma_{xy}$ = shear strains

p = 2π times frequency of sustained vibration

$\alpha = p/V = 2\pi/l$ where V is the radial velocity which the wave approaches asymptotically as it gets farther from its source and l is the corresponding wave length

c = half thickness of pavement

$a = \dfrac{p}{\alpha}\sqrt{\dfrac{\rho}{\lambda + 2G}}$, ratio of velocity of propagation to velocity of compressional waves in interior of pavement

$b = \dfrac{p}{\alpha}\sqrt{\dfrac{\rho}{G}}$, ratio of velocity of propagation to velocity of shear waves in interior of pavement; ordinate of Fig. 2

$m = \alpha\sqrt{1 - a^2}$

$n = \alpha\sqrt{1 - b^2}$

a_1, b_1, m_1, n_1 have the same definitions as a, b, m, and n, respectively, except that those with subscript unity refer to subgrade instead of pavement

$k = \sqrt{(1-a^2)(1-b^2)}$

$Re[\quad]$ = real part of expression in brackets

J_o, J_1 = Bessel functions of the first kind of order zero and unity, respectively

H_o^1, H_1^1 = Bessel functions of the third kind

REFERENCES

1. T. C. Powers, "Measuring Young's Modulus of Elasticity by Means of Sonic Vibration," Proc. Am. Soc. Testing Mats. V. 38, Part II, 460 (1938).

2. F. B. Hornibrook, "Application of Sonic Method to Freezing and Thawing Studies of Concrete," ASTM Bulletin, No. 101, 5 Dec. (1939).

3. W. T. Thomson, "Measuring Changes in Physical Properties of Concrete by the Dynamic Method," Proc. Am. Soc. Testing Mats. V. 40, 1113 (1940).

4. Leonard Obert and Wilbur I. Duvall, "Discussion of Dynamic Methods of Testing Concrete with Suggestions for Standardization," Proc. Am. Soc. Testing Mats. V. 41, 1053 (1941).

5. R. K. Bernhard, "Highway Investigation by Means of Induced Vibrations," Engr. Exp. Station Series, Penn. State Coll. V. 33, Bull. No. 49 (1939).

6. Bartlett G. Long, Henry J. Kurtz and Thomas A. Sandenaw, "An Instrument and a Technic for Field Determination of the Modulus of Elasticity and Flexural Strength of Concrete (Pavements)"; ACI Journal, Jan. 1945; *Proceedings* V. 41, p. 217.

7. B. G. Long and H. J. Kurtz, "Effect of Curing Methods upon the Durability of Concrete As Measured by Changes in the Dynamic Modulus of Elasticity," Proc. Am. Soc. Testing Mats. V. 43, 1051 (1943).

8. Horace Lamb, "On Waves in an Elastic Plate," Roy. Soc. Proc. 1916-17, Ser. A, V. 93, p. 114.

9. S. Timoshenko, "Transverse Vibrations of Bars of Uniform Cross-Section," Phil. Mag. V. 43, 125 (1922) (Series 6).

10. A. E. H. Love, *Some Problems of Geodynamics* (Cambridge Univ. Press, 1911).

11. A. E. H. Love, *Mathematical Theory of Elasticity*, 4th ed. (Cambridge Univ. Press, 1927), p. 288.

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Estimating 28-Day Strength of Concrete from Earlier Strengths—Including the Probable Error of the Estimate*

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Member American Concrete Institute

SYNOPSIS

This paper presents a method for estimating the 28-day strength of concrete from earlier strengths. Using a simple basic formula, its coefficient is adjusted by applying the method of least squares to a small number of data obtained from the mix under consideration. The method is noteworthy because it demonstrates that: only limited data are required for estimating purposes; earlier strengths in weighted combination can be used to estimate 28-day strength with increased accuracy; the formula can be computed accurately with a 10-in. slide rule; and, because it presents a criterion for judging accuracy of estimates.

INTRODUCTION

The writer has had occasion to be impressed with the cost involved in preparing and testing a large number of concrete specimens and, even more so, with the inconvenience and delay in construction caused by having to wait for 28-day results when earlier strengths were on hand. Application of existing formulas for estimating 28-day strengths from earlier results led to the conclusion that these formulas were designed to cover too wide a field. Inasmuch as 28-day concrete strength depends upon a number of variables such as type of mix, cement and aggregates, and methods of curing and testing, these formulas, whose constants were based on the data obtained from a theoretical average concrete, cannot be expected to yield reliable results except, of course, in cases where by chance the concrete under consideration is similar to that on which the constants were based.

The solution was to develop a general formula which would apply to all classes of concrete, and then to adjust the parameters of the formula by applying the method of least squares to a small number of data

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obtained from the specific mix under consideration. Another thought was that the formula had to be practical; that is, capable of accurate computation on a 10-in. slide rule. Finally, a criterion for judging the accuracy of the computations was deemed desirable.

The method which follows is believed to meet these conditions of generality, singularity and practicability. It is to be hoped that its use may lead not only to a saving in the number of specimens required, but also to avoidance of construction delays.

The hardening of concrete proceeds approximately exponentially, but the results of parallel measurements of strength are subject to a large dispersion which makes hazardous any prediction of 28-day strength by exponential extrapolation from earlier strengths. It is practical, however, to express 28-day strength as a linear or non-linear combination of the others. This is done in the ensuing discussion* in which two-parameter formulas are first fitted by least squares to a preliminary series of data; then reduced to one-parameter equations, and used thereafter to estimate 28-day strengths from observed 3- and 7-day strengths of parallel specimens. For different jobs, the values of the parameters will vary appreciably only if the conditions defining parallel specimens vary; otherwise their values should remain fixed.

Parallel specimens are defined here as concrete specimens of approximately identical mix, type of cement, aggregates, specimen dimensions, and curing.

Specimens may no longer be considered parallel when, for instance, a mix is changed from 1:1:3 to 1:3:6; when the cement is changed from Type 1 to High Early Strength; when the coarse aggregates vary from trap rock to crushed limestone, and the fine aggregates from Potomac river sand to Puerto Rico beach sand; when the dimensions of the specimens are changed from 6 x 12-in. to 3 x 6-in. cylinders; and when laboratory curing becomes field curing. On the other hand, water-cement ratio and slump may vary, without affecting parallelity.

1—FORMULAS AND APPLICATION

The method which follows is general, and may be used to estimate the 28-day strengths of concretes and mortars from *any* earlier strengths. The method may also be used to compute flexural and tensile as well as compressive strengths.

Let t = observed 3-day strength, psi.
s = observed 7-day strength, psi.
f = observed 28-day strength, psi.
f_e = predicted 28-day strength, psi.

*See part 2.—Development of Theory

We define the combination c as

$$c = \frac{t + 2s}{3} \qquad (1)$$

and seek to fit the data by means of the equation

$$f_e = Kd \qquad (2)$$

Equation (1) expresses the average relative weight of the 3-day strength t as compared to the 7-day strength s in estimating the predicted 28-day strength f_e. The writer has found that in cases where the 7-day strength could be used to predict the 28-day strength within a maximum probable error of, say 240 psi., the prediction from the 3-day strength resulted in a maximum probable error of 480 psi. This, therefore, is our justification for assigning a weight of 2 to the 7-day strength when using the 3-day and 7-day strengths in combination to estimate the 28-day strengths.

The coefficient K in equation (2) is determined by preliminary computations on data obtained from a specified number of parallel specimens from a given job. The preliminary data provide the equations

$$\begin{aligned} f_1 &= Kd_1 \\ &\vdots \\ f_n &= Kd_n \qquad (3) \end{aligned}$$

in which n is the set number.

The least squares solution of equation (3) is

$$K = \frac{\text{Summation } (fd)}{\text{Summation } (d^2)} \qquad (4)$$

Let us assume that*

$$d = 1 + 0.00221\, c \qquad (5)$$

for plotted data which exhibit substantially no curvature,

$$d = c + 25.9\, c^{1/2} \qquad (6)$$

for plotted data which exhibit only slight curvature, and

$$d = c - 0.0000692\, c^2 \qquad (7)$$

for plotted data which exhibit appreciable curvature.

In choosing an empirical formula to represent specific data, we first plot the given data by taking the observed 3-day t, 7-day s, or their combination c values as abscissas, and the corresponding observed 28-day f strengths as ordinates, and pass a curve through the plotted points. The features of such a curve are best brought out if the scales of the two axes are so chosen that the curve drawn makes approximately equal angles with them. This may be accomplished as follows: If the maxi-

*See part 2.—Development of Theory

mum required value along the ordinate is, say 6000 psi., and the ordinate scale chosen is such that 1-in. equals 1000 psi., and the maximum value along the abscissa axis is, say 4000 psi.; then the abscissa scale should be 1.5-in. equals 1000 psi. With the data plotted in this manner, the curve will show to the eye how f varies with t, s or c. If the plot develops as substantially a straight line, equations (1), (2), (4), and (5) are used; if slight curvature is exhibited, equations (1), (2), (4) and (6) may fit best; and if the plot shows appreciable curvature, equations (1), (2), (4) and (7) should be tried.

Exactly which set of formulas will apply best to a given set of data, so as to fit them well, and at the same time be of use in prediction, can be determined only by trial. In evaluating results obtained from the three sets of formulas given above, that one is the best which yields the smallest value of maximum probable error. Finally, if the three sets of equations furnish substantially the same maximum probable error, that one is the best which is the simplest to compute.

However, unless the curvature is very marked, the writer is inclined to believe that the straight line graph represented by equations (1), (2), (4) and (5) will provide substantially as much accuracy for estimating purposes as the other curves when dealing with a material as heterogeneous as concrete. In addition, it is the simplest to compute.

Maximum probable error

The probable error is a function of the *root-mean-square* (RMS) departure of the predicted 28-day strength f_e from the observed 28-day strength f. As used here, the term *maximum probable error* (PE_{max}) is defined to be that value of the probable error which will not be exceeded by the $(f - f_e)$ departures of 90 out of 100 parallel specimens. We find* from the standard probability curve that for such a probability ($P = 0.90$)

$$PE_{max} = 1.645 \left[\frac{N+2}{N-2}\right]^{1/2} \text{RMS} \qquad (8)$$

where N represents the total number of sets of parallel specimens used as a base.

Note that, because of the approximate nature of equation (8), it is not considered trustworthy for values of N less than 6.

The question naturally arises as to how many sets of parallel specimens are required as a base in order to obtain reliable predictions of 28-day strength from earlier strengths. It might be thought that if the coefficient K in equation (2) were based on extensive data and long experience, predictions of 28-day strength would be much more accurate than with a value of K obtained from a limited series of data.

*See part 2.—Development of Theory

The answer to this question* is that no value of K, however painstakingly chosen, can reduce the maximum probable error below a certain minimum, which value is closely approached by the use of a K obtained from a limited series of data. This is indeed a fortunate and significant conclusion.

Let us consider what this means. If, for example, we are working with a concrete mix which develops a 28-day strength of 3000 psi., and with a value of K based on most extensive data and long experience ($N = \infty$), the 28-day strength might possibly be predicted within an indicated maximum probable error of, say 240 psi. On the other hand, if K were based on $N = 10$ or $N = 6$ specimens, the maximum probable errors would be 264 or 276 psi., respectively. This means that the maximum probable error for $N = \infty$, $N = 10$ and $N = 6$ is 8.0 percent, 8.8 per cent and 9.2 percent of 3000 psi., respectively.

Here, therefore, is our justification for using only a relatively small number of specimens for predicting 28-day strength from earlier strengths.

We are now ready to illustrate the use of the foregoing equations on specific test data.

Problem 1.—Prediction of 28-day strength from observed 3- and 7-day strengths combined.

G. B. Sheldon, Jr., construction engineer, Public Buildings Administration, Washington, D. C., furnished the data shown in Table 1, concerning 15 sets of 3-, 7-, and 28-day (6 x 12-in.) concrete cylinders made during construction of the Hydrographic Building at Suitland, Maryland, from March 10 to May 22, 1942.

The materials used per cubic yard of concrete were: 517-lb. of Type I portland cement, 1430-lb. of bank-run sand, and 1870-lb. of ¾-in. bank-run gravel. The cylinders remained on the site for the first 24 hours, and were then taken to the Public Building Administration laboratory, and cured in a saturated atmosphere at 70 F. until tested.

Slumps ranged from 3½-in. to 6½-in. Average temperatures during the first 24 hours varied from 47 to 68 F. Also, the cylinder sets marked "v" were vacuum processed. The large range of strengths noted is probably due to the combined influence of variations in water content, first day curing temperatures, and vacuum processing.

The test results were arranged from low to high on the basis of 3-day tests. Five representative sets were then selected by Mr. Sheldon for purposes of prediction. The latter are identified as set numbers 11 to 15.

We now enter the pertinent data of cylinder sets 1 to 10 in Table 2 and compute the values of c. We note that the maximum value to be plotted along the ordinate is 4250, and that the maximum value along the abscissa is 3100. We therefore select an ordinate scale of 1-in. equals 1000 psi., and an abscissa scale of 4250/3100, or 1.4-in. equals 1000 psi. We then plot values of c of sets 1 to 10 against the corresponding values of f, in Fig. 1. Since the graph through the 10 points exhibits substantially

*See part 2.—Development of Theory

TABLE 1—COMPRESSIVE STRENGTHS OF 6 x 12-IN. CYLINDERS

Suitland Test Results

Set Number (n)	3-Day (t) psi.	7-Day (s) psi.	28-Day (f) psi.
1	980	1520	2230
2	1200	1720	2320
3	1240	2110	2660
4	1430	1760	2640
5v	1720	2540	3340
6v	1780	2250	3240
7	1980	2900	3900
8	2060	2900	3820
9	2250	2750	3550
10v	2730	3290	4250
11	1240	1920	2730
12v	1540	2040	2690
13	1910	2500	3470
14v	2220	2710	3800
15v	3170	4560	5560

TABLE 2—CALCULATION OF K FROM OBSERVED 3, 7 AND 28-DAY COMPRESSIVE STRENGTHS OF CYLINDER SETS 1 TO 10

Set No. n	3-Day t psi.	7-Day s psi.	28-Day f psi.	c psi.	d	d^2	df
1	980	1520	2230	1340	3.96	15.7	8830
2	1200	1720	2320	1550	4.43	19.6	10280
3	1240	2110	2660	1820	5.02	25.2	13350
4	1430	1760	2640	1650	4.65	21.6	12280
5	1720	2540	3340	2270	6.02	36.2	20100
6	1780	2250	3240	2090	5.62	31.6	18210
7	1980	2900	3900	2590	6.72	45.2	26200
8	2060	2900	3820	2620	6.79	46.1	25900
9	2250	2750	3550	2580	6.70	44.9	23800
10	2730	3290	4250	3100	7.85	61.6	33400
						347.7	192350

$f_e = Kd;\ d = 1 + 0.00221c \qquad c = \dfrac{t + 2s}{3};$

$$K = \frac{192350}{347.7} = 554$$

no curvature, we choose to work with equations (1), (2), (4) and (5). Using these to compute and complete the entries in Table 2, we find that $K = 554$.

