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Andrzej SKALNY
Ludger SZKLARSKI

DIGITAL CALCULATIONS OF DYNAMIC ELECTROMECHANICAL SYSTEMS ON THE GROUND OF STATE EQUATIONS IN THE CONTINUOUS AND DISCRETE TIME DOMAINS

Abstract. Basing on the Lagrange equations the electromechanical linear system is described using the vector state equations. The finished number of lumped elements is regarded. The models of different configurations the selected example is discussed; the digital representation of state variables of an electric drive is used, with the elasticity of mechanical elements of the system taken into account. For the electromechanical systems with an a.c. induction driving motor the dynamical model described by the vector difference equations is written and the numerical example for both deterministic and stochastic systems cases is discussed.

1. INTRODUCTION

The paper deals with the mathematical models of electromechanical systems with lumped parameters.The vector from of equations for different configurations of lumped parameters such as mass,electric elements and damping of the moving system.As an illustration of a model described by linear differential equations (in the continuous time domain) is the dynamical system with four points of moving lumped masses.As a matter of fact this is a d.c.drive(with mechanical part).

The machanical system may be described in the discrete time domain.The model with a.c.induction motor for both deterministic and stochastic loads has been discussed.Selected trancients for such models received from the simulation technique are enclosed.

2. DETERMINISTIC ELECTROMECHANICAL SYSTEM DESCRIBED BY THE VECTOR STATE EQUATIONS IN THE CONTINUOUS TIME DOMAIN

Mechanical part.
The stationary linear mechanical system is being described by the equations:

$$\underline{K}\ddot{\underline{q}} + \underline{D}\dot{\underline{q}} + \underline{P}\,\underline{q} = \underline{Q} \tag{1}$$

where: q-vector of generalized coordinates,

Q-vector of generalized forces,

and matrices $\underline{K},\underline{P},\underline{D}$ are symmetrical positive definite.

Usually this equation is being reduced to the form where the coefficients of components of generalized accelerations equals to one,i.e.

$$\ddot{\underline{q}} + \underline{K}^{-1}\underline{D}\,\dot{\underline{q}} + \underline{K}^{-1}\underline{P}\,\underline{q} = \underline{K}^{-1}\underline{Q} \tag{2}$$

The equation (2) may be presented by following block system,with zero initial conditions,as shown in Fig.1.

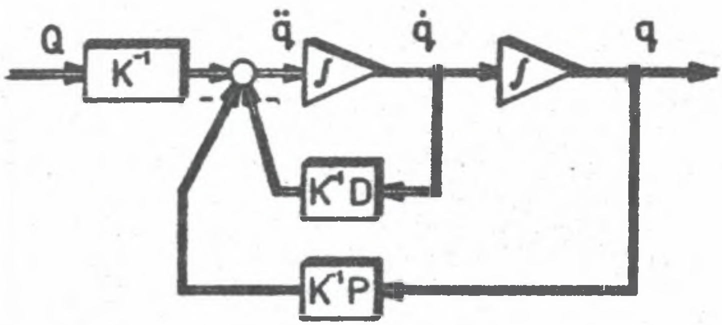

Fig. 1. Mathematical blok system of the equation of mechanical moving port

Basing on above system (Fig.1),and introducing following notation

$\underline{X}_1(t) = \underline{q}\,(t)$ - displacements vector

$\underline{X}_2(t) = \dot{\underline{q}}\,(t)$ - velocity vector

$$\Rightarrow \begin{bmatrix} \underline{X}_1(t) \\ \underline{X}_2(t) \end{bmatrix} = \underline{X}(t)$$ state variables vector

$\underline{U}(t) = \underline{Q}(t)$ - input,

we receive the vector state equation

$$\dot{\underline{X}}_1(t) = \underline{X}_2(t)$$

$$\dot{\underline{X}}_2(t) = -\underline{K}^{-1}\underline{P}\,\underline{X}_1(t) - \underline{K}^{-1}\underline{D}\,\underline{X}_2(t) + \underline{K}^{-1}\underline{U}(t)$$

