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Czesław M. RODKIEWICZ

University of Alberta, Edmonton, Canada

ON THE CHARACTERISTICS OF THE THRUST BEARING WITH ACCELERATING SLIDER

Abstract. Thrust bearing characteristics are examined in the motion generated by the accelerating slider. The selected case is the situation where the fluid within the lubricating oil film initially is at rest and at time zero the infinitely-wide slider assumes a velocity which is a function of time. Numerical solution to the governing differential equations is obtained for the constant and time dependent acceleration. The corresponding previously published analytical solutions are compared with those numerical results.

Introduction

In this paper it is of interest to examine the thrust bearing characteristics in the motion generated by the accelerating slider. The case at hand is the situation where the lubricating film initially is at rest and at time zero the infinitely-wide slider assumes a velocity which is some function of time. Subsequently, due to the viscous effects penetrating through the oil film the velocity and pressure fields are generated yielding the lifting force. The time history of these quantities if obtained numerically and compared with the corresponding analytical results of Ladanyi [1], and Lyman and Saibel [2].

Ladanyi assumed in his analysis that the acceleration at any point within the film, due to the change of speed of the moving surface, is linearly proportional to the distance from the stationary surface. This assumption reduced the equation of motion to a form which could be integrated. Lyman and Saibel developed asymptotic solutions for small and large values of time. Those describe the transient pressure and yield, for the case of constant acceleration of the moving surface, expressions for pressure and load capacity in closed form.

Basic equations

The governing equations for the two-dimensional thrust bearing, assuming incompressible fluid of constant properties, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \varrho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\delta p}{\delta x} + \mu\frac{\partial^2 u}{\partial y^2} \tag{1,2}$$

where: the differentiation symbol δ indicates that the gauge pressure $p = p(x,t)$; t is time; x and y are (see Fig. 1) coordinates parallel and perpendicular to the slider, respectively; u and v are velocities in x and y direction, respectively; ϱ is density and μ is dynamic viscosity.

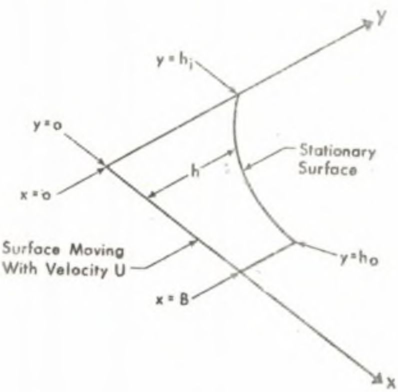

Fig. 1. Coordinate system

On the grounds of references [3, 4, 5] it is now assumed that the cases under consideration are such that the second and third term of Eq. (2) can be neglected. For such a situation Eq. (2) may be written in a simplified dimensionless form, namely:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\delta \bar{p}}{\delta \bar{x}} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \tag{3}$$

where: $\bar{u} = u/V$ (V = reference velocity); $\bar{t} = \nu t/h_0^2$ (ν = kinematic viscosity); $\bar{p} = ph_0^2/(B\mu V)$; $\bar{x} = x/B$, $0 \leqslant x \leqslant B$; $\bar{y} = y/h_0$, $0 \leqslant y \leqslant h$. The associated boundary conditions become

$$\bar{u}(\bar{x},0,\bar{t}) = \bar{U}, \ \bar{u}(\bar{x},\bar{h},\bar{t}) = \bar{u}(\bar{x},\bar{y},0) = \bar{p}(0,\bar{t}) = \bar{p}(1,\bar{t}) = 0 \tag{4}$$

with $\bar{h} = h/h_0$, and with $\bar{U} = \bar{a}\bar{t}$, where the dimensionless acceleration $\bar{a}$ $(= ah_0^2/(\nu V))$ is a constant or function of time. This implies that the velocity of the slider at zero time is zero. The case where the fluid within the lubricating film is initially also at rest, however, where at time zero the infinitely-wide slider suddenly assumes a constant velocity was treated in Ref. [6]. In addition the transient response of an infinitely-wide slider bearing subjected to tangential acceleration of the thrust ring has been investigated numerically by Kettleborough [7].