In Table 2A, we estimate the 28-day strengths of cylinder sets 11 to 15 from their respective observed 3- and 7-day strengths. The results and departures are as in Table 2B.

To determine the maximum probable error, we first calculate the value of RMS from the observed 3-, 7-, and 28-day strengths of sets 1 to 10. This is done in Table 3, with RMS = 112 psi. Then, by equation (8), with $N = 10$, the value of the maximum probable error is found to be

Fig. 1 (Problem 1)—Weighted 3- and 7-day values (c) plotted against 28-day strengths (f) to determine shape of curve through points.

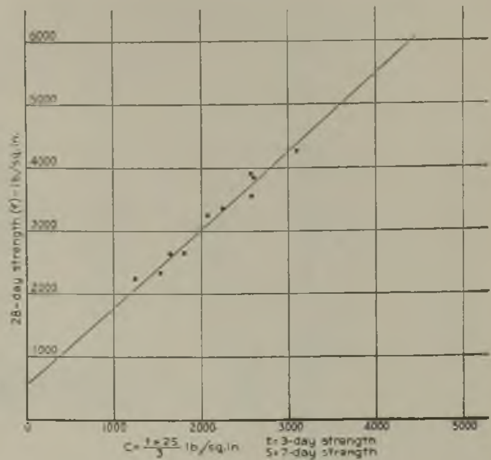


TABLE 2A—CALCULATION OF 28-DAY COMPRESSIVE STRENGTH OF CYLINDER SETS 11 TO 15 FROM OBSERVED 3- AND 7-DAY STRENGTHS

Set No. n	3-Day t psi.	7-Day s psi.	c psi.	d	Predicted 28-Day f_e psi.
11	1240	1920	1690	4.74	2630
12	1540	2040	1870	5.13	2840
13	1910	2500	2300	6.08	3370
14	2220	2710	2550	6.64	3680
15	3170	4560	4100	10.06	5570

$f_e = Kd;\quad K = 554;\quad d = 1 + 0.00221c;\quad c = \dfrac{t + 2s}{3}$

$PE_{max} = 225$ psi

In the prediction of the 28-day strengths of parallel specimens from combined 3- and 7-day strengths, it is probable, therefore, that, in the case of 90 out of 100 cylinders, the individual errors will not exceed 225 psi. Examination of the errors ($f - f_e$ values) in sets 1 to 15 supports this conclusion, all of the departures being smaller than PE_{max}, with the largest, 180 psi occurring in set 7.

TABLE 2B

n	Observed 28-day Strength, f psi.	Predicted 28-day Strength, f_e psi.	Departure $f - f_e$ psi.
11	2730	2630	+ 100
12	2690	2840	— 150
13	3470	3370	+ 100
14	3800	3680	+ 120
15	5560	5570	+ 10

TABLE 3—CALCULATIONS OF RMS FROM OBSERVED 3, 7 AND 28-DAY STRENGTHS OF CYLINDER SETS 1 TO 10.

Set No. n	3-Day t psi.	7-Day s psi.	c psi.	d	Predicted 28-Day f_e psi.	Observed 28-Day f psi.	$f - f_e$ psi.	$(f - f_e)^2$
1	980	1520	1340	3.96	2190	2230	+40	1600
2	1200	1720	1550	4.43	2450	2320	−130	16900
3	1240	2110	1820	5.02	2780	2660	−120	14400
4	1430	1760	1650	4.65	2580	2640	+60	3600
5	1720	2540	2270	6.02	3340	3340	0	0
6	1780	2250	2090	5.62	3110	3240	+130	16900
7	1980	2900	2590	6.72	3720	3900	+180	32400
8	2060	2900	2620	6.79	3760	3820	+60	3600
9	2250	2750	2580	6.70	3710	3550	−160	25600
10	2730	3290	3100	7.85	4350	4250	−100	10000

$f_e = Kd; \quad d = 1 + 0.00221c; \quad c = \dfrac{t + 2s}{3}$

$K = 554$

Sum = 125000
Mean = 12500
RMS = 112

TABLE 4—CALCULATION OF K FROM OBSERVED 3 AND 28-DAY COMPRESSIVE STRENGTHS OF CYLINDER SETS 1 TO 10

Set No. n	3-Day t psi.	28-Day f psi.	$t^{1/2}$	d	d^2	df
1	980	2230	31.3	1790	320x10⁴	399x10⁴
2	1200	2320	34.6	2100	441	487
3	1240	2660	35.2	2150	462	572
4	1430	2640	37.8	2410	581	636
5	1720	3340	41.5	2800	784	935
6	1780	3240	42.2	2870	824	930
7	1980	3900	44.5	3130	980	1221
8	2060	3820	45.4	3240	1050	1238
9	2250	3550	47.4	3480	1211	1235
10	2730	4250	52.2	4080	1665	1734
					8318x10⁴	9387x10⁴

$f_e = Kd;\quad d = c + 25.9c^{1/2};\quad c = t$

$$K = \frac{9387 \times 10^4}{8318 \times 10^4} = 1.13$$

TABLE 4A—CALCULATION OF 28-DAY COMPRESSIVE STRENGTHS OF CYLINDER SETS 11 TO 15 FROM OBSERVED 3-DAY STRENGTHS

Set No. n	3-Day t psi.	$t^{1/2}$	d	Predicted 28-Day f_e psi.
11	1240	35.2	2150	2430
12	1540	39.2	2560	2890
13	1910	43.7	3040	3440
14	2220	47.1	3440	3890
15	3170	56.3	4630	5230

$f_e = Kd;\quad K = 1.13;\quad d = c + 25.9c^{1/2};\quad c = t$

Problem 2.—Prediction of 28-day strength from observed 3-day strength.

For some purposes, for example in connection with the use of high early strength cement, or the vacuum concrete process, it may suffice to make an estimate of the 28-day strength based only on the observed 3-day strength, t. Utilizing the cylinder sets previously given in Table 1, we enter the pertinent data in Table 4 and plot Fig. 2. Since the graph through the 10 points exhibits a slight curvature, we choose to work with equations (1), (2), (4) and (6). We note, however, that in equation (1), $c = t$; therefore, t is used throughout instead of c.

Using the above equations to compute the entries in Table 4, we find that $K = 1.13$.

In Table 4A, we estimate the 28-day strengths of cylinder sets 11 to 15 from their respective observed 3-day strengths. The results and departures are as in Table 4B.

To determine the maximum probable error, we calculate the value of RMS from the observed 3- and 28-day strengths of sets 1 to 10. This is done in Table 5, with RMS = 238 psi. Then, by equation (8), with $N = 10$, the maximum probable error is found to be

$$PE_{max} = 479 \text{ psi.}$$

Examination of the $(f - f_e)$ departures in sets 1 to 15 shows that the largest, 380 psi, occurs in set 9.

Fig. 2 (Problem 2)—3-day strengths (t) plotted against 28-day strengths (f) to determine shape of curve through points.

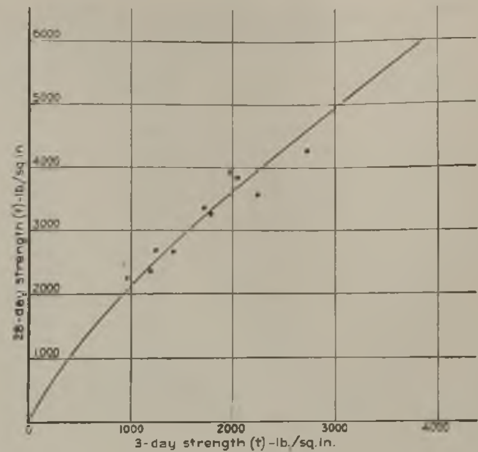


TABLE 4B

n	Observed 28-day Strength, f psi.	Predicted 28-day Strength, f_e psi.	Departure $f - f_e$ psi.
11	2730	2430	+ 300
12	2690	2890	− 200
13	3470	3440	+ 30
14	3800	3890	− 90
15	5560	5230	+ 330

TABLE 5—CALCULATION OF RMS FROM OBSERVED 3 AND 28-DAY STRENGTHS OF CYLINDER SETS 1 TO 10

n	d	f_e	f	$(f-f_e)$	$(f-f_e)^2$
1	1790	2020	2230	+ 210	441 00
2	2100	2370	2320	− 50	25 00
3	2150	2430	2660	+ 230	529 00
4	2410	2720	2640	− 80	64 00
5	2800	3160	3340	+ 180	324 00
6	2870	3240	3240	0	0
7	3130	3540	3900	+ 360	1296 00
8	3240	3660	3820	+ 160	256 00
9	3480	3930	3550	− 380	1444 00
10	4080	4610	4250	− 360	1296 00

$f_e = Kd \quad K = 1.13$

$d = c + 25.9c^{1/2}$

$c = t$

Sum = 5675 00
Mean = 567 50
RMS = 238

Problem No. 3—Prediction of 28-day strength from observed 7-day strength.

On most jobs it may suffice to have an estimate of the 28-day strength based only on the observed 7-day strength, s. Using the cylinder sets previously given in Table 1,

TABLE 6—CALCULATION OF K FROM OBSERVED 7 AND 28-DAY COMPRESSIVE STRENGTHS OF CYLINDER SETS 1 TO 10

Set No. n	7-Day s psi.	28-Day f psi.	d	d^2	df
1	1520	2230	4.36	19.0	9720
2	1720	2320	4.80	23.0	11140
3	2110	2660	5.66	32.1	15060
4	1760	2640	4.89	23.9	12910
5	2540	3340	6.61	43.7	22080
6	2250	3240	5.97	35.7	19340
7	2900	3900	7.41	54.9	28900
8	2900	3820	7.41	54.9	28300
9	2750	3550	7.08	50.1	25100
10	3290	4250	8.27	68.4	35100
$f_e = Kd;\quad d = 1 + 0.00221c;\quad c = s;$				405.7	207650

$$K = \frac{207650}{405.7} = 512$$

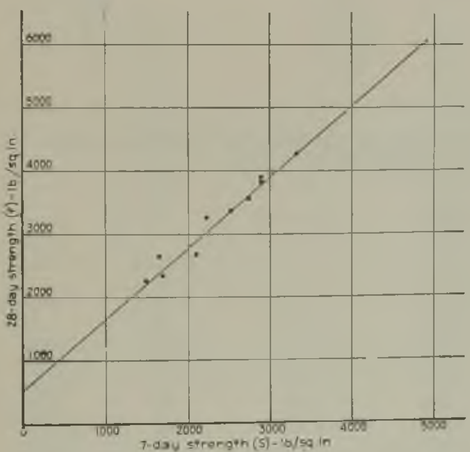


Fig. 3 (Problem 3)—7-day strengths (s) plotted against 28-day strengths (f) to determine shape of curve through points

we enter the pertinent data in Table 6 and plot Fig. 3. Since the graph through the 10 points exhibits substantially no curvature, we again choose to work with equations (1), (2), (4) and (5). We note, however, that in equation (1), $c = s$; therefore, s is used throughout instead of c.

Using the above equations to compute the entries in Table 6, we find that $K = 512$.

In Table 6A, we estimate the 28-day strengths of cylinder sets 11 to 15 from their respective observed 7-day strengths. The results and departures are as in Table 6B.

To determine the maximum probable error, we calculate the value of RMS from the observed 7- and 28-day strengths of sets 1 to 10. This is done in Table 7, with RMS = 121 psi. Then, by equation (8), with $N = 10$, the value of the maximum probable error is found to be

$$PE_{max} = 243 \text{ psi.}$$

TABLE 6A—CALCULATION OF 28-DAY COMPRESSIVE STRENGTHS OF CYLINDER SETS 11 TO 15 FROM OBSERVED 7-DAY STRENGTHS

Set No. n	7-Day s psi.	d	Predicted 28-Day f_e psi.
11	1920	5.24	2680
12	2040	5.51	2820
13	2500	6.53	3340
14	2710	6.99	3580
15	4560	11.08	5670

$f_e = Kd$; $K = 512$; $d = 1 + 0.00221c$; $c = s$

TABLE 6B

n	Observed 28-day Strength, f psi.	Predicted 28-day Strength, f_e psi.	Departure $f - f_e$ psi.
11	2730	2680	+ 50
12	2690	2820	− 130
13	3470	3340	+ 130
14	3800	3580	+ 220
15	5560	5670	− 110

TABLE 7—CALCULATION OF RMS FROM OBSERVED 7 AND 28-DAY STRENGTHS OF CYLINDER SETS 1 TO 10

Set No. n	7-Day s psi.	d	Predicted 28-Day f_e psi.	Observed 28-Day f psi.	$f - f_e$	$(f - f_e)^2$
1	1520	4.36	2230	2230	0	0
2	1720	4.80	2460	2320	−140	19600
3	2110	5.66	2900	2660	−240	57600
4	1760	4.89	2500	2640	+140	19600
5	2540	6.61	3380	3340	− 40	1600
6	2250	5.97	3060	3240	+180	32400
7	2900	7.41	3800	3900	+100	10000
8	2900	7.41	3800	3820	+ 20	400
9	2750	7.08	3620	3550	− 70	4900
10	3290	8.27	4230	4250	− 20	400

$f_e = Kd$; $d = 1 + 0.00221c$; $c = s$; $K = 512$ Sum = 146500
Mean = 14650
RMS = 121

Examination of the $(f - f_e)$ departures in sets 1 to 15 shows that the largest, 240 psi, occurs in set 3.

Problem No. 4.—Prediction of 28-day strength from observed 7-day strength of nodulite concrete.

H. J. McGillivray, Chief of the Concrete Control and Testing Division, Hooker's Point Shipyard, Tampa, Fla., furnished the data given in Table 8, concerning 15 sets of 7- and 28-day (6 x 12-in.) concrete cylinders made during the construction of 24 self-propelled reinforced-concrete cargo vessels from June, 1943 to Sept., 1944.

The materials used per cubic yard of concrete were as follows: 9.2 bags of type 2 portland cement; 1370-lb. of coarse, medium and fine Nodulite, ranging in size from ½ inch to dust; 410-lb. of Lake Wales sand, ranging in size from ⅛ inch to dust; and 4.6-lb. of Plastiment. The average fineness modulus of the combined Nodulite sizes was 4.28. The fineness modulus for the sand was 1.98. Water was used in such quantity as to obtain slumps of from 3½ to 4½ inches in the mix.

Nodulite, a lightweight aggregate developed for use in the concrete ships built at Tampa, is made of Fuller's Earth, molded and burned in roughly spherical shapes in a plant at Ellenton, Fla., constructed for this purpose.

All cylinders were made in the concrete laboratory at the Yard from the field mix as delivered in buggies from the transit-mix trucks. The cylinders were cured in a fog-room at 70 F. until tested.

TABLE 8—COMPRESSIVE STRENGTHS OF 6 X 12-IN. CYLINDERS
Hooker's Point Shipyard Test Results

Set Number (n)	7-Day (s) psi.	28-Day (f) psi.
101	3460	5150
102	3890	5480
103	4010	5410
104	4200	5530
105	4200	5960
106	4390	6410
107	4510	6550
108	4790	6010
109	5090	6680
110	5170	6750
111	3680	5220
112	4330	6000
113	4820	6220
114	4970	6010
115	5000	6660

The test results in Table 8 were arranged from low to high on the basis of the 7-day tests, and five sets were then selected for purposes of prediction. The latter are identified as set numbers 111 to 115.