In more general form this equation will be

$$\dot{\underline{X}}(t) = \underline{A}\,\underline{X}(t) + \underline{B}\,\underline{U}(t) \tag{3}$$

where the state matrix:

$$\underline{A} = \begin{bmatrix} \underline{0} & \underline{I} \\ -\underline{K}^{-1}\underline{P} & -\underline{K}^{-1}\underline{D} \end{bmatrix} \quad \text{and} \quad \underline{B} = \begin{bmatrix} \underline{0} \\ \underline{K}^{-1} \end{bmatrix}$$ - inout matrix

The matrices $\underline{K},\underline{P},\underline{D}$ are determined for the system with a finite number of degrees of freedom,with lumped parameters,i.e.mass(inertia moment),elasticity,damping, J_i,k_i,r_i - respectively.

The series structure of mechanical part,with finite number of degrees of freedom is shown in Fig.2.

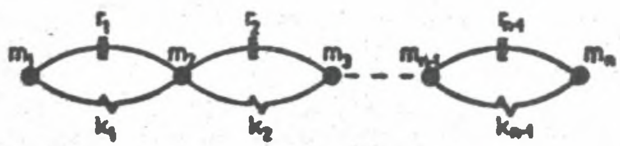

Fig. 2. Model of mechanical part with lumped parameterseseries structure

Such a structure is being characterized by the following forms of matrices $\underline{K},\underline{P},\underline{D}$

$$\underline{K} = \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & \ddots \\ & & & J_n \end{bmatrix} = \mathrm{diag}\left\{J_i\right\}$$ - matrix represents the influence of inertia of lumped masses of moving system,

$$\underline{P} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 & . & . & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 & . & . & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 & . & . & 0 \\ . & & & & & & & . \\ . & & & & & & & 0 \\ 0 & . & . & 0 & -k_{n-1} & k_{n-1} \end{bmatrix}$$

this matrix represents the influence of elastic linear deformations,

$$\underline{D} = \begin{bmatrix} r_1 & -r_1 & 0 & 0 & 0 & \dots & 0 \\ -r_1 & r_1+r_2 & -r_2 & 0 & 0 & . & . & 0 \\ 0 & -r_2 & r_2+r_3 & -r_3 & 0 & . & . & 0 \\ . & & & & & & & . \\ . & & & & & & & 0 \\ 0 & . & . & 0 & -r_{n-1} & r_{n-1} \end{bmatrix}$$

matrix represents the dissipation in the system.

The dynamics of the system is being characterized by the oscillations about the steady state.The state equation of the dynamical system will be written introducing the new state variables φ_i and $\dot{\varphi}_i$, i.e.for the displacements $\varphi_1=\theta_1-\theta_2$; $\varphi_2=\theta_2-\theta_3$, . . . , $\varphi_{n-1}=\theta_{n-1}-\theta_n$, and for the velocities $\dot{\varphi}_1=\dot{\theta}_1-\dot{\theta}_2$; $\dot{\varphi}_2=\dot{\theta}_2-\dot{\theta}_3$, . . . , $\dot{\varphi}_{n-1}=\dot{\theta}_{n-1}-\dot{\theta}_n$.

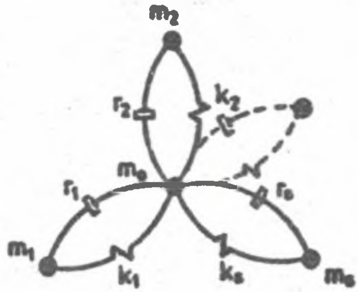

Fig. 3. The forked structure of mechanical part with lumped parameters

The forked structure for n moving lumped mass points is shown in Fig.3. In this structure n=1+s.

