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INVESTIGATING THE UNSTABLE REGIONS IN THE TRAVEL  
OF CONVEYANCES ALONG THE UNDERGROUND MINE SHAFTS

**Summary.** In Chechoslovakia cases are known when at certain conveying speeds the conveyance of an hoisting equipment vibrates excessively, which may well lead to the destruction of timber guides, or at least to excessive abrasion if there is dynamic pressure between rigid guides of a conveyance and steel guides.

Conveyances are led on guides (two on each side). From the dynamic point of view this mechanical equipment represents rather a complex system. The conveyance may vibrate and the vibrations may be of one or more types and shapes:

- a) longitudinal or transverse vibrations - as a mass attached to a steel rope longitudinally/transversally flexible,
- b) transverse or torsional vibrations in the guide plane as an elastic body placed on timber or steel guides (resp. guides made from another special material) attached to bending elastic beams - sets.

When moving on guides the conveyance is parametrically excited (due to their periodically changing rigidity) and vibrates. In dependence on the speed the vibration may be transverse, torsional, or combined. The frequency of the parametric vibration is given by the set dividers on which the guides are attached, and by the hoisting speed.

Farther, the dynamic equivalent computing model is described in the present paper solving the problems mentioned above.

## 1. INTRODUCTION

Owing to the periodical variation of shaft lining structure rigidity, the conveyance - shaft lining system may, under certain conditions, become unstable. This instability may manifest itself excessive oscillation of skip and the oscillation may well lead to the destruction of shaft lining elements. In suffering from these unfavourable phenomena, however, the skips and the shaft lining structures comply with all the requirements of the Mine Regulations BP 12/82.

In Czechoslovak underground practice situations are known, in which defects in plants occurred as excessive fractures of roller guides on conveyances or as the destruction shaft lining (timber guides). Such phenomena have never been explained properly, and they have always been removed using some compensatory mining mechanical methods without being sufficiently analysed theoretically. The first paper dealing with parametric vibration of cages was published in Czechoslovakia only in January, 1986 [2]. Getting out of our own experiences and considering the quick development of computing technics and devices it is possible to analyse unstable region already in this way to prevent dangerous events in vertical transport.

## 2. THEORETICAL ANALYSIS

Conveyances (cages, skips, etc.) are mostly placed guides supported by suspension gears with dividers. For conveyances of middle and great capacities wheels are placed in the lower part. Thus, we have four components of a wheel guide equipment. Moreover, the conveyances are placed on fixed auxiliary guides which are activated in special cases only. The wheel guide equipment may be either spring-loaded or unsprung.

## 3. COMPUTING MODEL

To investigate unstable regions the planar computing model given in fig. 1 is used. This model represents an oscillation system with two degrees of freedom.

The computing model holds for the following conditions:

- the conveyance travels on a backlash-free guide equipment
- guides represent a system of supports not transferring the bending moment from one field into another
- the conveyance travels at constant rate  $v$
- owing to the fact that there is one type of conveyances only there is also one conveyance guide only acting in one field
- the conveyance behaves as an ideal solid body
- the guides are ideally straight and vertical

## 4. EXPRESSION FOR THE ELASTICITY OF CONVEYANCE GUIDE EQUIPMENT

The overall rigidity of the conveyance guide equipment depends on the rigidity of rollers, guides and sets. It is the function of conveyance position  $y$  and we calculate it solving the relation

$$k(y) = \frac{k_b \cdot k_v(y)}{k_b + k_v(y)}, \quad (1)$$

where:

$k_b$  = spring constant of the roller ( $\text{Nm}^{-1}$ )

$k_v(y)$  = spring constant of the lining (guide and sets ( $\text{Nm}^{-1}$ ))

For vertical distance of sets being constant, this function may be considered for periodic function, so that the following holds:

$$k(y) = k(y + l_s) \quad (2)$$

As it satisfies also other necessary conditions, the function may be expanded in the following Fourier series:

$$k(y) = \frac{a_0}{2} + a_1 \cdot \cos 2\pi \frac{y}{l_s} + a_2 \cdot \cos 4\pi \frac{y}{l_s} + \dots + a_j \cdot \cos 6\pi \frac{y}{l_s} + \dots \quad (3)$$

For elastic constants of lower and upper guide equipments the relation holds:

$$k_d(y) = k_h(y-h), \quad (4)$$

where:

$h$  - the distance between rollers axes (m).