Utilizing the cylinder sets 101 to 110 given in Table 8, we enter the pertinent data in Table 9, and plot Fig. 4. Since the graph through the 10 points exhibits appreciable curvature, we choose to work with equations (1), (2), (4) and (7). We note, however, that $c = s$, in equation (1); hence, s is used throughout instead of c.

Using the above equations to complete the entries in Table 9, we find that $K = 1.98$.

In Table 9A, we estimate the 28-day strengths of cylinder sets 111 to 115 from their respective observed 7-day strengths. The results and departures are as in Table 9B.

To determine the maximum probable error, we calculate the value of RMS from the observed 7- and 28-day strengths of sets 101 to 110. This is done in Table 10, with RMS = 275 psi. Then, by equation (8), with $N = 10$, the maximum probable error is found to be

$$PE_{max} = 554 \text{ psi.}$$

TABLE 9—CALCULATION OF K FROM OBSERVED 7 AND 28-DAY COMPRESSIVE STRENGTHS OF CYLINDER SETS 101 TO 110

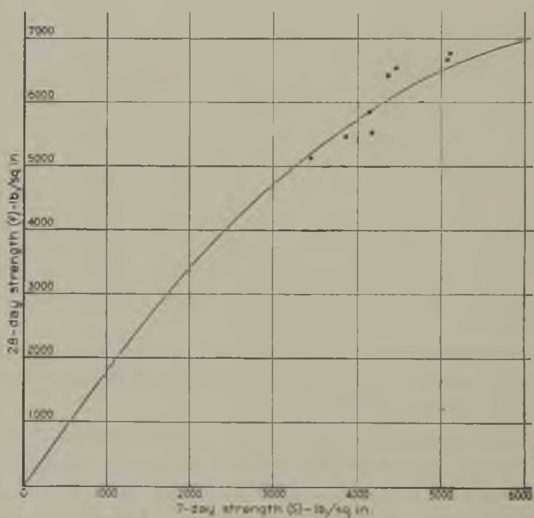
Set No. n	7-Day s psi.	28-Day f psi.	s^2	d	d^2	df
101	3460	5150	119×10^6	2640	697×10^4	1359×10^4
102	3890	5480	151	2840	807	1556
103	4010	5410	161	2900	841	1569
104	4200	5530	176	2980	888	1648
105	4200	5960	176	2980	888	1776
106	4390	6410	193	3050	930	1955
107	4510	6550	203	3100	961	2030
108	4790	6010	229	3210	1030	1929
109	5090	6680	259	3300	1089	2200
110	5170	6750	267	3320	1102	2240
					9233×10^4	18262×10^4

$f_e = Kd \qquad d = c - 0.0000692c^2$

$c = s$

$$K = \frac{18262 \times 10^4}{9233 \times 10^4} = 1.98$$

Fig. 4 [(Problem 4)—7-day strengths (s) plotted against 28-day strengths (f) to determine shape of curve through points



Examination of the $(f - f_e)$ departures in sets 101 to 115 shows that the largest, 440 psi, occurs in cylinder sets 114.

Recommended procedure

With the mix, type of cement and aggregates, specimen dimensions, curing and testing techniques determined, the following sequence of operations is recommended:

TABLE 9A—CALCULATION OF 28-DAY COMPRESSIVE STRENGTHS OF CYLINDER SETS 111 TO 115 FROM OBSERVED 7-DAY STRENGTHS

Set No. n	7-Day s psi.	s^2	d	Predicted 28-Day Strength, f_e psi.
111	3680	135×10^5	2750	5450
112	4330	187 "	3040	6020
113	4820	232 "	3210	6360
114	4970	247 "	3260	6450
115	5000	250 "	3270	6470

$f_e = Kd$; $K = 1.98$; $d = c - 0.0000692c^2$; $c = s$

TABLE 9B

n	Observed 28-day Strength, f psi.	Predicted 28-day Strength, f_e psi.	Departure $(f - f_e)$ psi.
111	5220	5450	− 230
112	6000	6020	− 20
113	6220	6360	− 140
114	6010	6450	− 440
115	6660	6470	+ 190

TABLE 10—CALCULATION OF RMS FROM OBSERVED 7 AND 28-DAY STRENGTHS OF CYLINDER SETS 101 TO 110

n	d	f_e	f	$f - f_e$	$(f - f_e)^2$
101	2640	5230	5150	− 80	6400
102	2840	5620	5480	− 140	19600
103	2900	5740	5410	− 330	108900
104	2980	5900	5530	− 370	136900
105	2980	5900	5960	+ 60	3600
106	3050	6040	6410	+ 370	136900
107	3100	6140	6550	+ 410	168100
108	3210	6360	6010	− 350	122500
109	3300	6530	6680	+ 150	22500
110	3320	6570	6750	+ 180	32400

$f_e = Kd$ $K = 1.98$
$d = c - 0.0000692c_2$
$c = s$

Sum = 757800
Mean = 75780
RMS = 275

a. Make ten (but not less than six) sets of 3-, and/or 7-day, and 28-day specimens, varying the slump from 0.5 to 8-in., and record the test results.

b. Use the data from the above series to plot earlier strengths against 28-day strengths to proper scales.

c. Draw a curve through the plotted points and then select the appropriate sets of equations as discussed previously. To repeat, the straight line graph represented by equations (1), (2), (4) and (5) is recommended for practically all cases encountered in practice.

d. Calculate the values of K and PE_{max}.

e. Draw a job curve representing $f_e = Kd$. This will be used thereafter to estimate 28-day strengths from earlier strengths.

f. Parallel to the above-mentioned curve, draw two other dotted curves representing the upper and lower limits defined by PE_{max}. Use these as a guide in evaluating the quality of concrete control on the job.

g. Thereafter, for every 100 specimens made to represent earlier strengths, make 10 specimens to be broken at 28 days. These are to be used as a check on the continuing accuracy of both K and PE_{max}.

2—DEVELOPMENT OF THEORY

Formulas

Case 1.—Plotted data exhibit substantially no curvature.

We seek to fit the data by means of the equation

$$f_e = A + Bc \qquad (9)$$

The values of the parameters A and B are to be determined from preliminary measurements on a job. The preliminary data provide the equations

$$\begin{aligned} f_1 &= A + Bc_1 \\ &\cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \\ f_n &= A + Bc_n \end{aligned} \qquad (10)$$

n being the set number.

The least squares solution of equation (10) is

$$A = \frac{TQ - CP}{NQ - C^2} \qquad (11)$$

and

$$B = \frac{NP - CT}{NQ - C^2} \qquad (12)$$

into which we have introduced for simplicity the notations

$$\begin{aligned} C &= \text{Summation of } c \\ T &= \text{Summation of } f \\ P &= \text{Summation of } c^2 \\ Q &= \text{Summation of } cf \end{aligned} \qquad (13)$$

If we let $A = K$, and $B = MK$, equation (9) becomes

$$f_e = Kd; \text{ where } d = 1 + Mc \qquad (14)$$

We have transformed the parameters from A and B to K and M. If now we cease to regard M as an adjustable parameter, but fix it in advance, we will have only K to adjust.

The determination of M should logically be based on comprehensive data and long experience. In lieu thereof, the writer believes that a likely value is $M = 0.00221$. Substituting this value for M in equation (14), we obtain

$$f_e = Kd \quad \text{(2)}$$

in which

$$d = 1 + 0.00221\, c \quad \text{(5)}$$

Therefore, with M fixed in advance, we have a one-parameter formula, the value of which is readily determined by a short series of data. Thus, at any stage on a given job, we can check the value of K from the last few pours and thereby quickly detect changes in field conditions.

Case 2.—Plotted data exhibit slight curvature.

We seek to fit the data by means of the two-parameter equation

$$f_e = Ac + Bc^{1/2} \quad \text{(15)}$$

of which the least squares solution is

$$A = \frac{CP - YZ}{CQ - Z^2} \quad \text{(16)}$$

and

$$B = \frac{QY - PZ}{CQ - Z^2} \quad \text{(17)}$$

where

C = Summation of c
P = Summation of cf
Q = Summation of c^2
Y = Summation of $c^{1/2}f$
Z = Summation of $c^{1.5}$

By using a technique similar to that shown in Case 1, the writer has determined that the corresponding one-parameter equation is

$$f_e = Kd \quad \text{(2)}$$

in which

$$d = c + 25.9c^{1/2} \quad \text{(6)}$$

Case 3.—Plotted data exhibit appreciable curvature.

We seek to fit the data by means of the two-parameter equation

$$f_e = Ac + Bc^2 \quad \text{(18)}$$

of which the least squares solution is

$$A = \frac{PS - RW}{QS - R^2} \quad \text{(19)}$$

and

$$B = \frac{QW - PR}{QS - R^2} \quad \text{(20)}$$

where

P = Summation of cf
Q = Summation of c^2
R = Summation of c^3
S = Summation of c^4
W = Summation of c^2f . . . . (21)

By using a device similar to that shown in Case 1, the writer has determined that the corresponding one-parameter equation is

$$f_e = Kd \qquad (2)$$

in which

$$d = c - 0.0000692c^2 \qquad (7)$$

Probable error

The probable error is a function of the standard error (SE), which in turn is simply the *root-mean-square* (RMS) departure of the predicted 28-day strength (f_e) from the observed 28-day strength (f). However, three RMS values must be distinguished:

RMS(N_1), or the RMS for N_1 pours, the values of A and B, or K being obtained directly from the pours.

RMS($N \propto$) or the RMS for an infinitely large number of pours, using *true* values of A and B, or K.

RMS(N_2) or the RMS to be encountered in future work, the values of A and B, or K being based on experience on similar jobs, or on N_2 pours.

For our purposes, the significant RMS is the third, therefore

$$SE = \text{RMS}(N_2) \qquad (22)$$

We have available an approximate formula for RMS(N_2) expressed in terms of RMS($N \propto$), or

$$\text{RMS}(N_2) = \left[\frac{N_2 + 2}{N_2}\right]^{1/2} \text{RMS}(N \propto) \qquad (23)$$

We also have available an approximate formula for RMS($N \propto$) expressed in terms of RMS(N_1), or

$$\text{RMS}(N \propto) = \left[\frac{N_1}{N_1 - 2}\right]^{1/2} \text{RMS}(N_1) \qquad (24)$$

Combining equations (22), (23) and (24) to obtain a general formula, we have

$$SE = \left[\frac{N_2 + 2}{N_2} \times \frac{N_1}{N_1 - 2}\right]^{1/2} \text{RMS}(N_1) \qquad (25)$$

We may distinguish two cases of interest: If $N_1 = N_2$, equation (25) becomes

$$SE = \left[\frac{N + 2}{N - 2}\right]^{1/2} \text{RMS}\ (N) \qquad (26)$$

and, if $N_2 = \infty$, equation (25) becomes

$$SE_{min} = \left[\frac{N}{N-2}\right]^{1/2} \text{RMS}(N) \quad \text{(27)}$$

Let us now inquire by how much the accuracy of SE in equation (26) would be increased if it were based on unlimited data and long experience instead of a small number of observations. The answer to this question may be obtained by analyzing the ratio obtained in dividing equation (26) by equation (27), or

$$\frac{SE}{SE_{min}} = \left[\frac{N+2}{N}\right]^{1/2} \quad \text{(28)}$$

It is of interest to compute this ratio for several values of N. This is done in Table 11. It should be kept in mind that, because of inherent approximations, equation (28) is not considered reliable for values of N less than 6; hence, lower values were not computed.

TABLE 11—INCREASE IN ACCURACY OF STANDARD ERROR WITH INCREASE IN NUMBER OF CYLINDER SETS USED TO COMPUTE A AND B, OR K

Cylinder Sets, N	Error in SE percent
6	15
8	12
10	10
20	5
100	1
Infinity	0

Table 11 informs us that no practical end is served in using a large number of data to evaluate A and B, or K; therefore, the use of more than a limited series of data for this purpose is not indicated.

We now adopt the concept of *maximum probable error* (PE_{max}), and define it as that value of the probable error which will not be exceeded by the individual departures of 90 out of 100 parallel specimens. For such a probability ($P = 0.90$), we find from the standard probability curve that

$$PE_{max} = 1.645\ SE = 1.645 \left[\frac{N+2}{N-2}\right]^{1/2} \text{RMS} \quad \text{(8)}$$

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REFERENCES

1. Gowen, J. W., Leavitt, H. W., and Evans, W. S., "The Prediction of the 28-Day Breaking Strengths of Mortars from Their 7-Day Results," Bull. 10, 1925, University of Maine.

2. Gowen, J. W., and Leavitt, H. W., "Relation of 7-Day to 28-Day Compressive Strength of Mortar and Concrete," Bulletin 17, 1926, University of Maine.

3. Leavitt, H. W., "Predictive Values and Variability of Field Concrete Cylinders from Maine Gravel Aggregates," Bulletin 33, 1937, University of Maine.

4. Leavitt, H. W., "A Method of Evaluating Concrete Data by Correlation," A. S. T. M. Bulletin, March, 1938.

5. Slater, W. A., "Relation of 7-Day to 28-Day Compressive Strength of Mortar and Concrete," ACI *Proceedings* V. 22, p. 437, 1925.

6. Fisher, R. A., "Statistical Methods for Research Workers," Section 26, 8th Edition, 1941, Oliver and Boyd, London.

7. Merriman, M., "Method of Least Squares," 8th Edition, 1937, John Wiley & Sons, Inc., New York.

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Job Problems and Practice

Five cash awards—$50.00, $25.00 and 3 of $10.00 each are to be made for the best contributions to this department in the current volume year—Sept. 1944 to June 1945.

In JPP many Members may participate in few pages. So, if you have a question, ask it. If an answer is of likely general interest, it will be briefed here (with authorship credit unless the contributor prefers not). But don't wait for a question. If you know of a concrete problem solved —in field, laboratory, factory, or office—or if you are moved to constructive comment or criticism, obey the impulse; jot it down for JPP. Remember these pages are for informal and sometimes tentative fragments—not the "copper-riveted" conclusiveness of formal treatises. "Answers" to questions do not carry ACI authority; they represent the efforts of Members to add their bits to the sum of ACI Member knowledge of concrete "know-how."

Concrete in New Caledonia (41-165)

By Capt. K. K. HANSEN*

American soldiers in New Caledonia have found that French colony as backward in matters of paving and plumbing as their fathers found France itself in World War I—and as their brothers are probably finding France today. But in one respect, observant GI's have found this out-of-the-way island amazingly ahead of the United States itself.

Concrete is used in New Caledonia not only for foundations, curbing, culverts, bridges and the like, but also for walls, signposts, fences, telephone and telegraph poles, gates and even village bulletin boards. The only use to which it does not seem to be put is for sewers, which are entirely lacking even in Noumea, the capital city. Noumea normally has a population of 12,000, but has a business district as flourishing as that of an average American city of 50,000, largely because it is the only town of any size on the island, and is the mercantile center for the Loyalty and New Hebrides islands as well as New Caledonia.