For this structure the matrices $\underline{K}$,$\underline{P}$,$\underline{D}$ are as follows:

$$\underline{K} = \begin{bmatrix} J_0 & & & & \\ & J_1 & & & \\ & & J_2 & & \\ & & & \ddots & \\ & & & & J_s \end{bmatrix} \qquad \underline{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \dots & p_{1n} \\ p_{21} & p_{22} & 0 & \dots & 0 \\ p_{31} & 0 & p_{33} & \cdot & \cdot \\ \cdot & \cdot & & \cdot & 0 \\ p_{n1} & 0 & \cdot & 0 & p_{nn} \end{bmatrix}$$

where : $p_{11} = k_1 + k_2 + \dots + k_s$

$p_{1j} = -k_{j-1}$ for $j=2,3,\dots,n$

$p_{i1} = -k_{i-1}$ for $i = 2,3,\dots,n$

and

$p_{22} = k_1$

$p_{33} = k_2$

$p_{nn} = k_s$

And similarly the matrix $\underline{D}$:

$$\underline{D} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & \dots & d_{1n} \\ d_{21} & d_{22} & 0 & \dots & 0 \\ d_{31} & 0 & d_{33} & \cdot & \cdot \\ \cdot & \cdot & & \cdot & 0 \\ d_{n1} & 0 & \cdot & 0 & d_{nn} \end{bmatrix}$$

whose elements d_{ij} are being determined from the relations

$$d_{11} = r_1 + r_2 + \dots + r_s$$

$$d_{1j} = -r_{j-1} \text{ for } j= 2,3,..,n \;; \quad d_{i1} = -r_{i-1} \text{ for } i = 2,3,..,n$$

and

$$d_{22} = r_1$$

$$d_{33} = r_2$$

$$d_{nn} = r_s$$

Electromechanical system

Let us discuss some driving system,as an example. The mechanical part of this system is shown in Fig.4. J_1 is the inertia moment of lumped mass of a separately

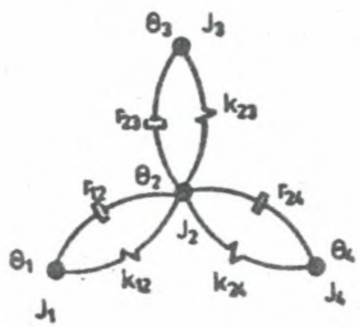

Fig. 4. The example of a forked mechanical part of a driving system

excited d.c.motor armature.The remaining three points of moving lumped masses are parts of driven mechanism.The symbols $\theta_1,\theta_2,\theta_3$ and θ_4 denote the steady-state displacements.Introducing the generalized variables $\varphi_{12}= \theta_1- \theta_2$, $\varphi_{23}=\theta_2- \theta_3$ and $\varphi_{24}= \theta_2 - \theta_4$, the equations of system dynamics will be as follows

$$\dot{I}(t) = a_1 I(t)+ a_2\dot{\theta}_1(t)+ b_1 U(t)$$

$$\ddot{\theta}_1(t)= a_3 \dot{\varphi}_{12}(t)+ a_4 \varphi_{12}(t)+ a_5 I(t)$$

$$\ddot{\varphi}_{12}(t)= a_6\dot{\varphi}_{12}(t)+ a_7\varphi_{12}(t)+ a_8\dot{\varphi}_{23}(t)+ a_9\varphi_{23}(t)+ a_{10}\dot{\varphi}_{24}(t)+ a_{11}\varphi_{24}(t)+a_{12}I(t)$$

$$\ddot{\varphi}_{23}(t)= a_{12}\dot{\varphi}_{23}(t)+ a_{13}\varphi_{23}(t)+ a_{14}\dot{\varphi}_{24}(t)+ a_{15}\varphi_{24}(t)+ a_{16}\dot{\varphi}_{12}(t)+ a_{17}\varphi_{12}(t) + b_2 m_{ob} \qquad (4)$$

$$\ddot{\varphi}_{24}(t)= a_{18}\dot{\varphi}_{24}(t)+ a_{19}\varphi_{24}(t)+ a_{20}\dot{\varphi}_{23}(t)+ a_{21}\varphi_{23}(t)+ a_{22}\dot{\varphi}_{12}(t)+ a_{23}\varphi_{12}(t)$$

The coefficients a_i of the above equations are some functions of system parameters:

R_a - motor armature circuit resistance,
L_a - motor armature circuit induction,
k_e, k_m - motor constants,
$J_{1,2,3,4}$ - inertia moments,
k_{12}, k_{23}, k_{24} - elasticity constants,
r_{12}, r_{23}, r_{24} - damping constants,
$I(t), U(t), m_{ob}$ - instantaneous values of armature current, armature voltage and torque resp.