In the boundary conditions (4) we require the quantity $\bar{h}(\bar{x})$ which describes the shape of the stationary part of the thrust bearing with reference to the moving plane slider. The following two shapes are assumed

$$\bar{h} = \bar{x} + (1 - \bar{x})H, \quad \bar{h} = \exp\left[(1-\bar{x})\ln H\right] \tag{5,6}$$

where $H = h_i/h_0$. Equation (5) represents linear configuration used by Ladanyi and Eq. (6) gives the exponential shape treated by Lyman and Saibel.

The solution will result in the time dependent distribution of velocity, pressure and in the dimensionless load capacity which is given by

$$\bar{W} = \int_0^1 \left(\int_0^{\bar{x}} \frac{\delta\bar{p}}{\delta\bar{x}}\, d\bar{x}\right) d\bar{x} \tag{7}$$

The condition that space inertia terms are negligible have been used by Ladanyi [1] for the case where the infinitely-wide slider is accelerating at a uniform rate. Furthermore, in solving eq. (3) he assumed that the variation of acceleration is linear across the film:

$$\frac{\partial\bar{u}}{\partial\bar{t}} = \frac{\bar{h} - \bar{y}}{\bar{h}} \frac{d\bar{U}}{d\bar{t}} \tag{8}$$

The same problem, but without making the above assumption, was treated by Lyman and Saibel. However, an interesting result is found for their bearing, as given by (6), when the load capacity is obtained with the assumption (8). It is given by

$$\bar{W} = \frac{(H^2-1)}{2(H\ln H)^2}\left[1 - \frac{6H^2\ln H}{(H+1)(H^3-1)}\right]\bar{U}$$

$$+ \left[\frac{(2-\ln H)}{4\ln H} - \frac{1}{2(H^3-1)}\right]\frac{d\bar{U}}{d\bar{t}} \tag{9}$$

where $\bar{W} = Wh_0^2/(B^2\mu V)$.

Numerical method

Numerical computations [8] with the selected time and space steps ($\Delta\bar{t}$, $\Delta\bar{x}$ and $\Delta\bar{y}$ respectively) have been made beginning with the "first step" assumption of a quantity for $\bar{c}$, $\bar{c} = \delta\bar{p}/\delta\bar{x}$, at $\bar{x} = 1$. With this $\bar{c}$ fluid velocities were computed in the range $0 < \bar{y} < \bar{h}$. Then the quantity $\bar{Q}$ (dimensionless rate of flow) associated with equation (1) was computed:

$$\bar{Q} = \int_0^{\bar{h}} \bar{u}\, d\bar{y} \tag{10}$$

This quantity, if correct, should be a function of $\bar{t}$ only. For each interval of time, $\bar{t} = n\Delta\bar{t}$ $(n = 1,2,\ldots)$, marching was done from $\bar{x} = 1$ to $\bar{x} = 0$ in steps of $\Delta\bar{x}$. At each step of $\bar{x}$, such a new value of $\bar{c}$ would be taken which would preserve invariance of $\bar{Q}$.

After each sweep of the range $0 < \bar{x} < 1$ the integral of $\bar{c}$ from $\bar{x} = 0$ to $\bar{x} = 1$ was computed. In view of the conditions (4) it should be equal to zero. If not, the procedure would go to the "first step" and assume a new value of $\bar{c}$ at $\bar{x} = 1$. This iteration was continued until some desired tolerance was reached. At this time $\bar{W}$ was computed. Then $\bar{u}$, $\bar{p}$ and $\bar{W}$ were stored and the "first step" commenced again at the next time step.

Results and discussion

The results are presented in Fig. 2 through 6. The dimensionless acceleration $\bar{a}$ was assumed constant or equal to $\bar{t}$. In the case of constant acceleration the results, due to linearity of the posed problem, can be expressed by one set of curves by letting $V = ah_0^2/\nu$. The corresponding graphs are Fig. 2, 3 and 5.