The multiplicity of uses to which concrete is put in New Caledonia is the result of preference, and not because there is a lack of other build-

*Courtesy Public Relations Bureau, War Department.

Fig. 1—U. S. Army truck enters La Foa on the new concrete bridge spanning the La Foa River, which, in time of flood, is often higher than the bridge.

Fig. 2—This view of the La Foa River bridge, with an ancient French Renault crossing it, shows the old steel bridge, built in 1908 and now condemned.

Fig. 3—Post-office gate, sign, fence and the post-office itself in La Foa are all of concrete.

Fig. 4—The side gate to the post-office is also of concrete. "P. T. T." is for "Poste-Telephonie-Telegraphie".

Fig. 5—Signpost on the outskirts of La Foa, a village of perhaps 250 people, 80 miles north of Noumea, is cast in one piece of solid concrete, and stands as high as a man's shoulders. The supports and background for the matrix letters are natural concrete; the letters are painted a light blue and the two horizontal bars dark blue.

Fig. 6—Noumea house with a concrete fence and foundation.

Fig. 7—Even the village bulletin board, in front of the post-office, is of concrete, with a blackboard insert.

Fig. 8—Telephone poles on "U. S. No. 1", New Caledonia are cast in one piece, cross-trees and all, of concrete.

ing materials. The island is about as far south of the equator as the Hawaiian islands are north, and compares to the latter climatically. Many hardwoods are found in the mountains, and the remainder of the island is virtually covered with "niaouli" or paper-bark trees, a species of eucalyptus so resistant to rot and insects—of which there are comparatively few, if one excepts mosquitoes, as compared to other Pacific islands—that it lasts for years when used for fence posts and rails, and even as pilings in salt water, where it will remain sound indefinitely.

The island is rich in iron, chrome and nickel, and there are also a number of deposits of gleaming white sand from which the Melanesian natives make a dazzling white plaster for their wattle houses, which are roofed with paper-bark thatch. The French prefer houses of concrete, with the ubiquitous galvanized-iron roof of the tropics.

Concrete is the favored building material not only in Noumea but in the tiny towns along both coasts of the island. Each village has concrete signposts at its approaches, modernistic in design and built for the ages—signposts which would be a decided improvement on the wooden eyesores which are a blemish on the environs of most American towns.

As one enters the towns themselves, concrete work is in evidence on every hand, some of it, in decorative fences built with concrete on an armature of iron rods, strikingly beautiful. Bridges, too, throughout the island, are of concrete, with no railings because of the torrential tropical rains and the frequent flash floods. The bridges are built so that water can flow over as well as under them, without hindrance.

The accompanying photographs illustrate some examples of New Caledonian artistry in concrete.

Bonding New Topping to Old Concrete Surfaces (41-166)

By F. B. HORNIBROOK* and NATHAN GREEN*

Not infrequently a situation is called to notice where a new mortar or concrete topping had been applied to an old concrete surface and as a result of the failure in bond of the new to the old concrete an unsatisfactory surface exists. Since a new topping undergoes changes in volume as it ages and dries not matched by similar changes in the old concrete to which the topping is applied, strains may be expected to develop at the interface. This, of course, tends to cause separation of the two concretes. A brief series of tests is described, undertaken to ascertain the order of the bond strength between an "old" concrete and a new

*National Bureau of Standards, Washington, D. C.

TABLE 1—TENSILE STRENGTHS DEVELOPED IN BOND OF "NEW" TO "OLD" CONCRETE

(by casting a new topping to a matured concrete surface and after curing of the topping, allowing it to dry to permit drying shrinkage to occur)

Surface Preparation	Bonding Treatment	Tensile strength[1] of bond av. 3 specimens psi	Actual range in measured bond strength psi
Original wood-floated surface	Cement-water slush coat, w/c = 0.5	40	34 to 46
" " " "	1:1 cement to fine sand slush coat, w/c = 0.5	45	38 to 53
" " " "	1:1 cement to powdered iron[2] and rusting agent mix, w/c = 0.5	45	35 to 53
" " " "	Cement-water slush coat plus $\frac{1}{16}$ in. of 1:2 mortar	30	24 to 30
Troweled surface, etched with 1:2 HCl	Cement-water slush coat, w/c = 0.5	35	29 to 43
Chipped to expose coarse aggregate	Cement-water slush coat, w/c = 0.5	95[3]	84 to 111
" " " " "	1:1 cement to fine sand slush coat, w/c = 0.5	95[3]	85 to 103
" " " " "	1:1 cement to powdered iron and rusting agent mix, w/c = 0.5	95[3]	96 to 97
" " " " "	Cement-water slush coat, w/c = 0.5 (topping mixed with 5 gal. sack extra water allowed to stand 1 hr., remixed then applied)	120	119 to 122

(1) Reported to nearest 5 lbs/sq in.
(2) A proprietary compound recommended by the manufacturer for bonding
(3) Some aggregate pulled out of topping by bond coat

topping after the drying shrinkage of the new topping had been allowed to take place. A few variations in the bonding procedure were also tried to determine the resulting effect on the bond strength.

The test specimens were each prepared as follows: A 6-in. cube was cast and after damp-curing for 7 days was allowed to dry at room temperature for 35 days. This cube represented the "old" concrete. The top face (as cast) was roughened by chipping or acid etching and just prior to application of the new concrete, the roughened surface was soaked with water for 3 hours then air-dried one hour after removal of excess water. The scheduled slush coat was then applied with vigorous scrubbing by means of a stiff-bristled brush, the six-inch cube-mold fitted to the specimen to overlap slightly, and a second cube cast onto this prepared surface. The specimen, consisting of the two joined cubes, was then damp-cured 7 days, air dried 21 days, then tested in tension.

The grips for the tension test were provided by embedding to a depth of about 3 in. in the "rear" of each cube, 8 symmetrically-placed threaded steel rods. The half of each cube containing the threaded bars was filled with a 1:2 mortar, and the "face" half of the first cube (the "old" surface) was filled with a concrete having a nominal cement factor of 6 bags per cu. yd. and ¾ in. maximum size gravel aggregate. The "face" half of the second cube was filled with a topping mix proportioned 1:1 1/3:2 parts by weight of cement, sand, and No. 4 to ⅜-in. pea gravel, with a nominal water content of 4½ gal. per bag. This topping is patterned after the recommendations of the Committee 804 report, "Suggested Recommended Practice for Wearing Surfaces for Floors" (A.C.I. Jl. Sept., 1938; *Proceedings* V. 35, p. 21).

Table 1 lists the various bonding procedures used and the tensile strengths attained. In all cases the break occurred in the bond of the "new" to the "old".

From the results in this table it is evident that under the conditions of this test even the most favorable bonding procedure failed by a wide margin of developing strengths approximating the normal tensile strength of concrete. Considering the various bonding procedures it is indicated that (1) a thorough chipping and roughening of the old surface was essential; (2) none of the slush coats tried was significantly more effective than the plain cement-water application; and (3) that pre-aging the topping mix for an hour then remixing improved the bond strength materially. This latter procedure, while probably impractical on a large job, should be well worth while on small patching jobs.

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Current Reviews

of Significant Contributions in Foreign and Domestic Publications, prepared by the Institute's Reviewers

Decrease of the detrimental influence of organic impurities in concrete sand

INGE LYSE, Kgl Norske Videnskab. *Selskab Forh.* 16, 43-6 (1943); *Chem. Zentr.* 1943, II, 2197. *Chemical Abstracts*, V. 38, No. 21, Nov. 10, 1944. HIGHWAY RESEARCH ABSTRACTS

By speeding up the binding and hardening process the detrimental influence of organic impurities can be decreased. The speeding up is accomplished by use of $CaCl_2$, $MgCl_2$, $AlCl_2$, and $FeCl_2$. $CaCl_2$ proved to be best.

Bridge Foundations

W. A. FAIRHURST, *Concrete and Constructional Engineering*, V. 39, p. 304-307
Reviewed by GLENN MURPHY

This installment includes several drawings for the Inverbervie bridge which is a multiple span girder bridge on a curve of about 420 ft. radius, and the bridge at Guardbridge, also a multiple span girder. Details of foundations, piers, hinges, and rockers are shown.

Flexural strength of concrete

STANTON WALKER, before annual meeting of Board of Directors of Nationl Sand & Gravel Assn., Jan., 1945 —mimeographed.

Discusses in elementary terms significance of flexural strength of concrete, particularly in relation to road slabs. Reviews data from various sources on the several factors affecting flexural strength, including angularity of aggregates, surface texture of aggregates, strength of coarse aggregate, etc. Suggests that flexural strength is better measure of deterioration of concrete than compressive strength.

Bridge foundations

W. A. FAIRHURST, *Concrete and Constructional Engineering*, V. 39, No. 11 (Nov., 1944), pp. 277-279
Reviewed by GLENN MURPHY

The author continues his series with photographs, drawings and brief descriptions of three arch bridges. The Glen Bridge, Dumfermline has a span of nearly 190 ft. for a 28-ft. roadway with 6-ft. walks on each side. The Kemnay Bridge, Aberdeenshire, carries a 19-ft. roadway ad has a clear span of 122 ft. The Linlithgow Bridge has a clear span of 108 ft. and an overall width of 53 ft. Details of foundations are shown for each of the structures.

Lightweight concrete aggregates

Concrete and Constructional Engineering, V. 39, No. 11 (Nov., 1944), pp. 289-290

Reviewed by GLENN MURPHY

A new edition of Building Research Bulletin No. 15 "Lightweight Concrete Aggregates" by Dr. F. M. Lea is available from H. H. Stationery Office, London (Price 3 d.). Data are given comparing the properties of concretes made with coke breeze, pumice, foamed slag and expanded slate aggregates. Some values of compressive strength, unit weight, shrinkage, expansion, and thermal conductivity are abstracted in this article.

Concrete caissons at the Normandy harbor

Concrete and Constructional Engineering, V. 39, No. 11 (Nov., 1944), pp. 280-283

Reviewed by GLENN MURPHY

This is a general descriptive article concerning the 146 caissons which were constructed in England and towed across the channel to provide two artificial harbours each approximately the size of Dover Harbour for the landing of troops and equipment on a large scale. The caissons were constructed in six sizes ranging from 1672 to 6044 tons displacement. Their construction required approximately 330,000 cu. yd. of concrete, 31000 tons of steel, and 1500000 yd. of forms.

Experimental houses built with lightweight concrete

Concrete and Constructional Engineering, V. 39, No. 11 (Nov., 1944), pp. 284-285

Reviewed by GLENN MURPHY

Of several experimental houses built at Northolt by the Ministry of Works two pair are of steel frame construction with concrete slab filling, one pair is "no-fines" concrete, one pair is concrete with foamed slag aggregate, and one pair is concrete with expanded clay aggregate. These houses, constructed to provide information for postwar housing schemes, are to be compared with houses with 11-in. cavity brick walls. All houses have a floor area of 850 sq. ft. Details of construction and mixtures are given. One feature was the use of expanded metal (with a diamond mesh ½-in. by ¼-in.) for form The mesh retained the concrete satisfactorily and the rough texture provided an excellent key for plastering and external rendering.

Construction of a bridge over a railway

L. E. HUNTER, *Concrete and Constructional Engineering*, V. 39, No. 7 (July, 1944), pp. 177-181

Reviewed by GLENN MURPHY

This article describes some of the construction details involved in the erection of a bridge over a railway and highway. The 18-in. road slab of the bridge is supported by steel joists with a clear span of 26 ft. 4 in. over the railroad and a clear span of over 15 ft. 6 in. over the highway. The joists are supported by 3 ft. piers at the ends and a 4 ft. thick intermediate pier between the railroad and highway. The piers are supported on bearing slabs approximately 2 ft. 6 in. thick. Footings for the bearing slabs were obtained by driving sheet piling through the underlying soil and peat to a sand bottom about 15 ft. below the surface. A description of the system of forms used in the concreting is given in detail.

Concrete surface finishes, renderings and terrazo

W. S. GRAY and H. L. CHILDE, London, 1944 (Concrete Publications, Ltd.), 9½ in. by 6½ in., pp. 137, figs. 132, 8*s*. 6*d*.

The book is a practical treatise on present methods of carrying out surface finish treatments for concrete whereby both color and texture are improved. Many photographs serve to illustrate modern trends in the treatment of concrete surfaces including the development of "architectural concrete" through careful attention to design and surface finish. Chapter headings are as follows:—Causes and prevention of blemishes. Cement washing and rubbing down. Concrete in architecture. Pre-cast slabs as shuttering. Sliding shutters for special concrete faces. Exposed aggregate surfaces. Paints, stains, and gunite. Pre-cast units. Renderings and kindred surfaces. Interior finishes. Terrazzo and mosaic. Artificial marble.

Shear reinforcement in column bases

W. T. Marshall, *Concrete and Constructional Engineering*, V. 39, No. 9, Sept., 1944, pp. 219-223

Reviewed by Glenn Murphy

The author presents a method of design of reinforcement for column footings based on the assumption that the critical section in shear is on a surface extending downward from the base of the column at an angle of 45 deg. with the horizontal. The assumption is based on the results of tests conducted by the author and published in the *Journal of Inst. C. E.*, March, 1944. The proposed design involves two sets of reinforcing bars: one set being the conventional tensile reinforcement in the bottom of the footing; the other, a set along the bottom of the fotting under the column then extending upward and outward at 45 deg. to the top of the footing, and then along the top of the footing to the outside. Hooks are provided on each set. Design calculations are given for two examples.

Comparison of standard and proposed tension grips and test pieces for testing portland cement mortars

Matthew McNeary, Bulletin 39, Maine Technology Experiment Station, University of Maine, April, 1944.

Highway Research Abstracts

The standard tension briquet, adopted by the American Society for Testing Materials, does not give a true measurement of the tensile strength of portland cement mortar because of (1) stress concentrations at the grips, (2) higher stresses at the outside of the neck than at the center, and (3) cross stresses at the neck, which obviate a condition of pure tension at the break. Photo-elastic studies have shown that all three of these conditions could be improved by using rubber grips instead of the metal roller type and by elongating the neck of the briquet by inserting a straight section one inch long.

Several hundred briquets of both the standard and elongated type were made and tested in both the standard grips and the rubber grips with the following results: 1. The changed briquet shape and the rubber grips each separately caused an appreciable increase in the apparent tensile strength of the mortar. 2. The rubber grips proved to have the important effect of narrowing the range of variation of the strength of three briquets taken from the same three-gang mold.

Overlapping tensile reinforcing rods in concrete beams

R. H. Evans, *Concrete and Constructional Engineering*, V. 39, No. 7 (July, 1944), pp. 167-175

Reviewed by Glenn Murphy

This paper presents the test results obtained from a series of concrete beams in which the reinforcing steel consisted of hooked bars 2 or 3 ft. longer than half the span. The bars were lapped (a) where the bending moment was constant, (b) where the shear was constant.

Fifteen beams were tested, each having a cross section 5 in. by 10 in. and a span of 100 in. with loads at the third points. A 1:2:4 mixture with high alumina cement was used for all beams. Various arrangements and sizes of high strength steel reinforcing bars were used. Strains along the steel were measured with a 4-inch strain gage in several of the beams immediately after the beam cracked in tension and again just before final failure.