Introducing the state variables:

$x_1(t)=I(t)$; $x_2(t)=\theta_1(t)$; $x_3(t)=\varphi_{12}(t)$; $x_4(t)=\dot{\varphi}_{12}(t)$; $x_5(t)=\varphi_{23}(t)$;
$x_6(t)=\dot{\varphi}_{23}(t)$; $x_7(t)=\varphi_{24}(t)$; $x_8(t)=\dot{\varphi}_{24}(t)$

we receive the state vector and autput equations:

$$\dot{\underline{X}}(t) = \underline{A}\,\underline{X}(t) + \underline{B}\,U(t) + \underline{G}\,m_{ob}(t) \qquad (5)$$
$$\underline{Y}(t) = \underline{C}\,\underline{X}(t)$$

In this equation U(t) denotes the scalar reference, while $m_{ob}(t)$ is the disturbance.

3. INVESTIGATION OF THE ELECTROMECHANICAL SYSTEM IN THE DISCRETE TIME DOMAIN

Discrete models.

The transition from the model described in continuous time domain to one in the discrete time domain may be achieved in different way. The choice of the method of discretisation should be made with great care, among others, it depends on the tipe of discussed system and its properties and demanded accuracy of numerical calculations.

In the engineering practice, quite reasonable results may be abtained, for linear systems by applying the discretisation of differential equations replaing first and second derivatives by finite differences with chosen discretisation time(sampling time). But still better mathematical model will be received by solving the state equations in a continuous time domain. If the matrices of the equations are known, as well as their eigenvalues and eigenvectors, the matrices of discrete form of state equations may be received. This method may be applied to the linear dynamical system, as well as to the stationary and non-stationary ones. This problem has been developed in following publications (4,5).

The discrete mathematical models may be received by means of different integral transformations of continuous functions and equations. In this paper, however the differential transformation has been used. The discrete description of electric driving system with a.c. cage rotor supplied from the current-source inverter. For the simlicity reasons the dynamics of the driving system may be written with the driven mechanism torque reduced to the motor shaft. It means, that the model of the motor described by the equations in X-Y coordinate system, besides, we may assume that both of stator and rotor windings coincide.

Assuming following relations $\omega_{syn}(t)=\omega_K(t)$, what means that the coordinates X-Y rotate in synchronism with the stator current space vector; $\omega_r(t)=\omega(t)/p$, where $\omega_r(t)$ - angular velocity of motor rotor, $\omega(t)$ - angular velocity of magnetic field, p - number of pairs of poles.

Besides: $T_r = L_r R_r$ - time constant of rotor circuit, then the equations describing the dynamics of the electric drive are as follows:

$$\frac{d\Psi_{rX}(t)}{dt} = \frac{L_m}{T_r} i_{sX}(t) - \frac{1}{T_r}\Psi_{rX}(t) + \omega_{syn}(t)\Psi_{rY}(t) - p\omega_r(t)\Psi_{rY}(t)$$

$$\frac{d\Psi_{rY}(t)}{dt} = \frac{L_m}{T_r} i_{sY}(t) - \frac{1}{T_r}\Psi_{rY}(t) - \omega_{syn}(t)\Psi_{rX}(t) + p\omega_r(t)\Psi_{rX}(t) \qquad (6)$$

$$\frac{d\omega_r(t)}{dt} = \frac{3pL_m}{2L_r J}\Psi_{rX}(t)\, i_{sY}(t) - \frac{3pL_m}{2L_r J}\Psi_{rY}(t)\, i_{sX}(t) - \frac{1}{J} m_{ob}$$

$$\frac{d\,\theta(t)}{dt} = \omega_r(t)$$

The following state variables and components of reference vector were chosen:

$x_1(t)=\Psi_{rX}(t)$ - projection of rotor flux space vector to the 0-X axis,

$x_2(t)=\Psi_{rY}(t)$ - projection of rotor flux space vector on the 0-Y axis,

$x_3(t)=\omega_r(t)$

$x_4(t)=\theta(t)$

$u_1(t)=i_{sX}(t)$ - projection of the space vector of stator current to the 0-X axis,

$u_2(t)=\omega_{syn}(t)$

$u_3(t)=i_{sY}(t)$ - projection of stator current space vector to 0-Y axis.

As it was mentioned previously, the case will be discussed when $u_3(t)=i_{sY}=0$. For C-class functions from above described continuous model the discrete model is received by means of Taylor's differential, for total argument k=0,1,2,...

$$X_1(k+1) = -\frac{H}{(k+1)T_r}X_1(k) - \frac{Hp}{k+1}\sum_{l=0}^{k} X_3(l)X_2(k-l) + \frac{HL_m}{(k+1)T_r}U_1(k) + $$
$$+ \frac{H}{(k+1)}\sum_{l=0}^{k} U_2(l)X_2(k-l)$$

$$X_2(k+1) = -\frac{H}{(k+1)T_r}X_2(k) + \frac{Hp}{k+1}\sum_{l=0}^{k} X_3(l)X_1(k-l) - \frac{H}{k+1}\sum_{l=0}^{k} U_2(l)X_1(k-l) \tag{7}$$

$$X_3(k+1) = -\frac{3HpL_m}{2(k+1)L_rJ}\sum_{l=0}^{k} X_2(l)U_1(k-l) - \frac{H}{(k+1)J}M_{ob}(l)$$

$$X_4(k+1) = \frac{H}{k+1}X_3(k)$$

In these equations $X_1(k),X_2(k),X_3(k),X_4(k),U_1(k),U_2(k)$ and $M_{ob}(k)$ are discrete forms of previously discussed state variables of components of reference vecto-r and load torque. Constant H is of time dimension.

Applying the Cauchy product properties the algebraic twists are received in equation (7).In order to describe this model in state space,the following vectors are defined as follows:

$$\begin{bmatrix} X_1(k) \\ X_2(k) \\ . \\ X_j(k) \end{bmatrix} = \underline{X}(k) \text{ - state variable vector,}$$

$$\begin{bmatrix} U_1(k) \\ U_2(k) \\ . \\ U_i(k) \end{bmatrix} = \underline{VU}(k) \text{ - reference vector}$$

Some additional vector resulting from the approximation of algebraic twists are:

$$\sum_{l=0}^{k} X_j(l)X_w(k-l) \overset{def}{=} \underline{XR}_j^T(k)\underline{XR}_w \overset{def}{=} \underline{XX}_\chi(k) \Rightarrow \begin{bmatrix} XX_1(k) \\ XX_2(k) \\ \cdot \\ \cdot \\ XX_\chi(k) \end{bmatrix} = \underline{XN}(k)$$

$$\sum_{l=0}^{k} U_i(l)X_j(k-l) \overset{def}{=} \underline{UR}_i^T(k)\underline{XR}_j(k) \overset{def}{=} \underline{UX}_\nu(k) \Rightarrow \begin{bmatrix} UX_1(k) \\ UX_2(k) \\ \cdot \\ \cdot \\ UX_\nu(k) \end{bmatrix} = \underline{UX}(k)$$

where: χ, ν - number of products of pairs of variables $X_j(k)$ and $X_j(k), U_i(k)$ in the equations (7),respectively. In the vectoe form the discrete model may be described as follows:

$$\underline{X}(k+1) = \underline{AL}(k,H)\ \underline{X}(k) + \underline{AN}(k,H)\ \underline{XN}(k) + \underline{BL}(k,H)\ \underline{VU}(k) + \underline{BN}(k,H)\underline{UX}(k) \qquad (8)$$

In discussion of electromechanical systems often the problem of stochastic disturbances may be encountered.Their nature and point of entering into the system may be different. If the load torque $m_{ob}(t)$ containes two components: deterministic $m_0(t)$ and stochastic one $m_{os}(t)$,in the discrete model of the drive appear discrete T-transforms.