## 5. EQUATION OF CONVEYANCE MOTION

For the motion of centre of gravity in the direction of  $z$ -axis and rotation of the conveyance about this point the following equation of motion holds:

$$m \cdot \ddot{z} + b_p \cdot \dot{z} + k_h [z + (h-l) \cdot \varphi] + k_d [z - l \cdot \varphi] = 0$$

$$I \cdot \ddot{\varphi} + b_r \cdot \dot{\varphi} + k_h \left[ \dot{z} + (h - l) \dot{\varphi} \right] \cdot (h - l) - \\ - k_d \left[ z - l \varphi \right] \cdot l = 0, \quad (5)$$

where:

- $m$  - conveyance mass (kg),
- $I$  - moment of inertia of the conveyance with respect to its centre of gravity ( $\text{kg} \cdot \text{m}^2$ ),
- $b$  - damping coefficient of advance motion,
- $b_r$  - damping coefficient of rotary motion,
- $z$  - coordinate for the displacement of the conveyance centre of gravity,
- $\varphi$  - deflection angle of the conveyance,
- $l$  - distance between the conveyance centre of gravity and lower roller axis (m).

Solving this we obtain

$$\ddot{z} + \frac{b}{m} \cdot \dot{z} + \frac{1}{m} (k_h + k_d) \cdot z + \frac{1}{m} \left[ k_h \cdot h - (k_h + k_d) \cdot l \right] \cdot \varphi = 0 \\ \ddot{\varphi} + \frac{b_r}{I} \cdot \dot{\varphi} + \frac{1}{I} \left[ k_h \cdot h - (k_h + k_d) \cdot l \right] \cdot z + \frac{1}{I} \cdot \\ \cdot \left[ k_h (h - l)^2 + k_d \cdot l^2 \right] \cdot \varphi = 0 \quad (6)$$

Applying equations (3) and (4) and the basic trigonometric formulae, we obtain:

$$a) \quad k_h + k_d = a_0 + a_1 \left( 1 + \cos 2\pi \frac{h}{l_0} \right) \cdot \cos 2\pi \frac{y}{l_0} + \\ + a_2 \left( 1 + \cos 4\pi \frac{h}{l_0} \right) \cos 4\pi \frac{y}{l_0} + \dots + \\ + a_1 \sin 2\pi \frac{h}{l_0} \sin 2\pi \frac{y}{l_0} + \\ + a_2 \sin 4\pi \frac{h}{l_0} \cdot \sin 4\pi \frac{y}{l_0} + \dots$$

$$b) \quad k_h \cdot h - (k_h + k_d) l = \frac{a_0}{2} (h - 2l) + \\ + a_1 \left[ h - l \left( 1 + \cos 2\pi \frac{h}{l_0} \right) \right] \cos 2\pi \frac{y}{l_0} +$$

$$\begin{aligned}
& + a_2 \left[ h - l \left( 1 + \cos 4\pi \frac{h}{l_0} \right) \right] \cos 4\pi \frac{y}{l_0} + \dots \\
& - a_1 l \cdot \sin 2\pi \frac{h}{l_0} \sin 2\pi \frac{y}{l_0} - \\
& - a_2 \sin 4\pi \frac{h}{l_0} \cdot \sin 4\pi \frac{y}{l_0} - \dots \\
& k_h (h - l)^2 + k_d l^2 = \frac{a_0}{2} \left[ (h - l)^2 + l^2 \right] + \\
& + a_1 \left[ (h - l)^2 + l^2 \cdot \cos 2\pi \frac{h}{l_0} \right] \cos 2\pi \frac{y}{l_0} + \\
& + a_2 \left[ (h - l)^2 + l^2 \cdot \cos 4\pi \frac{h}{l_0} \right] \cos 4\pi \frac{y}{l_0} + \dots + \\
& + a_1 l^2 \cdot \sin 2\pi \frac{h}{l_0} \sin 2\pi \frac{y}{l_0} + \\
& + a_2 \cdot \sin 4\pi \frac{h}{l_0} \cdot \sin 4\pi \frac{y}{l_0} + \dots
\end{aligned}$$