The results show that simple overlapping of the reinforcing rods does not form an effective joint. At low loads, cracks appear at the ends of the overlaps and develop into diagonal tension cracks, resulting in collapse of the beam at loads well below those carried by a beam with continuous reinforcement. The recommendation is made that if full length rods are not available the shorter lengths welded together rather than simply overlapped.

Post-war building studies No. 8—Reinforced concrete structures

MINISTRY OF WORKS, London, 1944 (H. M. Stationery Office), 9½ in. by 6 in., pp. 12, 6*d*.

The Reinforced Concrete Structures Committee which was convened by the Institution of Structural Engineers in September, 1942, here presents its recommendations for ensuring in the postwar period the maximum economy of material and the most rapid methods of reinforced concrete construction having due regard to other interrelated building services such as lighting, ventilation, plumbing, etc. The report comprises the following sections:—1. Consideration of Code of Practice; a new national Code of Practice for reinforced concrete, containing a power of waiver and to be revised every 3 years, is recommended. 2. Loads on floors and roofs. 3. Stresses in steel and concrete. The stresses in concrete in compression due to bending may be raised by 10 per cent above those given in the code issued in 1939 by British Industries National Council (see *B.S.A.*, 1940, No. 660). 4. Improvement in design and construction methods. Emphasis is laid on the employment of only properly qualified and trained persons for the design and construction of reinforced concrete. Suggestions for effecting economy of time are also enumerated. 5. Loan periods. 6. Reinforced concrete foundations. 7. Prestressed and vibrated concrete. 8. Welding; it is recommended that, at present only, butt welding be allowed for mild steel. 9. Composite construction. 10. Code of Practice: certain modifications to the existing B.I.N.C. Code are recommended and enumerated. The two appendices list the Institution of Structural Engineers' schedule of symbols recommended for use in reinforced concrete and structural steelwork calculations, respectively.

Expansion of clay and concrete drain tile due to increase of temperature and moisture content

DALTON G. MILLER and CHARLES G. SNYDER, *Agricultural Engineering*, May, 1944, V. 25, No. 5. (Abstracted by M. S. Kersten). HIGHWAY RESEARCH ABSTRACTS

Comparisons of the amount of expansion in clay and concrete drain tile due to temperature increase and change of moisture content is provided by the results of tests on products from 16 plants in Minnesota and Iowa.

The average coefficient of thermal expansion for the clay tile pipe was 0.0000025 per degree Fahrenheit. The average expansion due to wetting from a room dry condition to 28 days submerged was 0.00005-ft. per ft. of length; the average absorption in this time was 9.7 per cent. Thus the expansion due to each per cent of moisture absorbed is equivalent to that of a 2-deg. F. rise in temperature.

For the concrete pipe the average thermal coefficient was 0.0000030 per deg. F. and the average expansion due to a 10.1 per cent absorption was 0.00046 ft. per ft. A moisture increase of 1 per cent gives an expansion equivalent to that caused by a 15-deg. F., rise in temperature.

The authors conclude that expansion of clay tile after installation due either to increase of temperature or to increase in moisture content is not of sufficient magnitude to impair the effectiveness of the tile line through closure of the joints between individual tile. The same may be said of dry-tamped concrete tile as regards expansion due to temperature changes, but some precaution should be exercised not to install concrete tile when too dry.

Shape, size, and shrinkage

A. D. Ross, *Concrete and Constructional Engineering*, V. 39, No. 8 (Aug., 1944), pp. 193-199

Reviewed by Glenn Murphy

The objectives of the investigation reported were to (a) determine qualitatively the shrinkage in specimens of widely differing dimensions and shape, (b) correlate shrinkage and loss of weight by drying, (c) investigate differential shrinkage, (d) test the applicability of the diffusion and surface emission equations to mortar drying as a porous solid, and (e) determine the magnitude of autogenous shrinkage. Specimens were made of a 1:2 standard Portland cement mortar with a water-cement ratio of 0.40 by weight, and were stored in a room maintained at a mean temperature of 71 F. and a mean relative humidity of 53 per cent. Linear measurements were taken between steel balls cast in the specimens, using a frame with a built-in micrometer. Autogenous shrinkage was measured by displacement, immersing the specimens in mercury. Specimens were 9 in. long with rectangular, square, triangular, circular, and annular cross-sections. From two to four sizes of each shape were used to obtain different values of the surface-volume ratio. Shrinkage and loss of weight curves for times up to 160 days are shown.

The conclusions were that size and shape have a pronounced effect upon the magnitude of the shrinkage, and that the variation is largely a function of the surface-volume ratio The rate of shrinkage is not a fixed property of the material, but may be evaluated in terms of the surface-volume ratio. Shrinkage extends only a short distance into the interior of large masses.

It is unfortunate that destruction of equipment and specimens by enemy action prevented completion of the project.

1944 book of A.S.T.M. standards

In three parts (1235 specifications and tests, 6030 pages, American Society for Testing Materials ($30.00 or $10.00 per part.)

The new 1944 Book of A.S.T.M. Standards, issued in three parts contains in latest approved form all of the society's widely used specifications and tests for materials—1235 specifications and standard methods which cover more than 6000 pages.

All specifications, whether formal standards or tentative are given. The three parts, are: *Part I, Metals.*—Ferrous and non-ferrous metals (all A and B and some E serial designations) except methods of chemical analysis. General testing methods (E serial designations;) *Part II, Nonmetallic Materials*—Constructional.—Cementitious materials, concrete and aggregates, masonry building units, ceramics, pipe and tile, thermal insulating materials (all C serial designations); wood and wood preservatives, paints, varnishes and lacquers, road materials, waterproofing and roofing materials, soils (certain D serial designations). General testing methods, thermometers (E serial designations); *Part III, Nonmetallic Materials—General.*—Fuels, petroleum products, electrical insulating materials, rubber, textiles, soaps and detergents, paper, plastics, water (remainder of D serial designations). General testing methods, thermometers (E serial designations).

This 1944 Book includes all emergency standards and emergency alternate provisions which have been widely used to expedite production and procurement of important materials (a separate volume, Chemical Analysis of Metals, includes standards and recommendations for both ferrous and non-ferrous metals. These methods are not included in the Book of Standards.) Each part of the 1944 Book has a complete subject index (from 36 to 44 pages).

To keep the books up to date, a supplement will be issued to *each* part late in 1945. As a service with the 1944 Book of Standards there is a complete 200 page Index to Standards, which is furnished without additional charge and a copy accompanies the purchase of each part or complete set. (1945 Supplement will cost $4.00 per part).

Effect of tar, ammonium fluosilicate, and sodium hydroxide on the alkali resistance of concrete

E. C. E. Lord, *Pub. Roads*, 1944, 23 (11), 282-96.

Exposure tests have been made on the protective value of water-gas tar, ammonium fluosilicate and sodium hydroxide, either alone or in combination, for concrete exposed to the action of sulphate waters. The tests were made on concrete specimens (3-in. by 6-in. cylinders) having water/cement ratios of 0.7 and 0.8 by volume and using two different cements, one having a low alumina-iron ratio and the other a high alumina-iron ratio. The specimens were given an intial curing in the moist room of 7 days and 28 days, followed by curing in laboratory air for periods of 1 and 7 days for those specimens to be treated with tar, and 7 and 28 days for those specimens to be treated with ammonium fluosilicate and sodium hydroxide. The protective materials were then applied by immersion and by brush coating followed in some cases by a seal coat of coal tar applied with a brush. The treated specimens, when dry, were stored, some in 5 per cent sulphate solution in the laboratory, and some in sulphate waters (2.4 to 2.7 per cent sulphates) of Medicine Lake, South Dakota, for periods of several years. The results of these exposure tests are shown in photographs and tables and are discussed. The trend of performance of the specimens stored in the laboratory solutions and of those stored in Medicine Lake were in comparatively close agreement. Concrete made with the low alumina/iron ratio was more resistant to sulphate action than that made with the high alumina/iron ratio. Water-gas tar applied either by immersion or by brush gave the same degree of protection which was greatly increased by the addition of a seal coat of coal tar. Immersion beyond 1 hour or the application of more than 4 brush coats produced no appreciable benefit. Ammonium fluosilicate, either alone or combined with tar, did not give adequate protection; sodium hydroxide, either alone or combined with tar, gave somewhat better protection but was still distinctly inferior to the tar treatment alone. Character and curing of the concrete influenced the resistance to sulphate waters; concrete made with low water/cement ratio was more resistant than concrete with a high water/cement ratio. Concrete cured 1 day in air prior to treatment was more resistant than concrete cured 7 days.

Precast concrete bridge

L. J. Eichelgrun, *Concrete and Constructional Engineering*, V. 39, p. 293-303

Reviewed by Glenn Murphy

Several general applications of precast concrete to bridge construction are described. Most of the structures discussed in the article were built or designed for the Great Western Railroad. The first application is the use of precast concrete slabs for replacing decks on bridges. In the example described, standard slabs 8 inches thick and 3 feet wide were used for a 60-ft., single track railroad bridge deck. The slabs were laid on a hardwood backing placed between the protruding unit heads of the girders as a cement mortar backing was found to be unsatisfactory. The slabs were necessarily heavily reinforced.

The second application makes use of precast girders in new construction. In one example listed U-shaped girders are used. The girders are laid side by side giving a beam-and-slab effect in cross section. As the girders act independently, a thick layer of ballast is recommended to help distribute the wheel load. Another example utilizes the standard girder-and-slab construction. In a third example the slab was cast integral with the two main girders and the unit lifted into place after the forms had been stripped. The bridge described had a length of 47 feet and the slab and girder unit weighed about 16 tons.

Precast concrete units have been used for foot bridges. In one type of construction a T-beam is used, the top of the T forming the 5-foot footway. The parapets are attached to the sides of the T-beam. In another type of construction narrow beams about 6 feet deep run longitudinally on each side, the upper part of the beams forming the parapet. The walkway consists of a 5-foot slab expanding between the beams. The slab rests on shoulders 8 inches above the bottom of the beams.

Precast U-shaped sections have been used as launders (flumes). In the example shown, the sections have an overall height of 5 ft. 7 in., a width of 4 ft. 3 in. and a length of 21 ft. 6 in. The walls are 6 inches thick and the base 5 inches thick. The sections were waterproofed with two coats of tar.

The use of precast slabs for forms is mentioned and several design and construction hints are given. Drawings and photographs are included.

Rate of sedimentation

Ind. Eng. Chem. 36, 618-624, 840-847, 901-907 (1944);
(Bulletin 3, Portland Cement Assn Research Laboratory)

Three papers report the results of a study of the fundamentals of sedimentation in thick suspensions such as portland cement pastes. This study was a direct outgrowth of the work reported in 1939 by Powers on the bleeding of portland cement paste, mortar and concrete.* In that study the bleeding of paste and concrete was shown to be a special case of sedimentation, and equations were developed showing the role played by water content and surface area. The equations contained an empirical constant for which only a tentative interpretation could be offered. When in 1940 Dr. Steinour took over the work, his aim was to determine whether the empirical constant merely serves to compensate for fundamental misconceptions embodied in the equation or whether it represents physical characteristics of the suspension and if so to determine what the characteristics are.

Dr. Steinour confirmed the fundamental concepts of the original work but showed the need for modifying some of the mathematical expressions. However, Dr. Steinour's papers go much beyond the matter of confirming earlier work. They provide a general understanding of the behavior of thick suspensions that should be valuable wherever such suspensions are encountered.

The first of the three papers deals with non-flocculated suspensions of spheres; the second deals with uniform-size angular particles in both the flocculated and non-flocculated states. Comparisons of the behaviors of these suspensions with those of non-flocculated spheres showed the effects of particle angularity and flocculation. The third paper takes up sedimentation of various pulverized minerals embracing a wide range of particle sizes. In general the sedimentation of these mineral powders was found to follow the same laws as the simplified systems dealt with in the first two papers.

To the investigator of problems in cement and concrete, these papers not only round out information on the phenomenon of bleeding, but they give also insight into the structure of fresh cement pastes. For example, the studies indicate that a normal portland cement paste is not composed of individual clusters of cement grains; rather, the particles are linked together into one continuous "floc" in which the individual particles are evidently held in place by interparticle forces.

*J. Am. Concrete Inst., June 1939; *Proceedings* V. 35, p. 465 (Bulletin 2, PCA Research Laboratory June 1939).

Contribution to the question of the lime-solubility of portland cement and mixed cement on a portland cement basis

R. Zollinger, *Zement*, 1943, 32 (17/18), 187-96.

The investigation described was carried out in connection with the selection of a cement for concrete exposed to the action of water containing free carbon dioxide. The 12 test cements included portland cement, "Erz" cement, iron-portland and portland-blastfurnace cements, also trass cement, containing portland cement clinker: trass in the proportions 70:30 or 60:40, and cement with added "Thurament," and artificial trass. The problem to be studied was the extent to which the protective influence of the reactive silica added to the cement results from chemical reaction as distinct from the physical effect of increased density due to the colloidal volume increase. The latter effect was excluded by the use of porous test specimens into the interior of which the water could penetrate. The cement-sand (1:5.5) specimens were prepared in the form of slabs, 20 cm. by 20 cm. by 2 cm., and were exposed in separate glass tanks to a continuous flow of water containing carbon dioxide which was changed when the lime content exceeded 0.2 g./l. At intervals during a period of 210 days the quantity of lime dissolved from the slabs was determined from analysis of the tank water. On conclusion of the experiments the specimens were removed from the water and, the disintegrated material having been brushed off, were stored in damp air for 7 days. The specimens were then weighed and the loss of weight obtained, the original weight being known. The loss of weight of the residual slabs exceeded the weight of the removed, disintegrated material; the amount of the additional loss of weight was an indication of the extent to which the slabs had been attacked. The data of loss of weight and lime dissolved and the lime solubility curves of the test cements are shown. The tensile strength of the test slabs at the end of the test period of 234 days and of other slabs at the end of the same period was determined by bending strength tests. The strength test results showed not only a redction of strength in each case but also a complete rearrangement of the strength values of the different cements. A second series of experiments was carried out with slabs of the same cements at the end of a storage period of 6 months in damp air to allow time for a chemical reaction similar in nature to the protective reaction to take place. The results obtained with each test series are considered and the test cements placed in order according to the degree of corrosion, lime solubility and the effect on tensile strength. A decisive statement respecting the value of the individual cements, on the basis of the experimental data, it is pointed out, would be misleading. It may be said definitely, however, that in no case is the chemical protective action adequate to prevent an attack by corrosive waters. Chemical reactions with protective effect undoubtedly take place. The age of the concrete is not without importance, but complete protection is not attained with increasing age and different types of cement differ considerably in their behaviour. Where, in practice, resistance to the action of corrosive water is observed it is due in part to the physical, waterproofing effect of the colloidal compounds.

Highway Research Board—synopses of papers, 24th Annual Proceedings

Highway Research Board Abstracts

The 24th Annual Meeting of the Highway Research Board, scheduled for Cincinnati Ohio on Nov. 22-25, 1944, was first postponed—later cancelled.