$m_{ob}(t) \;I\; M_{ob}(k)$

$m_0\ (t) \;I\; M_0\ (k)$

$m_{os}(t) \;I\; M_{os}(k)$.

In order to discuss the behaviour of such a drive,as a dynamical system,the de-scription of the system basing on the solutions of equations(7) will be especially suitable.

The vector state equation in this case will be as follows:

$$\begin{bmatrix} X_1(k+1) \\ X_2(k+1) \\ X_3(k+1) \\ X_4(k+1) \end{bmatrix} = \begin{bmatrix} -\frac{H}{(k+1)T_r} & 0 & 0 & 0 \\ 0 & -\frac{H}{(k+1)T_r} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \\ X_3(k) \\ X_4(k) \end{bmatrix} + \begin{bmatrix} 0 & -\frac{Hp}{k+1} \\ \frac{Hp}{k+1} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} XX_1(k) \\ XX_2(k) \end{bmatrix} +$$

$$+\begin{bmatrix} 0 & 0 & -\frac{H}{k+1} \\ 0 & -\frac{H}{k+1} & 0 \\ -\frac{3HpL_m}{2(k+1)L_rJ} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} UX_1(k) \\ UX_2(k) \\ UX_3(k) \end{bmatrix} + \begin{bmatrix} \frac{HL_m}{(k+1)T_r} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{H}{(k+1)J} & -\frac{H}{(k+1)J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1(k) \\ M_o(k) \\ M_{os}(k) \end{bmatrix} \qquad (9)$$