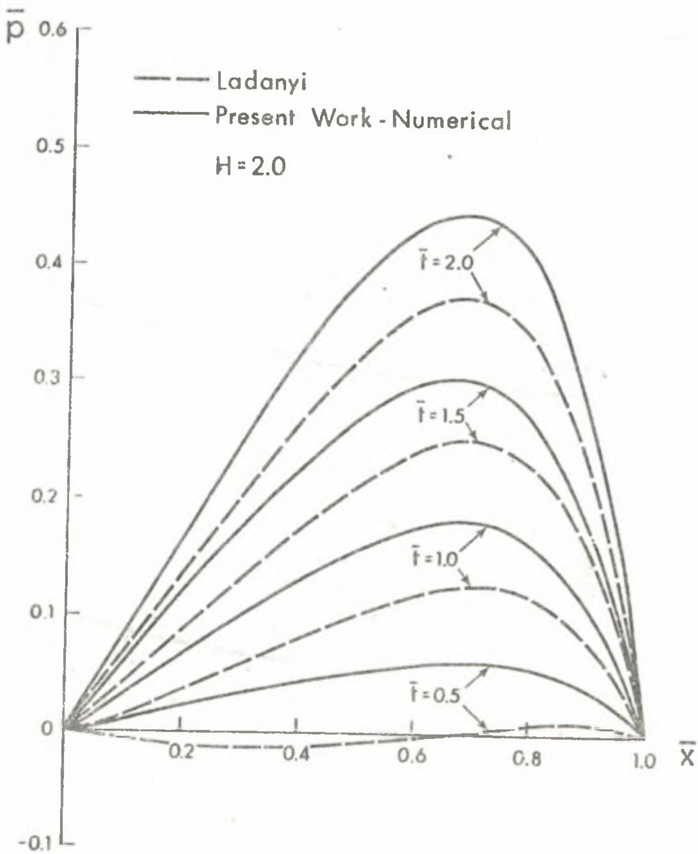

Fig. 2. Linear bearing pressure distribution at various times for $\bar{a} = \text{const}$