The major part of the value of the year's research work will be preserved by publication in Vol. 24 of H. R. B. Proceedings.

A special issue of Highway Research Abstracts contains summaries of practically all of the papers presented for publication. Synopses of those of major interest in relation to concrete, follow:

Structural efficiency of transverse weakened-plane joints

E. C. Sutherland, Senior Highway Engineer and H. D. Cashell, Associate Highway Engineer, Public Roads Administration (reported by Highway Research Board, Committee on Rigid Pavement Design, R. D. Bradbury, Chairman).

The investigation was made as a cooperative research between the States of California, Kentucky, Michigan, Minnesota, Missouri and Oregon and the Public Roads Administration. In each State an experimental pavement several miles in length, embodying the experimental features has been constructed and kept continuously under observation. All of the projects have been described before the Board with the exception of that in California (see Proceedings of the 20th and 21st Annual Meetings). In addition to these experimental pavements in service in the several States the original program included a study of the structural efficiency of transverse joints of the weakened-plane type to be made by the Public Roads Administration. The first description of this work is in the current symposium.

Briefly, the experimental features common to the six State projects consist of a series of plain and reinforced concrete sections in which the expansion joint spacing is varied from 120 to 5,280 feet. All of the plain concrete sections have transverse contraction joints at relatively close spacing (15 to 25 feet) while the reinforced sections have expansion joints at 120-foot spacing with one intermediate contraction joint.

In general, load transfer devices were used in all expansion joints but were used in only part of the contraction joints of a given project in order to determine whether or not load transfer is needed with closely spaced contraction joints of the weakened-plane type. Several of the States included in their projects additional experimental features of design that were of particular interest to them. These features are described in the reports published.

During the three or, in three cases, four years since these pavements were constructed measurements and observations have been made of the following:

1. Daily and seasonal variations in temperature.

2. Daily, seasonal and progressive or permanent changes in the widths of the expansion and contraction joints.

3. Changes in elevation of the pavement, especially with respect to faulting at the joints.

4. The general condition of the pavement and joints.

Progress reports describing the condition of the pavements in the respective States and presenting the data collected up to this time have been prepared by each of the States participating in this investigation and are being presented in this symposium. In addition, the California project is described for the first time.

The authors of these reports feel that it is too early to draw final conclusions regarding the merits of the different designs, but they have made some interesting observations with respect to the data and the significance of certain trends indicated by the data. Some of the tentative indications that appear in the present reports are:

1. Where expansion joint fillers of a plastic type are used there has been a progressive closing of the expansion joints and a progressive opening of the contraction joints with time. The greatest progressive change in the width of the joints occurs during the first year after the pavement is laid and it appears to continue so long as there is expansion space available. The magnitude of the progressive opening at the contraction joints appears to be related to the spacing of the expansion joints, being less for the greater expansion joint spacings.

2. Wood expansion joint filler as used in one project appears to restrain the progressive changes in joint width.

3. The seasonal changes in width of the contraction joints seem to increase as the spacing between these joints is increased, but this variation may not be linear if the expansion of the pavement is restrained during periods of high temperature.

4. In pavements laid at temperatures less than those which normally occur in the summer, the seasonal changes in the widths of the contraction joints seem to be less in pavements with limited expansion space than in pavements with greater expansion space.

5. While a limited amount of faulting has been noted in some of the pavements, the amount is not sufficient to justify any conclusions regarding the necessity for the use of dowels to prevent faulting in weakened-plane joints.

As an integral part of the general investigation the effectiveness of aggregate interlock for reducing the critical stresses caused by loads, acting in the vicinity of weakened-plane joints, has been studied by the Public Roads Administration on a test pavement near Washington, D. C. This pavement consisted of six 30- by 20-foot sections of 8-inch uniform thicknesses, each divided longitudinally by a deformed metal plate center joint and transversely by a weakened-plane joint. The type and maximum size of coarse aggregate in the concrete was varied in the different sections so that the effect of these variables on the efficiency of weakened-plane joints might be studied.

In making the tests to determine the efficiency of the joints, loads were applied at the joint edges, free edges and at the interior of the slabs and the critical strains caused by these loads were measured. Tests at the joint edges were made with the joints in a closed condition (under compression) and at various controlled openings.

It was found that all of the weakened-plane joints were effective in reducing the critical load stresses when closed and under compression, but that aggregate interlock was not a dependable method of stress reduction when the joints were open small amounts.

Structural Behavior of Concrete Airfield Pavements, R. R. Philippe, U.S. Engineer Department, Cincinnati Testing Laboratory. The years immediately before the war and those since its beginning have witnessed a tremendous development of all types of military aircraft. Not the least spectacular of these developments is the advancement of the heavy land-based bomber and cargo ship. Today we read of building pavements for 150 kip wheel loads, a six-fold rise in as many years from a wheel load standpoint. Experts in aircraft design speculate sagely that it would not be safe to guess that heavy aircraft of the future will weigh less than a million pounds.

These heavy aircraft, when used as bombers, must possess long range and large bomb carrying capacity. This demand, coupled with the structural difficulties of multiple landing gear, has resulted in a marked tendency on the part of plane designers to concentrate severely the loads of planes on the minimum number of wheels, thereby multiplying the need for heavier and heavier pavements.

The Corps of Engineers has met the challenge of these designs by accelerated testing of extrapolated designs. The investigations for rigid pavements have been centered largely in the Ohio River Division, where its unit, the Cincinnati Testing Laboratory, has been designated as the Engineer Department's Rigid Pavement Laboratory.

The tests, to date, have been designed to determine the effect of impact of a landing wheel of variable load on a rigid pavement, to measure the reactions of a pavement under a set of idealized conditions as assumed by Westergaard's theory, and to study the effect of accelerated traffic on rigid pavements. Impact effects were studied by means of flight tests and drop tests, wherein the reactions both in the plane and in the pavement were correlated. Static loading tests were employed to check the validity of theoretical assumptions, and later to correlate the effects of repeated loading to the results observed in traffic tests. Accelerated traffic tests with wheel loads ranging from 20 to 60 kip have been conducted on thirty-seven designs, and all preparations have been made to traffic test nineteen basic designs with a 150 kip wheel load.

The paper is a description of the methods employed in testing and presenting the factual data of the designs tested. It establishes for record the basis for future presentation and discussion of results. A few of the more startling results are illustrated to stimulate interest by the reader.

Progress Report of the Committee on Durability of Concrete, M. O. Withey, Chairman, University of Wisconsin. This committee, an outgrowth of the Project Committee on Durability of Concrete as Affected by the Cement, was formed early in 1940. The object in its formation was to consider and investigate factors related to the durability of concrete. A reasonably rapid method for ascertaining the resistance of concrete to freezing and thawing has long been desired by materials testing engineers, but to date such procedures as have been stipulated in specifications have failed to obtain a large following.

Since the Project Committee on Durability of Concrete as Affected by the Cement had made some tests in which the effects of different rates of freezing on resistance of mortars to freezing and thawing had been observed, it seemed desirable to give further study to this important subject. In arriving at this decision the committee was well aware that the results of further fundamental research on the causes of deterioration due to freezing and thawing might materially modify such testing procedures as the committee might now use or prescribe. Nevertheless the committee felt that the uncertainty of the time at which such fundamental information would be available and the immediate need of a suitable testing procedure were ample justification for the prosecution of the proposed program.

Program.—Essentially the program adopted involved: (1) a comparison of the relative severity of a carefully specified coordinating freezing and thawing test as practiced in different laboratories. (2) a comparison of the effects of the freezing and thawing procedures commonly used in these laboratories (local procedures). (3) a comparison of the severity of the coordinating test procedure with the local laboratory procedures.

Tests providing these comparisons were made in seven laboratories, and this report describes the testing procedures, discusses the results and contains the following conclusions: 1. Under the conditions of these tests the electronic vibrating devices used provided a convenient and rapid means of determining the change in the dynamic modulus of elasticity of the specimens tested. 2. Within the limits of these tests the average relation of the percentage decrease in modulus of rupture (R) to the percentage decrease in the dynamic modulus of elasticity (E) due to freezing and thawing was $R = 1.5E$ for either the local or coordinating test procedures. Considering the data of laboratories performing the entire B and C test programs the individual laboratory average $R - E$ relations were within 20 per cent of the above grand average. 3. The relation between the percentage increase in the modulus of rupture (R') and the percentage increase in dynamic modulus of elasticity (E') of the moist-cured control beams was much more variable than the relation between the decreases of these properties in freezing and thawing. Based on changes after 28-days the average relation was $R' = 1.2E'$. 4. For the types of concrete tested the relation between the reductions in flexural strength and in dynamic modulus of elasticity was sufficiently reliable to measure the rate of deterioration of the flexural strength under the methods of freezing and thawing used. 5. The flexural strength is much more sensitive to the deteriorating effects of freezing and thawing than is the compressive strength. 6. Within the limitations of these tests the relation of the reduction in the compressive strength to the reduction in the dynamic modulus of elasticity due to freezing and thawing was so variable in the tests conducted that the reduction in the dynamic modulus could not be used as a measure of the reduction in compressive strength. 7. Loss in weight does not provide a criterion of the early deterioration in flexural strength due to freezing and thawing. 8. Specimens frozen in air as in the coordinating program showed little surface deterioration due to breakdown of the mortar. Some specimens exhibited spalling over unsound coarse aggregate particles. 9. Specimens frozen in contact with water evinced deterioration at the corners and edges and over the portions partially immersed in water. 10. Although there are exceptions, a comparison of the local test procedures on the basis of the B and C specimens indicates that in general those procedures in which the rates of freezing from 32 to 15 F. were fast caused failure more quickly than those in which the rates were slow. 11. Those local test procedures having fastest rates of freezing and producing quickest failures did not discriminate clearly between the concretes made of satisfactory and those made of poor coarse aggregate, whereas the procedures in which the rates were somewhat slower and the number of cycles to failure greater provided good discrimination. 12. None of the freezing and thawing procedures tried provided a small dispersion in the number of cycles required for failure and a sufficiently high degree of discrimination to qualify as a standard method. Of the procedures used, the coordinating program, the local Wisconsin Highway Commission, and the Missouri Highway Department are the best, but all exhibit too great dispersions of individual test values to be considered satisfactory. With still better control of the variables it is believed that these dispersions can be reduced and a standard procedure established. 13. The accumulated data emphasize the necessity for regulating carefully the methods of making and curing specimens, the air content of the specimens, the degree of saturation of the aggregate at the time of making, and the degree of saturation of the concrete at the time of freezing. 14. Curing of concrete in 70-deg. water subsequent to deteriorating freezing and thawing treatments produces a marked recovery in the dynamic modulus of elasticity but a much less pronounced recovery in flexural strength. 15. In these tests neither the absorption nor the rate of absorption data for the concrete correlated with the differences which the beams exhibited in resistance to freezing and thawing.

Thermosetting Synthetic Resin Paints for Concrete Pavement Markings, Floyd O. Slate Research Engineer, Joint Highway Research Project, Purdue University. The purpose of this study was to find the causes of failure of present concrete highway paints, and to find new or different paints better able to withstand these causes of failure.

The effect of passage of water vapor through a paint film on concrete was studied by means of evaporation tests. It was found that the passage of water upward upon evaporation caused the deposit of salt crystals at the concrete surface. The effect of this crystal growth on paint films was studied. Photographic records are reproduced in part. Paint lines put down in the field were observed carefully for type and cause of failure.

Synthetic resin paints, chosen as likely to be superior to standard highway paints, were subjected to water, alkali, and abrasion resistance tests. Field and laboratory comparisons were made with the standard paints. Thermosetting synthetic resin paints were baked directly on concrete pavement by means of infra-red lamps.

Influence of Various Curing Methods on some Physical Properties of Portland Cement Concrete, H. C. Vollmer, Research Associate, National Bureau of Standards. An investigation has been undertaken to compare the effectiveness of various curing materials and procedures. This investigation includes the use of burlap, the use of several liquid curing compounds and the use of calcium chloride both integrally and as a surface application. The study includes tests of concretes cast and cured at 70 F and at a relative humidity of 50-60 per cent, concretes cast and cured at 100 F. and at a relative humidity of 25-35 per cent, and concretes cast and cured under field conditions. The evaluation tests include flexural strengths of beams, compressive strengths of beam ends and resistance to abrasion of the cured surfaces. This report, presented as a progress report, presents only results of tests on the concretes cast and cured at 70 F.

The use of damp burlap for 18 hours and with calcium chloride used either integrally in the concrete or spread on the surface of the concrete upon removal of the burlap, resulted in 28 day flexural strengths of the same order as obtained with wet burlap applied for 3 days, the accepted standard for highway construction. The use of surface calcium chloride applied as soon as the bleeding water disappeared (no burlap at all) resulted in strengths only some 7 per cent lower than those obtained by the procedure requiring burlap. The 28 day flexural strengths obtained by the use of liquid curing compounds were 16 to 19 per cent lower than those obtained with the 3 day burlap curing procedure. Flexural strengths obtained by continuous damp curing of the specimens for 28 days, which however is not a practical method under field conditions, were higher than obtained by any of the other procedures described, and strengths obtained on specimens receiving no curing treatment were considerably lower.

Tests of the resistance to abrasion of the top surface (cured surface) of the specimens indicated that the use of 1½ per cent calcium chloride integrally and 1½ lb. per sq. yd. applied to the surface as soon as the bleeding water disappeared (no burlap) and all specimens cured by employing the surface application of calcium chloride whether in conjunction with burlap or not, resulted in a higher wear resistance than specimens cured with wet burlap applied for three days; however the wear resistance of specimens with liquid curing membranes or with no curing were somewhat less than the specimens cured with wet burlap applied for three days.

Maintenance Methods for Preventing and Correcting the Pumping Action of Concrete Slabs, Rex M. Whitton, Engineer of Maintenance, Missouri State Highway Department. This paper describes experience in Missouri with the correction of pumping at joints in concrete pavements by the use of a semi-fluid soil-cement mixture forced under the slab with a mudjack. The slurry used consisted of four sacks of cement per cubic yard of topsoil and 50 to 55 per cent of water. The spacing of the holes for elimination of pumping and correction of faulting is discussed. The equipment and personnel required is given in detail. On an average section of pavement, it was found that 354 holes per mile and 35.45 cu. yd. of slurry were required per mile of road. The average cost of the work was $24.78 for drilling 1½-inch holes and $256.66 per mile for the material and pumping operation.

In a study of the deflections of the pavement under moving loads, it was noted that deflections increased immediately after filling the voids under the pavement with the soil-cement slurry and then decreased after a period had elapsed. For a 12,000-lb. rear axle load, the deflections were reduced as much as 0.007 in. and for a 16,000-lb. wheel load as much as .011 in. between measurements made 9 days and 153 days after mudjacking.

It was found that a few of the mudjacked slabs resumed pumping and that the work had to be supplemented with joint and crack water-proofing to keep surface water from reaching the subgrade. On pavements that were cracked extensively, the best method of waterproofing joints was by the use of a substantial bituminous surface or upper deck not less than 1 inch in thickness.