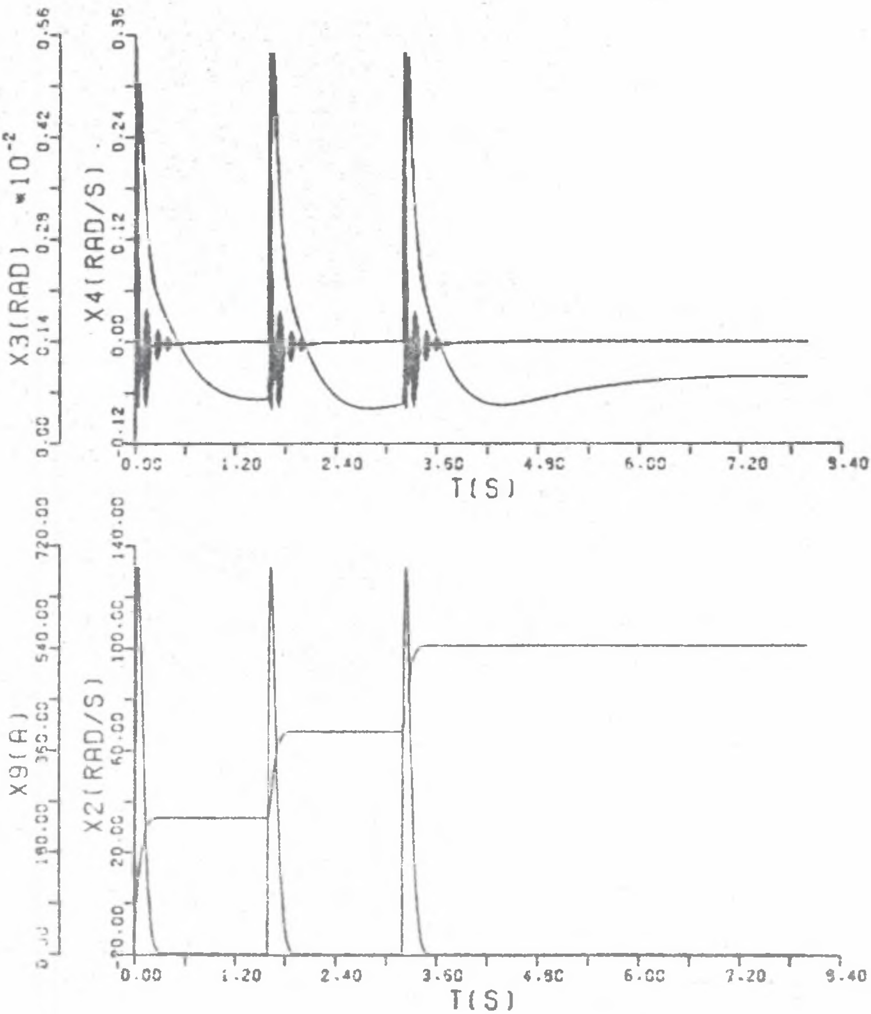

Fig. 5. Transients of state variables of fo ked driving system

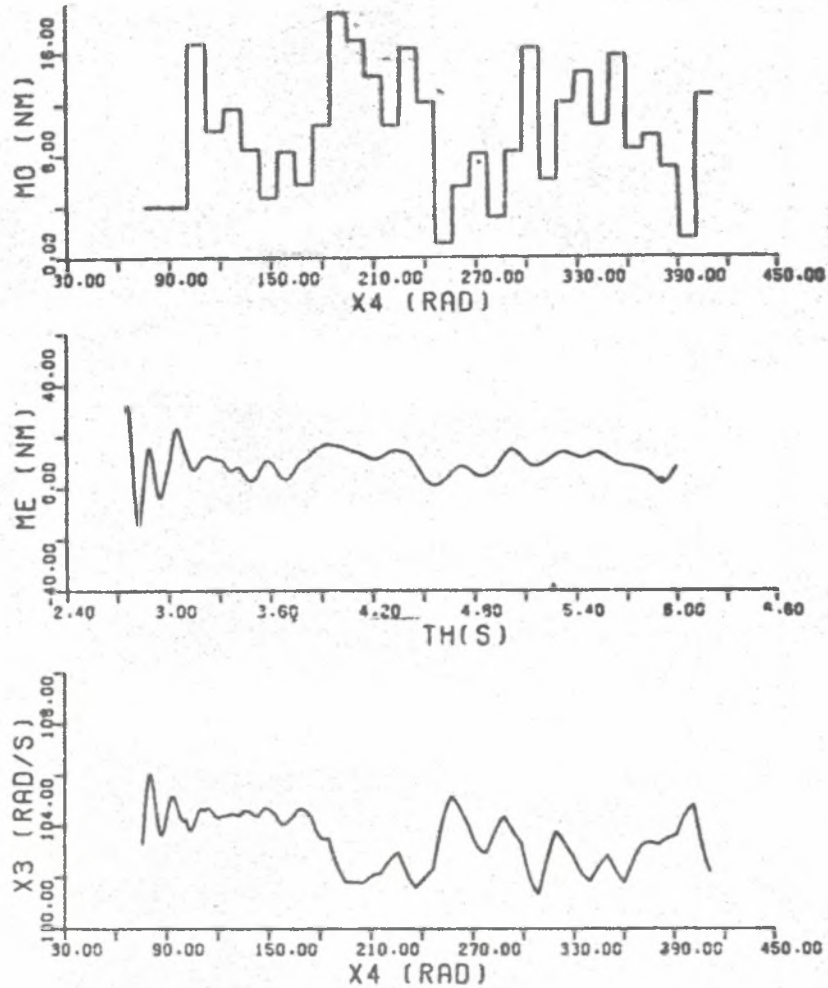

Fig. 6. Transients of torque and velocity of an a.c. induction motor with stochastic load

From the structure itself of the equations (7) and (9) the simple algorythms of numerical simulation will appear. The practice resulting from the application of such models conform their usefulness. They deliver very convenient instrument in the synthesis of digital control of electromechanical systems and will be very useful in computer-aided desing of electric drives.