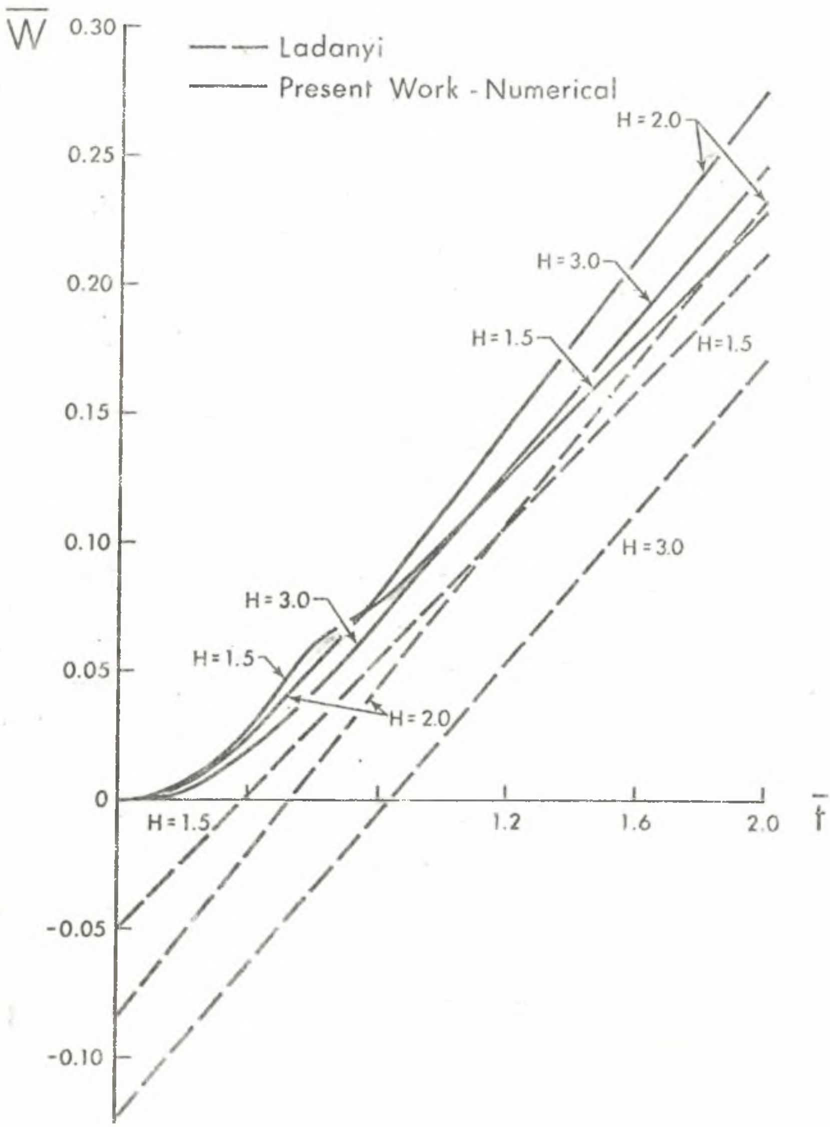

Fig. 3. Linear bearing load capacity vs time for $\bar{a}$ = const

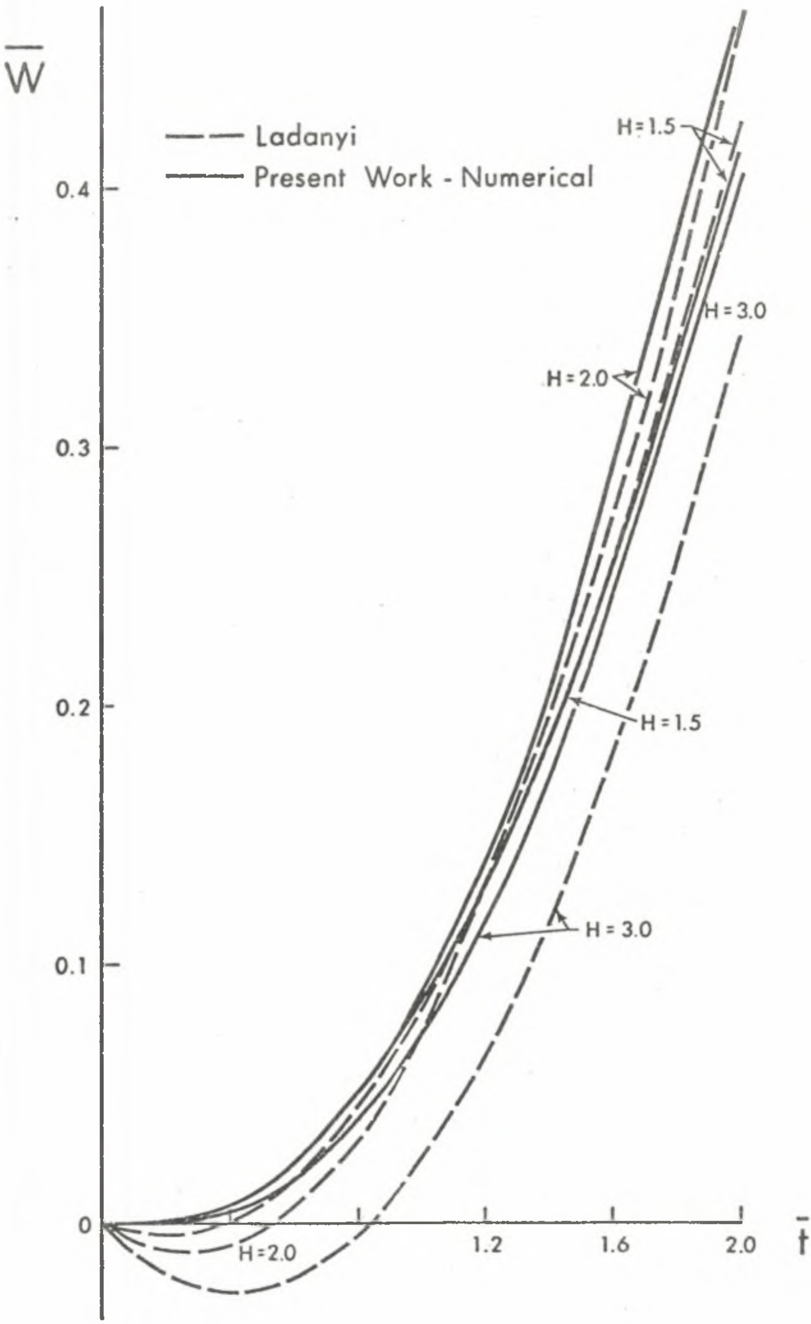

Fig. 4. Linear bearing load capacity vs time for $\bar{a} = \bar{t}$

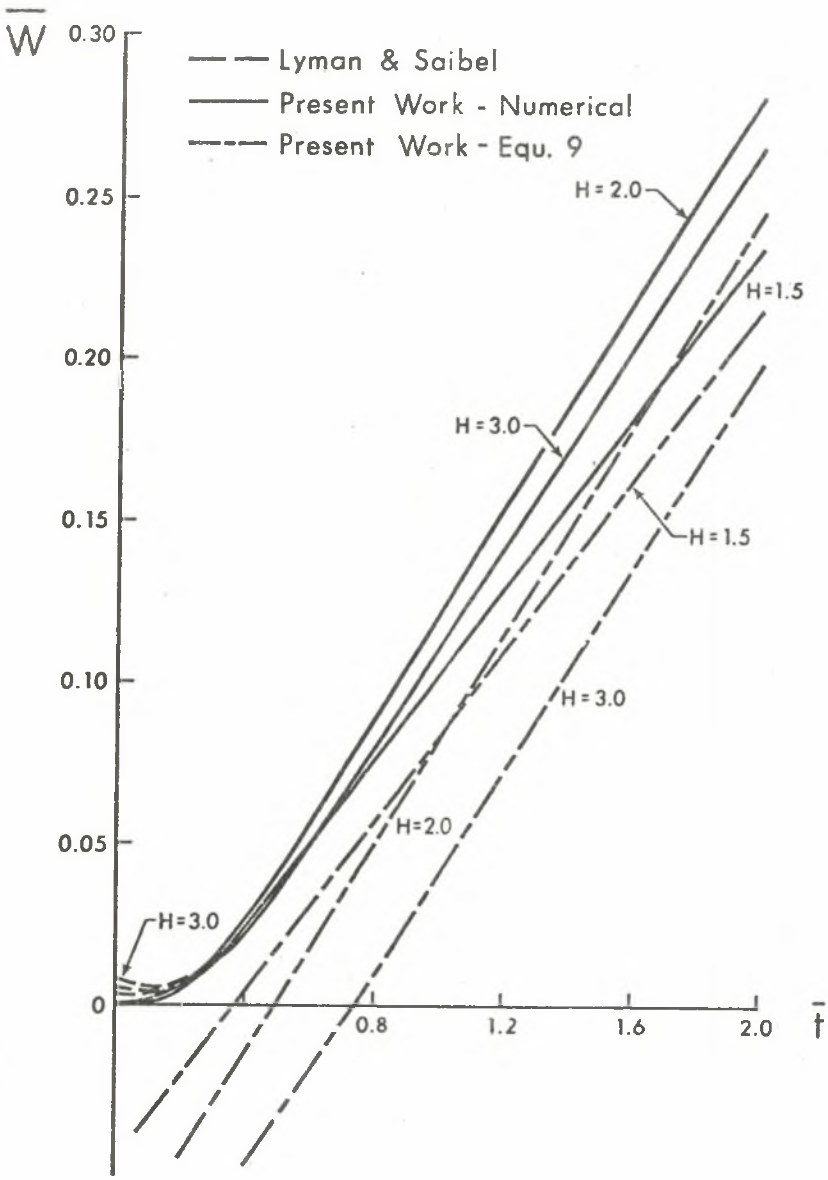

Fig. 5. Exponential bearing load capacity vs time for $\bar{a}$ = const

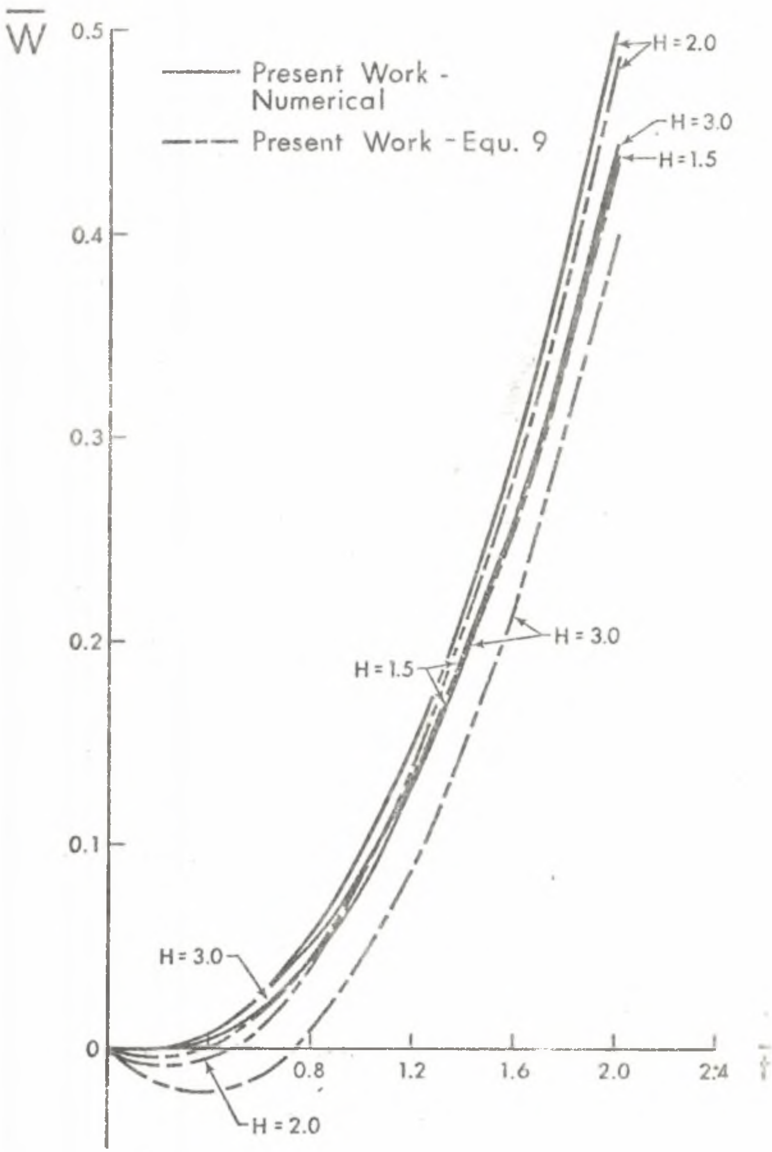

Fig. 6. Exponential bearing load capacity vs time for $\bar{a} = \bar{t}$

The quoted Lyman and Saibel closed form solution applies to the case of constant acceleration only. Their solutions admitting other types of accelerations require, due to the complexity of mathematics, numerical procedures. The question which arises is if it is not simpler to solve numerically the basic differential equations without having to go through the Laplace transforms as proposed by Lyman and Saibel.

Typical dependence of pressure on time, for the case of linear bearing, is shown in Fig. 2. At small times Ladanyi's solution indicates negative pressures which yield negative load capacities as demonstrated in Fig. 3 and Fig. 4. The negative pressures, negative load capacities, and decreasing $\bar{W}$ with $\bar{t}$ (when $\bar{a} = \bar{t}$), for very small $\bar{t}$, are the result of application of