After a study of design features from the viewpoint of a maintenance engineer, the author concludes that expansion or contraction joints should not be used in concrete pavements except at highway intersections, bridge ends or other locations where the pavement abutts a fixed object. The relation of the type of aggregate to the crack interval in concrete pavements in which no joints were placed is discussed.

Pumping of Concrete Pavements in New Jersey, Corrective Measures Employed, and Future Designs, William Van Breemen, Engineer of Special Assignments, New Jersey State Highway Department. Pumping at joints in concrete pavement slabs was first observed in New Jersey in 1930. It occurred on all pavement of standard design in which dowels ¾-in. in diameter were used for load transfer. A 100 per cent increase in the number of dowels did not eliminate the trouble. The use of crushed stone drains along the edge of the pavement was only partially effective in stopping pumping.

In 1932 a test road was built over a silty-clay soil. One joint with no load transfer device, two with six ¾-in. round dowels in the 10-foot width of pavement, two with twelve ¾-in. dowels, and several with various combinations of heavy rectangular and channel type dowels were placed in the slab. Continuous applications of heavy loads under adverse moisture conditions indicated that the use of a load transfer device composed of 2-in. channel-dowels was necessary to prevent faulting and subsequent pumping.

A recent survey of 60,000 channel-dowel joints on heavy duty highways disclosed only three failures that were caused by pumping. No faulting was found at these joints and the failures had occurred by sagging of the pavement. The stone drains along the edge of the pavement were partially clogged with subgrade soil.

A study of pavements laid on sub-bases composed of granular materials lead to the conclusion that their use minimized pumping, reduced damage due to frost action and increased load bearing capacity. All pavements built since 1939 are supported on a layer of bank-run sand, gravel or cinders 8 in. thick. To date, where granular materials have been used in conjunction with channel-dowel joints, pavements have remained true to grade, cracks are few and far between, and there have been no indications of pumping, even under heavy truck traffic.

A study of wood for use in expansion joints shows that for most varieties, loads in excess of 500 and less than 1000 psi will be required to cause compression of the fibers. If loading is continued, a point is reached where no further compression is obtained. Some varieties may be compressed to as much as 50 per cent of the original thickness. If dry wood is compressed to 50 per cent of its thickness, it will recover to about 65 per cent and remain at that thickness as long as it remains dry. Soaking in water will cause the wood to swell to 94 per cent, and for some varieties more than 100 per cent, of its original thickness. Repeated compression, drying and soaking will result in a permanent reduction in thickness. These tests indicate that wood as an expansion joint filler will have the following merits:

1. Unlike the conventional bituminous fillers, wood will not extrude, regardless of the extent of joint closure or infiltration. (This applies only to wood with the grain direction installed vertically).

2. Unlike other fillers, the wood is expected to retain sufficient swelling capacity and resiliency to prevent the detrimental accumulation and distribution of infiltrated material in the joint spaces which, in many locations, has caused rupturing of the concrete.

Investigation of Concrete Pavement Pumping, H. L. Krauser, Ohio Department of Highways. This paper describes the investigation of the pumping at transverse joints in concrete pavement slabs on a project 4.39 miles in length on U. S. 52 in Scioto County, O., near the village of Franklin Furnace.

The soils survey made prior to grading operations showed that on some sections the predominating soil was high in silt content and could be classified in the A-4 group. It was considered necessary to cover these areas with suitable granular material to a depth of 18 in. and provide tile drainage.

The pavement was built without reinforcing or load transfer devices. Records are also included on a short project built in 1941 in which complete reinforcing and load transfer devices were used. Observations were made of slab deflections under moving loads on both projects.

Pumping was found to be much more extensive and severe over the areas where no granular material was used. On the plain concrete section pumping was more severe at contraction joints of the premolded type. The severity on the reinforced section was about the same for expansion and contraction joints. On the plain concrete slabs without granular sub-base, 2 per cent of the expansion joints and 52 per cent of the contraction joints were pumping. On the plain concrete slabs placed on granular sub-base 1.5 per cent of the expansion and 16 per cent of the contraction joints were pumping. On the reinforced section, 71 per cent of the expansion joints and 91 per cent of the contraction joints were pumping.

On most of the work, the gradation of the backfill for the tile drains was from the ¾-in. to No. 4 sizes. It was observed that these drains silted up badly. On one section ⅜ in. to No. 8 material was used and only a small amount of silting was observed.

The conclusions of observations on these projects are as follows: 1. A sub-base composed of suitable granular material will appreciably reduce the pumping at joints in concrete pavements. 2. The use of small size backfill aggregate will extend the useful life of tile drains. 3. The use of load transfer devices prevents excessive permanent deformation at the joints between concrete slabs after pumping starts.

Use of Bituminous Materials as a Corrective Measure for Pumping Concrete Pavements, C. W. Allen and Harry E. Marshall, Ohio Department of Highways. The pumping of concrete pavement slabs on pavements in Ohio, which has developed since 1940, has followed the increase in volume of heavy truck traffic that has resulted from the concentration of war industries. A survey indicates that pumping has occurred mostly on soils of the A-4, A-6, or A-7 groups but is not confined exclusively to soils of these types. A study of the moisture contents of the subgrades at various depths indicates a maximum immediately beneath the pavements and a decrease with depth indicating that surface water is the chief source of the subgrade moisture that causes pumping. The use of transfer devices at joints and of granular subgrades have been found most effective in the prevention of pumping. The use of French drains in the shoulder was not effective in stopping pumping.

After experimenting with various soil-bituminous, portland cement mixtures and several grades of semisolid asphalts, it was found that an oil asphalt filler having a penetration of 30 to 45 at 77 F. was most satisfactory for filling the voids under pumping concrete slabs. This material is forced under the pavement by means of the hand spray equipment of a standard bituminous distributor through holes drilled in the pavement with a standard jackhammer and drill. The bituminous material forms a tight seal beneath the pavement and prevents the entrance of surface water and its stability is not affected by moisture from the subgrade. Although the costs of the asphalt is somewhat higher than any of the various soil-mixtures, a portion of this cost differential is equalized in the labor saved on the assembling and mixing of the various materials used in slurries.

Correcting Pavement Pumping by Mudjacking, R. E. Frost, Research Engineer, Joint Highway Research Project, Purdue University. This paper covers some field experiments designed to correct the pumping action of rigid pavement slabs. In 1942 a performance survey on a portion of U. S. 30 between U. S. 41 and Valparaiso, Indiana was made by representatives of the Joint Highway Research Project covering detailed analysis of pumping conditions on twenty-four miles of this road. Among other things the results of the survey showed that all of the experimental subgrade treatments (with the exception of the water-saturated section) were successful in minimizing or eliminating pumping.

As a result of this survey, a series of joints in a two-mile section near the Lake-Porter County line were selected for treatment by mudjacking. Treatment of these joints was performed in October and November of 1942. Four mixes were used:

1. Mix A; 77 per cent soil, 7 per cent RC-3, 16 per cent cement
2. Mix B; 77 per cent soil, 7 per cent Road Oil, 16 per cent cement
3. Mix C; 77 per cent soil, 7 per cent Tar, 16 per cent cement
4. Mix D; 79 per cent soil, 3 per cent Tar, 17 per cent cement

Even though work was hampered by cold weather and numerous equipment breakdowns, a total of 434 cubic feet of mix were pumped under fifty-pumping joints in twelve-working days. This is an average of 8.7 cubic feet of mix per joint.

Several performance surveys of this two-mile section have been made since treatment to determine the permanence of the treatments. The most recent survey (Oct.-Nov. 1944) shows that pumping had been reduced considerably. However, the installation of subgrade drains on U. S. 30 between S. R. 49 and S. R. 53 together with a particularly dry year (1944) made it difficult to rate the success on the basis of pumping alone. The settlement at the joints of both treated and untreated slabs shows considerable success for mudjack treatment. It was found that the average settlement of the outer edges of treated slabs was 0.093 inches as compared to 0.194 inches for the untreated slabs. Further, it was found that 68 per cent of the treated slabs had settled one-eighth inch or less as compared to 53 per cent for untreated slabs.

A crack survey showed that mudjacking had been successful in reducing the expected number of cracks that would normally occur on this two-mile section. The data further show that cracks less than 13 feet either side of a joint are caused by slab movement and pumping and that cracks in the middle third of a slab are from causes other than pumping. Of the four mixes used, Mixes "A" and "D" contained less cracks and had less than 0.04 inch settlement.

The results of this two-year study show the desirability for additional research in mudjacking.

ACI NEWS LETTER

Vol. 16 No. 5 JOURNAL of the AMERICAN CONCRETE INSTITUTE April 1945

The Institute's Forty-first Annual Convention

The forty-first Annual ACI Convention was the fourth of World War II. It was not the kind of annual convention to which the Institute looks forward in the peace years to come, nor the kind of convention which has made the annual meetings notable in former peace years.

The luncheon meeting at the Hotel New Yorker February 16, 1945 was much the same kind of meeting which characterized the 1943 convention in Chicago. This year seventy-two places were set (just one more than in Chicago in 1943); approximately two-thirds of those present were New York members. Those members in attendance who *travelled* to the meeting were almost exclusively confined to the more distant members of the Board of Direction and of the Advisory and Publications Committees.

There would undoubtedly have been a much larger attendance of local members but for unavoidable confusion about the restraints placed upon such gatherings under O. D. T. rulings. At first ACI had asked all New York members to attend if possible; then, facing a ruling that the *total* attendance at the New York meeting must be restricted to fifty (since the Institute had made no application for special dispensation) it was necessary to write to our New York members limiting the attendance of New Yorkers. Only in the last few days before the convention was it possible to advise local members in the region of the meeting that there was no restriction on their attendance, provided the attendance from a distance—the travelling membership—did not exceed fifty; this was the final O. D. T. ruling.

One pleasant ceremony was the award of Wason Medals by President Roy W. Crum, and unusual was the circumstance that in two instances the medals were received by ladies on behalf of their husbands.

Harrison F. Gonnerman, as previously announced in these pages, won the award for "Notable Research" reported in *Proceedings* V. 40 in his paper, "Tests of Concrete Containing Air Entraining Portland Cements—or Air Entraining Materials added to Batch at Mixer".

There being three authors of the paper chosen as the "Most Meritorious Paper", "Concrete Problems in the Construction of Graving Docks by the Tremie Method", there were, of course, awards in triplicate: To Commodore W. Mack Angas (CEC) U. S. N., Commander E. M. Shanley (CEC) U. S. N. R., and Lieutenant John A. Erickson (CEC) U. S. N. None of these medal winners could be present—all on distant Navy duty. The certificates of award were received by Mrs. Angas and by Mrs. Erickson for their husbands. Commander Shanley's award was received in his behalf by W. A. Durkin, Vice President, Walsh Construction Co., New York.

War conditions entered further into this ceremony in the fact that there were no medals for the winners of the awards for the "Most Meritorious Paper"—no bronze available from which to strike them off. So until bronze again becomes available for such purposes, winners of Wason medals will be receiving only the Certificate and their medals be deferred for postwar times.

Douglas E. Parsons became the new president of the Institute at the close of the New York meeting. He had served the Institute long and well, on both technical and administrative committees and as a member of the Board of Direction, and at many times arduously, and at some considerable personal sacrifice, there is reason to believe, on the Publications committee since 1940 and as chairman since 1941.

The report of the Tellers, R. R. Zipprodt and A. Burton Cohen, was presented by Mr. Zipprodt. All of the Nominating Committees' candidates for officers and directors were elected as follows:

President, Douglas E. Parsons, Chief of Masonry Construction Section, National Bureau of Standards, Washington, D. C.

Vice-President, Harrison F. Gonnerman, Manager, Research Laboratory, Portland Cement Association, Chicago, Ill.

Vice-President, Stanton Walker, Director of Engineering, National Sand and Gravel Association, Washington, D. C.

Director, First District (re-elected), Henry L. Kennedy, Manager, Cement Division, Dewey & Almy Chemical Co., Cambridge, Mass.

Director, Second District (re-elected), Myron A. Swayze, Director of Research, Lone Star Cement Corp., Hudson, New York.

Director, Third District, Alexander Foster, Jr., Vice-President, Warner Co., Philadelphia, Pa.

Director, Fourth District, Frank H. Jackson, Principal Engineer of Tests, Division of Physical Research, Public Roads Administration, Washington, D. C.

Director, Fifth District, Charles S. Whitney, Consulting Engineer, Milwaukee, Wis.

Director, Sixth District, Herbert J. Gilkey, Head, Dept. of Theoretical and Applied Mechanics, Iowa State College, Ames, Iowa.

Director-at-Large, Robert F. Blanks, Chief, Engineering & Geological Control & Research, Bureau of Reclamation, Denver, Colo.

The five to serve as the elective members of the 1945 Nominating committee, with the three presidents last past, are as follows: R. B. Young, chairman; T. C. Powers, J. C. Pearson, T. E. Stanton, Frank H. Jackson. (Professor Gilkey, among the five receiving the most votes had immediately declined to serve).

In acknowledging his election to the Presidency of the Institute, Mr. Parsons had these things to say:

"I appreciate that this election implies some small amount of confidence in me. It is not important that I do not share that confidence because the Institute is not run by the President. As you know, our capable, energetic and overworked Secretary, together with the Board members and members of the committees actually do most of the running of the Institute.

"Perhaps it is less well known that individual members who receive no recognition for the part they play in the operation of the Institute do a great deal to guide the officers and to keep the policies and practices within proper bounds.

"One of the things that the officers will miss most this year will be the suggestions and criticisms from members. Sometimes those criticisms are expressed in most blunt and forceful language and often they are addressed to an officer at a time when the man making the suggestion is unaware that he is talking to the one who may be responsible for the the blunder that is being condemned. That has happened to me on a number of occasions during conventions.

"I wish that there was some way that we could continue to have that candid advice and criticism. The only way that I know that we shall have the benefit of it would be for each of you and the other members of the Institute to let us hear the worst with the best and say it in writing. thank you!"

President Crum acknowledged the presence at the meeting of Lt. Peter J. Doanides of the Royal Helenic Navy, Greece, on a special mission in this country, and Prof. Boris G. Skramtaiev, Doctor of Technical Sciences; head of a technical institute in Russia, and in this country attached to the Government Purchasing Commission of the U. S. S. R. Professor Skramtaiev has been a member of the Institute for nine years,

Douglas E. Parsons, newly elected President of the American Concrete Institute

a contributor to the ACI Journal and has been in this country since May 1944 and expects shortly to be returning to Moscow. He had the following to say:

"It gives me great pleasure to be present here today, among leaders of the American Concrete Institute and its New York members. This is the first opportunity I have had to meet with American concrete men since 1936, the year when I joined the Institute.

"First, I would like to greet you cordially on behalf of all the Russian concrete specialists. We appreciate fully your achievements in the field of concrete, which you have developed so well. We very often use the results of your scientific work in our concrete practice in the Soviet Union, and the American names of Crum, Parsons, Abrams, McMillan, Gonnerman, Richart, and many others are famous among our Russian civil engineers.

"Meanwhile, we, in the Soviet Union, have also developed many different kinds of concrete, several new methods of winter work, and are making large quantities of precast reinforced concrete structures. We have also worked out a new theory for designing reinforced concrete, and now this theory, known as the Ultimate Theory, is the only official one in the Soviet Union.