Introducing the following constants:

$$A_0 = a_0$$

$$A_1 = a_1 \left( 1 + \cos 2\pi \frac{h}{l_0} \right)$$

$$A_n = a_n \left( 1 + \cos 2n\pi \frac{h}{l_0} \right)$$

⋮

$$B_1 = a_1 \sin 2\pi \frac{h}{l_0}$$

⋮

$$B_n = a_n \sin 2n\pi \frac{h}{l_0}$$

$$C_0 = \frac{a_0}{2} (h - 2l)$$

$$C_1 = a_1 \left\{ h - l \left( 1 + \cos 2\pi \frac{h}{l_0} \right) \right\}$$

⋮

$$C_n = a_n \left[ h - l \left( 1 + \cos 2n\pi \frac{h}{l_0} \right) \right]$$

$$\begin{aligned}
 D_1 &= -a_1 h \sin 2\pi \frac{h}{l_e} \\
 &\vdots \\
 D_n &= -a_n h \sin 2n\pi \frac{h}{l_e} \\
 E_0 &= \frac{a_0}{2} [(h-l)^2 + l^2] \\
 E_1 &= -a_1 [(h-l)^2 + l^2 \cdot \cos 2\pi \frac{h}{l_e}] \\
 &\vdots \\
 E_n &= a_n [(h-l)^2 + l^2 \cdot \cos 2n\pi \frac{h}{l_e}] \\
 F_1 &= a_1 l^2 \cdot \sin 2\pi \frac{h}{l_e} \\
 &\vdots \\
 F_n &= a_n \cdot l^2 \cdot \sin 2n\pi \frac{h}{l_e}
 \end{aligned} \tag{8}$$

$$\omega_1^2 = \frac{A_0}{m} \quad \omega_2^2 = \frac{E_0}{I} \quad \omega = 2 \frac{v}{l_e} \tag{9}$$

$$\frac{b_p}{m} = 2 \delta_1 \omega_1 \quad \frac{b_r}{I} = 2 \delta_2 \omega_2$$

where:

$v$  - running speed of the conveyance ( $m \cdot s^{-1}$ )

relative dampings with respect to the equation  $y = v \cdot t$  will simplify the equation of motion to the following form:

$$\begin{aligned}
 \ddot{z} + 2 \delta_1 \omega_1 \cdot \dot{z} + \frac{1}{m} (A_0 + A_1 \cos \omega t + A_2 \cdot \cos 2\omega t + \\
 + \dots + B_1 \sin \omega t + B_2 \cdot \sin 2\omega t + \dots) z + \\
 + \frac{1}{m} (C_0 + C_1 \cdot \cos \omega t + C_2 \cdot \cos 2\omega t + \dots + \\
 + D_1 \sin \omega t + D_2 \sin \cdot 2\omega t + \dots) \cdot \varphi = 0
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 \ddot{\varphi} + 2 \delta_2 \cdot \omega_2 \dot{\varphi} + \frac{1}{I} (C_0 + C_1 \cdot \cos \omega t + C_2 \cos 2\omega t + \\
 + \dots + D_1 \cdot \sin \omega t + D_2 \sin 2\omega t + \dots) = +
 \end{aligned}$$



$$+ \frac{1}{I} (E_0 + E_1 \cos \omega t + E_2 \cdot \cos 2\omega t + \dots + \\ + F_1 \sin \omega t + F_2 \cdot \sin 2\omega t + \dots) \varphi = 0$$

## 6. DEFINING THE UNSTABLE REGIONS

Equations of motion yield a system of homogeneous differential equations with periodic coefficients.