Ladanyi's assumption that the acceleration at any point within the oil film, due to the change in velocity of moving slider, is assumed to be linearly proportional to distance from the slider, as given by Eq. (8). This assumption is responsible for loss of the initial-value character which is reflected in negative $\bar{W}$ at small $\bar{t}$. The advantage of Ladanyi's assumption is in that there can be obtained rather simple closed form solution which, though in error at very small $\bar{t}$ compares favorably with the numerical results at larger times.

The exponential bearing results are presented in Fig. 5 and Fig. 6. These include the numerical solutions, the Lyman and Saibel curves and the graphical presentation of Eq. (9).

The Lyman and Saibel constant acceleration curves, shown in Fig. 5, very closely coincide with the numerically computed load capacity, except for the very small times. On the other hand the Ladanyi's method as applied to the exponential bearing indicates, again here, the negative load capacities at the small times. However, with the increase in time, as before, the error decreases.

For the case of the non-constant acceleration Lyman and Saibel did not produce closed form solutions which requires going through the Laplace transforms. At some stage the analytical method, in these cases, would have to be supplemented by some numerical procedure. However, the application of the Ladanyi's method provides a closed form solution which shows, in general, good agreement with the corresponding numerical results. This can be seen in Fig. 6. One may note that at small times the discrepancies are still large but decreasing with the increase of time. For example at $\bar{t} = 2,0$ the error is approximately 3% for $H = 2,0$ and $\bar{a} = \bar{t}$.

It is interesting to note that at very small times generation of the load capacity is such that, for a given time interval, it is increasing with the decreasing H (see for example Fig. 3). However, subsequently more like a steady state situation is developing, where the maximum load capacity is at approximately $H = 2,2$. In Fig. 3 this transition takes place at approximately $\bar{t} = 0,8$ (numerical results), i.e. the curve of $H = 1,5$ goes under the curve of $H = 2,0$. By investigating the development of the velocity profiles it is believed that this phenomenon is associated with the initial state of viscous effects penetrating into the oil film and eventually gaining full participation when reaching all of the stationary surface of the bearing.