The briet illustration of simulation of some numerical examples are shown diagrams,depicted in Fig 5.It shown the transients of velocities and ascillations in some chosen points (2) of Fig.4,as well as armature current (X9) of driving motor. The digital simulation was carried out basing on the equations (4) and (5). Fig.6 depicts the torque and velocity transients of an a.c.induction with cage rotor,according to the equation (9).

(Above problems have been solved with the problem RPBP 02.7)

LITERATURE

[1] Pełczewski W., Krynke M.: Metoda zmiennych stanu w analizie dynamiki układów napędowych. WNT, Warszawa 1984.

[2] Pukhov G.E.: Differential Transformations of Functions and Equations. ND, Kiev 1980.

[3] Skalny A.: Kształtowanie w dyskretnej przestrzeni stanu charakterystyk napędu par kinematycznych robota koparki. V Krajowe Seminarium "Rozwój Teoretycznych Podstaw Optymalizacji Zautomatyzowanych Układów Napędu El". Kraków-Karniowice 1987.

[4] Skalny A.: Dynamika układów elektromechanicznych o wielu stopniach swobody i estymacja w czasie dyskretnym przy zakłóceniach stochastycznych. PWN, Seria Wyd. Postępy Napędu El., Warszawa 1988.

[5] Skalny A.: Dyskretne metody analizy napędu elektrycznego w przestrzeni stanu. PWN. Seria Wyd. Postępy Napędu El. (w druku).

[6] Szklarski L., Skalny A.: Teoretyczne Zagadnienia Maszyn Wyciągowych Cz. I. PWN, Seria Wyd. Postępy Napędu El., Warszawa 1975.

[7] Szklarski L.: Teoretyczne Zagadnienia Maszyn Wyciągowych. Cz. II, PWN, Seria Wyd. Postępy Napędu El. Warszawa 1977.

[8] Szklarski L., Skalny A., Yahiatene A.: The Control Electric Drives of Complex Electromechanical System. ICAMC '86. Dubrovnik-Cavtat 1986.

Recenzent: Prof. dr hab. inż. Szczepan Borkowski

CYFROWE OBLICZENIA DYNAMICZNYCH SYSTEMÓW ELEKTROMECHANICZNYCH NA PODSTAWIE RÓWNAŃ STANU W CIĄGŁEJ I DYSKRETNEJ PRZESTRZENI CZASOWEJ

S t r e s z c z e n i e

Na podstawie równania Lagrange'a opisano elektromechaniczny system liniowy, stosując wektorowe równania stanu. Rozważono skończoną liczbę elementów skupionych. Przedyskutowano modele o różnych konfiguracjach na wybranych przykładach; zastosowano cyfrową reprezentację zmiennych stanu napędu elektrycznego, biorąc pod uwagę sprężystość mechanicznych elementów systemu. Dla systemu elektromechanicznego z indukcyjnym silnikiem na prąd stały opisano model dynamiczny za pomocą wektorowych równań różniczkowych i przedyskutowano przykład liczbowy dla przypadków systemów deterministycznych i stochastycznych.

ЦИФРОВЫЕ РАСЧЕТЫ ДИНАМИЧЕСКИХ ЭЛЕКТРОМЕХАНИЧЕСКИХ СИСТЕМ НА ОСНОВАНИИ УРАВНЕНИЙ СОСТОЯНИЯ В НЕПРЕРЫВНОМ И ДИСКРЕТНОМ ВРЕМЕННОМ ПРОСТРАНСТВЕ

Р е з ю м е

На основании уравнений Лягранжа описано электромеханическую линейную систему, применяя векторные уравнения состояния. Рассмотрено законченное количество совокупленных элементов. Обсуждено модели о разными конфигурациями на избранных примерах; применено цифровые репрезентации изменяемых состояния электропривода, принимая во внимание упругость механических элементов системы. Для электромеханической системы с индукционным двигателем постоянного тока описано динамическую модель при помощи векторных дифференциальных уравнений и обсуждено численный пример для детерминированных и стохастических систем.