Such equations have stable, or unstable, but also periodic solutions (1). It may be proved that periodic solutions separate stable from unstable regions.

To determine parameters of the system corresponding to the periodic solution, we set:

$$z = \sum_{i=1,3,5\dots} (a_i \sin \frac{1}{2} \omega t + b_i \cdot \cos \frac{1}{2} \omega t)$$

$$\varphi = \sum_{i=1,3,5\dots} (c_i \sin \frac{1}{2} \omega t + d_i \cdot \cos \frac{1}{2} \omega t) \quad (11)$$

Substituting in to (10), we obtain

$$\sum_{i=1,3,5\dots} (a_i \sin \frac{1}{2} \omega t + b_i \cdot \cos \frac{1}{2} \omega t) (\frac{A_0}{m} + \\ + \frac{A_1}{m} \cos \omega t + \frac{A_2}{m} \cos 2\omega t + \dots + \frac{B_1}{m} \sin \omega t + \\ + \frac{B_2}{m} \sin 2\omega t + \dots - \frac{1}{4} \omega^2) + 2\delta_1 \omega_1 \omega \cdot \\ \cdot \frac{1}{2} (a_i \cos \frac{1}{2} \omega t - b_i \sin \frac{1}{2} \omega t) + (c_i \sin \frac{1}{2} \omega t + \\ + d_i \cos \frac{1}{2} \omega t) (\frac{C_0}{m} + \frac{C_1}{m} \cos \omega t + \frac{C_2}{m} \cos 2\omega t + \dots + \\ + \frac{D_1}{m} \sin \omega t + \frac{D_2}{m} \sin 2\omega t + \dots) = 0$$

$$\begin{aligned}
& \sum_{i=1,3,5,\dots} (a_i \sin \frac{1}{2} \omega t + b_i \cos \frac{1}{2} \omega t) \left( \frac{C_0}{I} + \right. \\
& + \frac{C_i}{I} \cos \omega t + \frac{C_2}{I} \cos 2 \omega t + \dots + \frac{D_1}{I} \sin \omega t + \\
& + \frac{D_2}{I} \sin 2 \omega t + \dots \left. \right) + (c_1 \sin \frac{1}{2} \omega t + d_1 \cdot \cos \frac{1}{2} \omega t) \cdot \\
& \cdot \left( \frac{E_0}{I} + \frac{E_1}{I} \cos \omega t + \frac{E_2}{I} \cos 2 \omega t + \dots + \right. \\
& + \frac{F_1}{I} \sin \omega t + \frac{F_2}{I} \sin 2 \omega t + \dots - \frac{1^2}{4} (\omega^2) + \\
& + (c_1 \cos \frac{1}{2} \omega t - d_1 \cdot \sin \frac{1}{2} \omega t) \cdot 2 \delta_2 \frac{1}{2} \omega \omega_2 = 0
\end{aligned} \tag{12}$$

This system will be rearranged using the equations

$$\begin{aligned}
& \cos j \omega t \cdot \sin \frac{1}{2} \omega t = \\
& = \frac{1}{2} \left[ \sin \left( \frac{1}{2} + j \right) \omega t + \sin \left( \frac{1}{2} - j \right) \omega t \right] \\
& \cos j \omega t \cdot \cos \frac{1}{2} \omega t = \\
& = \frac{1}{2} \left[ \cos \left( \frac{1}{2} + j \right) \omega t + \cos \left( \frac{1}{2} - j \right) \omega t \right] \\
& \sin j \omega t \cdot \sin \frac{1}{2} \omega t = \\
& = \frac{1}{2} \left[ \cos \left( \frac{1}{2} - j \right) \omega t - \cos \left( \frac{1}{2} + j \right) \omega t \right] \\
& \sin j \omega t \cos \frac{1}{2} \omega t = \\
& = \frac{1}{2} \left[ \sin \left( \frac{1}{2} + j \right) \omega t - \sin \left( \frac{1}{2} - j \right) \omega t \right]
\end{aligned} \tag{13}$$

and its coefficients will be represented as a matrix (14).