Finally it is also interesting to note that at any given time the associated load capacity, prevailing at that time, is very small compared with the corresponding steady state lifting force. This indicates that the generation of the load capacity in considered motion is trailing behind its steady state counterpart by a large margin. For example, in the case of constant acceleration with $\bar{a} = 1$ and at the speed of U = 100 m/s, the load capacity can be found to be (using Fig. 5 and some typical oil in a small thrust bearing) equal to approximately 41 N/m. The time in which this was reached was approximately $2{,}0 \times 10^{-8}$s and the corresponding steady state load capacity was equal to approximately 4584 N/m. This was discussed in greater detail by Ladanyi [1] in 1948. He correctly noted that in some cases "the decrease in load capacity due to the temporal tangential acceleration is almost of the same magnitude as the steady-state load capacity. In such instances, the lubricating film may brake down and permit "metal-to-metal contact". He quoted the case of a reciprocating-engine piston ring, but concluded that this condition also exists for the general case of reciprocating bearings.

Conclusion

For the very small times neither Ladanyi's nor Lyman and Saibel's solutions reflect the physical expectation. At larger times, and for the case of constant acceleration, the Lyman and Saibel solution well coincides with the numerical results while the Ladanyi's approach reflects errors. These errors decrease with time. When the acceleration is not constant the Lyman and Saibel procedure becomes involved. However, the Ladanyi's method provides simple closed form solution. This solution when tested against the case of acceleration proportional to time, showed good agreement with the numerical results. Nevertheless, the negative load capacities at very small times can not be avoided.

In the cases where some errors can be tolerated the following, simple in use, load capacity formula, based on the Ladannyi's assumption, may be used

$$\bar{W} = 6\bar{U}\left[\int_0^1\left(\int_0^{\bar{x}}\frac{d\bar{x}}{\bar{h}^2}\right)d\bar{x} - \frac{\int_0^1 \frac{d\bar{x}}{\bar{h}^2}}{\int_0^1 \frac{d\bar{x}}{\bar{h}^3}}\int_0^1\left(\int_0^{\bar{x}}\frac{d\bar{x}}{\bar{h}^3}\right)d\bar{x}\right] + \frac{1}{2}\frac{d\bar{U}}{d\bar{t}}\left[\frac{\int_0^1\left(\int_0^{\bar{x}}\frac{d\bar{x}}{\bar{h}^3}\right)d\bar{x}}{\int_0^1 \frac{d\bar{x}}{\bar{h}^3}} - \frac{1}{2}\right]$$

where $\bar{U} = \bar{U}(\bar{t})$ and $\bar{h} = \bar{h}(\bar{x})$. However, it should be remembered that a reasonable answer will be obtained only at sufficiently high times. These times may be inferred from graphs of Fig. 3 through 6.

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О СВОЙСТВАХ УПОРНЫХ ПОДШИПНИКОВ С УСКОРИТЕЛЬНЫМ НАПРАВИТЕЛЕМ

Р е з ю м е

Свойства упорных подшипников исследовались в движении, вызванном ускорительным направителем. Был выбран случай, когда жидкость внутри маслофильма находится сперва в неподвижном состоянии и в нулевом положении бесконечно широкий направитель получает скорость, которая является функцией времени.

Численное решение системы дифференциальных уравнений, описывающих вышеуказанный процесс, было разработано для случая постоянного и переменного во времени ускорения.

O WŁAŚCIWOŚCIACH ŁOŻYSK OPOROWYCH Z PROWADNIKIEM PRZYSPIESZAJĄCYM

S t r e s z c z e n i e

Właściwości łozysk oporowych były badane w ruchu wywołanym przez prowadnik przyspieszający. Wybrany został przypadek, gdy płyn wewnątrz filmu olejowego jest początkowo w bezruchu i w chwili zerowej nieskończenie szeroki prowadnik uzyskuje prędkość, która jest funkcją czasu. Numeryczne rozwiązanie układu równań różniczkowych opisujących powyższy proces zostało opracowane dla przypadku przyspieszenia stałego i zmiennego w czasie.

Z wynikami otrzymanymi w omawianej metodzie numerycznej zostały porównane wyniki poprzednio publikowanych rozwiązań analitycznych.