"Next, I would like to take the liberty of suggesting that the new board of the Institute discuss the possibility of compiling a complete American bibliography pertaining to concrete. I am fully aware that this would be a difficult task, since you are already publishing very many books and articles, but I am sure you will agree that it is a very much needed piece of work.

"We have published a complete Russian bibliography on concrete, which I was glad to present to the American Concrete Institute. Upon my return to Moscow, I intend to continue this work in order to make this bibliography more complete by including all the Russian articles and books on concrete, cement and reinforced concrete constructions.

"It would seem to me highly desirable to establish a good-sized library at the American Concrete Institute for the use of all its members.

"May I close by wishing you the greatest success in all your work toward victory and also in the very extensive concrete jobs which you will be called upon to do after the war."

The address of President Crum in retiring from his office differed from previous presidential addresses. It had to do with the part of members of the Institute and others engaged in technical advancement with reference to our responsibilities in establishing a peaceful world in which such advancement may continue. The address appears on the first pages of this issue of the JOURNAL. Some who were present recall the address of Rear Admiral Ben Moreell (now Vice Admiral) on his retirement from the presidency of the Institute in 1942. The address was made from Washington by telephone and loud speaker hook-up at the convention, and sounded a note of responsibility of our membership in the war in which we became engaged such a short time before.

At a second meeting of the Board—the "new board"—following the luncheon meeting, several appointments were made. The election of Stanton Walker to a vice-presidency, one year before the completion of his three-year term as Director-at-Large left a vacancy to which Henry L. Kennedy, who had just been reelected Regional Director from the First District, was appointed. To fill the vacancy in the position of First District Director, the Board appointed Paul W. Norton, consulting engineer, Boston.

The resignation of Prof. Raymond E. Davis, after 6 years' service as chairman of the Advisory Committee, resulted in the election of Stanton Walker to fill that important place. Professor Davis remains on the committee in having been appointed to the chairmanship of Dept. 200.

On becoming President of the Institute, Mr. Parsons retired as chairman of the Publications Committee. Robert F. Blanks, Bureau of Reclamation, a member of the Board as Director-at-Large, was appointed to that very active committee.

The Board also appointed Harvey Whipple to succeed himself as Secretary-Treasurer.

WHO'S WHO in this ACI JOURNAL

Jacob Feld

ACI Member since 1944, presents his first paper (p. 441). Dr. Feld is a consulting civil engineer, New York City. He was born in Austria, attended the College of City of New York (B.S. 1918); attended Univ. Cincinnati (C.E. 1920, M.A. 1921, Ph. D. 1922, Mem. Phi Beta Kappa, Sigma Xi, Sigma Tau Phi).

In New York he was employed by the Public Service Comm., Queens Highway Dept. and Erie R.R. From 1919-1922 he was a Research Fellow, Institute C. E., Univ. of Cincinnati; in 1922 with Turner Construction Co., N.Y.; 1923, Long Island, R. R. as assistant bridge engineer; 1924, with Robinson & Steinman, bridge engineers; 1925, Barney-Ahlers Co., as concrete Engineer; 1925 to date, Consulting Engineer, bridges, buildings. His work has ranged through foundations, subways, radio towers, stadiums, dairy plants, an average of $5,000,000 work per year. He was consulting engineer for the contractors on the Tri-borough Bridge contracts, 205 St. Viaduct, grade crossing eliminations, bridge foundations, 1930-1935. He was in charge of construction of section 10 of the Sixth Ave. Subway for Brader Construction Corp. 1936-1939. He has done research in lateral earth pressure, foundations, tower analysis, long span cables, stadium designs, 1000 ft. guyed radio towers at Shanghai; is author of "Lateral Earth Pressure," A.S.C.E. Trans. 1923, "History of Earth Pressure Theories," Brooklyn Engineers Club *Proc.* 1929, "Unbraced Cables" Franklin Institute 1930 and numberous other papers. He is a member of A.S.C.E. (Collingwood Prize 1923), (F.), A.A.A.S., Member Brooklyn Engineers Club (A. T. White Prize 1928) Dir. 1927-1933, and of New York Academy Sciences.

D. R. Cervin

member of the Institute since 1943 presents his first Institute paper (p. 453). Mr. Cervin received his B.S. degree in architectural engineering from the University of Illinois in 1934; did graduate work in structural engineering at Illinois in 1936; and at the University of Iowa in 1937. He was with the United States District Engineer Office, Rock Island, Ill. 1934 to 1938 in connection with the design and construction of the locks and dams for the 9 ft. channel in the Mississippi River. In 1939 he was with A. H. Ebeling, Davenport, Iowa, architect, doing architectural engineering work on school buildings; in 1940 he was engaged in construction of the spill-way for the Santee Cooper project in South Carolina as office engineer for the McCarthy Improvement Co., and for the next year and a half he was with the special engineering division in the Canal Zone on the design of the third locks, a project which was terminated by the war. Another year and a half was spent with the United States Army District Engineer office in Jacksonville, Fla. as assistant head of the structural design section; his work included army camps, airports and appertenant works in the Florida area. Now, and for the past year he has been employed as a structural engineer for the National Advisory Committee for Aeronautics and his work includes the design and construction of the buildings required by the expanded research necessary for improving the field of aeronautics.

Gerald Pickett

who unfortunately we cannot claim as an ACI Member, still is not new to these pages. To the Feb. 1942 JOURNAL he contributed "The Effect of Change in Moisture Content on the Creep of Concrete under a Sustained Load". At that time it was noted in these columns that he was a graduate electrical engineer from Oklahoma Agricultural and Mechanical College in 1927; awarded a Ph.D. in mechanics by the University of Michigan in 1938; and aside from two years in the testing laboratory of the Brooklyn Navy Yard, he taught in the Department of Mechanics, Kansas State College until 1940—advanced courses in mechanics for graduate students in addition to regular courses to undergraduates. He joined the basic research division of the Portland Cement Association Research Laboratory, Chicago in 1940 as a research physicist, studying the mechanics of portland cement concrete.

Dr. Pickett has been interested in problems involving dynamics for several years and his paper on the "Dynamic Testing of Concrete" (p. 473) is a natural outgrowth of his earlier work in other fields of mechanics.

Jacob J. Creskoff

member of the Institute since 1937, is not new to these pages. To volume 33 he contributed "Earthquakes and Reinforced Concrete". At that time it was noted in this column that he was the author of "Dynamics of Earthquake Resistant Structures" (McGraw-Hill Book Co.), that he had worked for various Government agencies on aseismic design and that he had been consultant to the Treasury Department on the design of a new earthquake resistant San Francisco mint. He is a graduate from the civil engineering department Towne Scientific School, University of Pennsylvania. From 1937 to 1942 he was vice president of Vacuum Concrete Corp., Philadelphia; and from 1942 to date, resident vice president, McCloskey & Co., Washington D. C. and Tampa, Fla; actively identified with the McCloskey work on concrete ships. His present paper, as a contribution to the prediction of the strength of concrete appears p. 493.

Inge Lyse and Family escape from Norway

The many Institute friends of Prof. Inge Lyse will be pleased to know that he and his family who had been at Trondheim, Norway for several years, late in 1944 escaped with only what they could carry on their backs over the mountains into Sweden where they are safe with friends.

This news reached a number of Institute members at the time of the Institute's meetings in New York in February by way of a copy of a letter which Mrs. Lyse had written to friends in Bethlehem, Pa.

Returning from the New York meetings the Secretary of the Institute found a letter addressed to him January 9 from Stockholm in which Professor Lyse wrote "just a few words to let you know that I am here in Stockholm with my family and that we are all well. I had a narrow escape from the Gestapo, but the trip across the mountains solved the problem. I am at present working at the Swedish Cement and Concrete Research Institute, which has just moved into its new building at the Royal Technical Institute". He reports that his time with the Swedish Institute will be limited to that which he can spare from his engagement at the Norwegian Military Offices in Stockholm; "with best greeting to all my friends in the ACI".

More detailed and revealing of the times through which the Lyses had passed, since the German invasion of Norway, where Lyse was identified with Norway's Institute of Technology at Trondheim, are a few lines from Mrs. Lyse's letter which was written Dec. 26:

"The boys have been ill in bed since we came here, but are now perfectly all right again—their stomachs were unused to the rich food we get here, and besides meat, white bread and cheese and jam, they have been getting candy and chocolate, which was too much for their stomachs; but now we can eat everything without getting sick—and do we eat . . . Norway is absolutely emptied of all goods; and the joy to see lights again; you can't imagine what that means after having lived in a country constantly blacked out."

Professor Lyse, active member of ACI since 1926, three years after coming to the United States (see ACI Directory 1944, p. 70), in 1938 left Lehigh University, where he was in charge of Fritz Engineering Laboratory, to return to Norway's Institute of Technology, Trondheim, his alma mater, to be professor of reinforced concrete and solid bridges (see ACI Jl. N. L. p. 14, Sept. 1938).

Leo Nagel

Only recently did the Institute learn of the death of Leo Nagel, member of the Institute since 1939; resident of Detroit. He was a graduate of the Technical University of Vienna, Austria, 1915, and for 15 years before coming to America, he was managing director of one of the large engineering construction companies of Vienna, specializing in reinforced concrete in the Industrial field. In 1940, coming to the United States, he entered the organization of Albert Kahn, Inc., Detroit, where he was employed until the time of his death in August 1944.

The June 1945 JOURNAL will be notable in containing by direction of the Board all the ACI Standards promulgated since the inauguration of the present Standards Committee in 1937.

Alfred E. Lindau

The following is from the minutes of a meeting of the Institute's Board of Direction, New York, February 14 (See N.L. January, 1945):

"In the death of Alfred E. Lindau at Pearl Harbor, Hawaii, December 14, 1944, the American Concrete Institute lost one of its best known and consistently active workers. For three and a half decades he had been a leader in the thought and activities of the Institute, few of whose members had contributed so much over so many years from such a wide range of activity; few indeed, had been held in greater esteem by their fellow members for a high degree of intellectual integrity and a well stocked and finely discriminating mind.

"For five years (1939-1944) he had given strenuously and generously of his engineering knowledge to the war effort as Chief Civilian Engineering Assistant to Officer-in-Charge, Pacific Naval Air Bases.

"Alfred Lindau joined the Institute in 1909, was elected an Honorary Member in 1935. He had been a member of the Board of Direction, as a director in 1915 and 1916 and continuously a Board member from 1922 to 1935—as vice president, 1922 to 1923, president, 1924 and 1925, past president member 1926 to 1935. In returning to the ranks he continued a willing worker on many important tasks. He was unexcelled as an Institute convention presiding officer, bringing to the leadership of discussion not only his broad knowledge, but his dynamic spirit and a contageous interest.

"His lively interest in a wide range of human experience, his quick and deep understanding, his rare combination of objective judicial-mindedness with great warmth of personality and unfailing kindness won him the high respect and affection of a host of friends."

Honor Roll

February 1, 1945 to March 29, 1945

In the February issue we reported the 1945 Honor Roll only to January 29 and there were changes in the two days to January 31 which ended the scoring year.

Lewis H. Tuthill, top man, finished with a final score of 18 Members sponsored and F. E. Richart 13. H. F. Gonnerman increased his score from 2½ to 3½ and Jacob Fruchtbaum's name was added to the roll in his sponsoring one new member.

For several years the two top men each year have earned a worthwhile credit, as a Board of Direction award, for engineering literature purchased through the Institute.

By recent Board action the rules are changed and the two top men are to receive Certificates of Merit for important service to the Institute. The first entries, for the Certificate to be presented at the 1946 Convention (for the year ending January 31, 1945) are as follows:

A. Amirikian	3
J. H. Spilkin	3
C. Blaschitz	2
Charles S. Whitney	2
R. F. Dierking	1
Charles E. Wuerpel	1
Wm. G. McFarland	1

50-50 With Other Member:

Kanwar Sain
J. L. Savage

Subject to the possible exigencies of war, the Institute's 42nd annual convention is scheduled to be held in New York City, the week of February 18—a "full-blown" convention.

New Members

The Board of Direction approved 49 applications for Membership (43 Individual, 3 Junior, 3 Student) in January and February as follows:

Anderson, Boyd G., 4336 Livingston Rd., S.E., Washington D. C.

Arena, de la, Pedro Guash, Calle O esq. 25, Edit. Castro, Vedado, Havana, Cuba

Arnal M., Dr. Eduardo A., P. O. Box 263, Caracas, Venezuela, S. A.

Atkins, Clinton P., 115 Victory Lane, Leetsdale, Pa. (Student)

Atkinson, James L., P. O. Box 482, National City, Calif.

Aydin, Abdurrahman, U. P. O. Box 572, Princeton, N. J.

Barker, Austin, c/o U. S. Bureau of Reclamation, Mancos, Colo.

Bauman, E. W., National Crushed Stone Assoc., 1735 14th St. N. W., Washington, D. C.

Chatterley, Jay, 795 N. 8 E., Logan, Utah (Student)

Croll, Donald, 74 Hedley Place, Buffalo 8, N. Y.

Domínguez C., Dr. Atahualpa, Garita a Pepe Aleman No. 12-4, Caracas, Venezuela, S. A.

Evans, I. G., Building Research Station, Garston, Watford, Herts, England

Ewart, Raymond R., 2660 Quaker Bridge Rd., Box 45, Mercerville, N. J.

Finney, E. A., Michigan State College, Rm. 3, Olds Hall, East Lansing, Mich.

Furnival, George E., c/o Furnival-Rimmer Co., 54th and Lancaster Ave., Philadelphia 31, Pa.

Hom, Harry Goff, 4051 Front St., San Diego 3, Calif. (Junior)

de Lys-Gregson, H. St. J. R., c/o Messrs. Grindlay & Co. Ltd., 6 Church Lane, Calcutta, India

Handa, C. L., c/o The India Supply Mission, 635 F. Street, N. W., Washington, D. C.

Hansen, Waldemar C., c/o Universal Atlas Cement Co., Gary, Buffington Station, Ind.

Haws, Frank W., 37 E. 3rd So., Logan, Utah (Student)

Hepplewhite, Lt. W. T., c/o Director of Fortifications & Coastal Works, P. O. Box 2333, Cape Town, So. Africa

Holdampf, Carl R., 5844 N. Shoreland Ave., Milwaukee 11, Wisc.

Holm, J. C., c/o National Portland Cement Co., Brodhead Plant, R.F.D. No. 1, Bethlehem, Pa.

Howland, L. D., 1863 Meadowbrook Rd., Altadena, Calif.

Kaplan, Sgt. M. F., c/o Director of Fortifications & Coastal Works, P. O. Box 2333, Cape Town, So. Africa

Kleinman, Sgt. S., c/o Director of Fortifications & Coastal Works, P. O. Box 2333, Cape Town, So. Africa

Luke, Irvin H., Apartado 4068, San Juan, P. R.

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Witter, Harry C., 3059 Seaview Rise, Honolulu, T. H.

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Master Vibrator Co., Dayton 1, Ohio 430-1
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BAD TIMES

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