	$\cdot b_3$	$a_1$	$b_1$	$c_1$	$d_1$	$c_3 \dots$	
$\cos \frac{3}{2} \omega t$	...	...	...	...	...	...	
$\cos \frac{1}{2} \omega t$	1	...	$\delta_1 \omega \cdot \omega_1$	$\frac{A_0}{m} + \frac{A_1}{2m} \left(\frac{\omega}{2}\right)^2$	$\frac{D_1}{2m}$	$\frac{C_0}{m} + \frac{C_1}{2m}$	...
	2	...	$\frac{D_1}{2I}$	$\frac{C_0}{I} + \frac{C_1}{2I}$	$\delta_2 \omega \cdot \omega_2$	$\frac{E_0}{I} + \frac{E_1}{2I} \left(\frac{\omega}{2}\right)^2$	...
$\sin \frac{1}{2} \omega t$	1	...	$\frac{A_0}{m} - \frac{A_1}{2m} \left(\frac{\omega}{2}\right)^2$	$-\delta_1 \omega \cdot \omega_1$	$\frac{C_0}{m} - \frac{C_1}{2m}$	$\frac{D_1}{2m}$	...
	2	...	$\frac{C_0}{I} - \frac{C_1}{2I}$	$\frac{D_1}{2I}$	$\frac{E_0}{I} - \frac{E_1}{2I} - \left(\frac{\omega}{2}\right)^2$	$-\delta_2 \omega \cdot \omega_2$	...
$\sin \frac{3}{2} \omega t$	...	...	...	...	...	...	

This system has solution only if the determinant of matrix (14) equals zero. This problem has no solution for a non-finite system, however, it may be proved that central determinants provide a very good approximation.

For the particular problem under study only the definition of unstable determinant  $4 \times 4$  will do.

For easier calculation, the simplification will be introduced.

$$\frac{E}{2} = \eta \quad 2 \delta_1 \omega_1 = R_1 \quad 2 \delta_2 \omega_2 = R_2$$

$$\frac{A_0}{m} + \frac{A_1}{2m} = C_0 \quad \frac{A_0}{m} - \frac{A_1}{2m} = C_1 \quad \frac{E_0}{I} + \frac{E_1}{2I} = C_2$$

$$\frac{E_0}{I} - \frac{E_1}{2I} = C_3$$

$$\frac{C_0}{m} + \frac{C_1}{2m} = k_1 \quad \frac{C_0}{m} - \frac{C_1}{2m} = k_2 \quad \frac{C_0}{I} + \frac{C_1}{2I} = k_3 \quad \frac{C_0}{I} - \frac{C_1}{2I} = k_4$$

$$\frac{D_1}{2m} = B \quad \frac{D_1}{2I} = A$$

Using this denotation, the condition for solution has the form

$$\begin{vmatrix} R_1 \eta & C_0 - \eta^2 & B & k_1 \\ A & k_3 & R_2 \eta & C_2 - \eta^2 \\ C_1 - \eta^2 - R_1 \eta & k_2 & B & \\ k_4 & A & C_3 - \eta^2 & - R_2 \eta \end{vmatrix} = 0 \quad (16)$$

Developing the determinant, we obtain algebraic equation of the fourth degree in  $\eta^2$

$$H_0 (\eta^2)^4 + H_1 (\eta^2)^3 + H_2 (\eta^2)^2 + H_3 (\eta^2) + H_4 = 0,$$

$$H_0 = 1$$

$$H_1 = - (C_0 + C_1 + C_2 + C_3 - R_1^2 - R_2^2)$$

$$H_2 = \left[ C_0 \cdot C_1 + C_2 \cdot C_3 - 2AB - k_1 \cdot k_3 - k_2 \cdot k_4 + \right. \\ \left. + (R_1^2 - C_1 - C_0) R_2^2 - C_2 - C_3 \right]$$

$$H_3 = - \left[ C_2 C_3 (C_0 + C_1) + C_0 C_1 (C_2 + C_3) - AB (C_0 + C_1 + \right. \\ \left. + C_2 + C_3 - R_1 R_2 (k_2 k_3 + k_1 k_4 - AB) - \right. \\ \left. - R_1^2 \cdot C_2 \cdot C_3 - R_2^2 C_0 C_1 - k_1 k_3 (C_1 + C_3) - k_2 k_4 (C_0 + C_2) \right]$$

$$H_4 = C_0 C_1 C_2 C_3 + A^2 B^2 - AB (C_0 C_3 + C_1 C_3 + C_1 C_2) - \\ - k_2 k_4 \cdot C_0 C_2 - k_1 k_3 C_1 C_3 - k_3 k_4 B^2 - k_1 k_2 A^2 + k_1 k_2 k_3 k_4$$

From the roots of equation  $\eta$  the limiting value of speeds in separating stable and unstable regions may be determined from the relation

$$v = \frac{\eta l_0}{\tau}$$

To determine unstable regions of the travel of conveyances the computing program in FORTRAN was written for automatic computer EC 10-26.

A counterpart of this program is also the graphic output to DIGIGRAF enabling the unstable regions to be plotted for plants commonly seen in practice.

Unstable regions found out for the plant installed in Mine Darkov, the Ostrava-Karvina Coal District are given in figs. 2 and 3. As seen from the figures, damping reduces unstable regions.

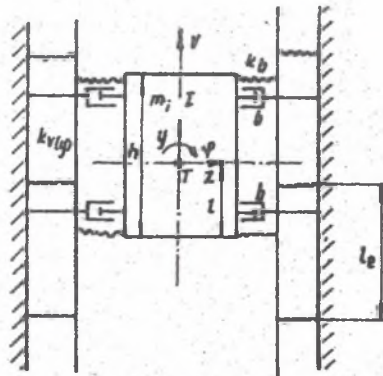


Fig. 1. Computing model

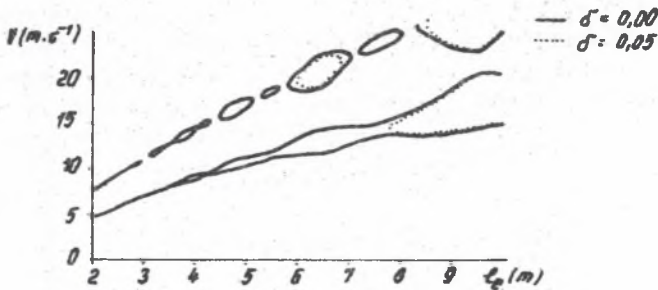


Fig. 2. Unstable regions  
Mine Darkov - Central full conveyance - 100 % guide

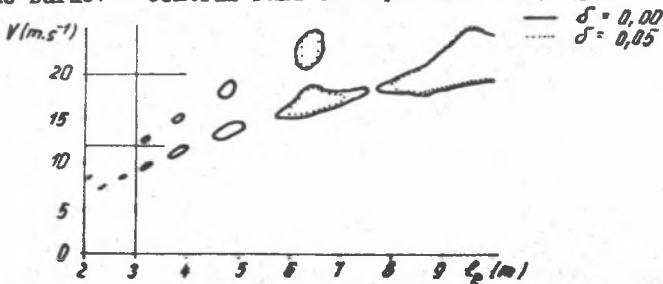


Fig. 3. Unstable regions  
Mine Darkov - Central empty conveyance - 100 % guide

## 7. CONCLUSION

In the hoisting plants being in operation or projected at present, excessive vibration of conveyances may occur at certain shaft lining parameters owing to variable rigidity of its guidance. Having analysed this problem theoretically, the given mass and geometric characteristics of conveyances and the type of rollers at critical speeds at which the travel is unstable may be determined in dependence on vertical distance of sets and chosen profiles of guides and sets. This enables to evaluate the tendency towards parametric vibration of the present plant and to propose, if required, necessary adaptations and to avoid in admissible changes of working parameters at reconstruction and finally, to give optimum design of skip - shaft lining system for the required hoisting speed when projecting new installations.

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## HADANIE OBSZARÓW NIEUSTALONYCH W RUCHU WÓZKÓW WZDŁUŻ SZYBÓW KOPALNI PODZIEMNYCH

### S t r e s z c z e n i e

W Czechosłowacji znane są przypadki, gdy przy określonych prędkościach transportu oprzyrządowanie wyciągu wykazuje nadmierne drgania. Mogą one doprowadzić do zniszczenia drewnianych prowadnic lub do nadmiernego ścierania, jeśli występuje dynamiczne ciśnienie między sztywnymi prowadnicami wózka a prowadnicami stalowymi.

Wózki są prowadzone przez prowadnice (dwie z każdej strony). Z matematycznego punktu widzenia, to mechaniczne oprzyrządowanie stanowi dość złożony układ. Wózek może drgać, a drgania mogą być jednego lub więcej rodzaju i kształtu.

- a) Drgania wzdłużne lub poprzeczne, jako masa mocowana do rury stalowej giętkiej wzdłużnej/poprzecznej.
- b) Drgania poprzeczne lub skrętne w płaszczyźnie prowadnicy, jako ciała sprężystego umieszczonego na drewnianych lub stalowych prowadnicach (odpowiednio prowadnicach wykonanych z innego specjalnego materiału), mocowanych do układu zginanych sprężystych belek.

Podczas ruchu na prowadnicach wózek jest wzbudzany paramechanicznie (w efekcie jego okresowo zmiennej sztywności). W zależności od prędkości drgania mogą być poprzeczne, skrętne lub kombinowane. Częstotliwość parametrycznego drgania jest zadana przez układ rozdzielczy, na których mocowane są prowadnice i przez prędkość wyciągu.

Dalej w artykule został przedstawiony dynamiczny równoważny model obliczeniowy, rozwiązujący wyżej wymienione problemy.

#### ИССЛЕДОВАНИЕ НЕОПРЕДЕЛЕННЫХ ЗОН В ДВИЖЕНИИ ТЕЛЕЖЕК ВДОЛЬ КРЕПЛЕНИЙ ПОДЗЕМНЫХ ШАХТ

#### Резюме

В ЧССР известны случаи, когда при определенных скоростях транспорта оснащение подъемников отличается чрезмерными колебаниями, которые могут вызывать прочу деревянных направляющих, а как минимум, их чрезмерное истирание, если выступает динамическое давление между жесткими направляющими тележки и стальными направляющими.

Тележки ведутся направляющими (по две с каждой стороны). С математической точки зрения — это механическое оснащение представляет собой довольно сложную систему. Тележка может колебаться, а колебания могут быть одного или больших видов и составов.

- a) продольные или поперечные колебания как масса прикрепленная к гибкой продольной/поперечной стальной трубе
- b) поперечные колебания или скручивающие колебания в плоскости направляющей как упругие тела помещенные на деревянных или стальных направляющих (соответственно на направляющих сделанных из другого специального материала) укрепленных в системе сгибаемых упругих балок.

Во время передвижения на направляющих тележка возбуждает парамеханические (в эффекте ее периодически изменяемой жесткости). В зависимости колебания могут быть поперечными, скручивающими или комбинированными. Частота параметрических колебаний задается распределительной системой на которой направляющие укрепляются и скоростью подъема.

Дальше в статье описывается динамически равновесная расчетная модель, решающая вышеописанные проблемы.