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MATHEMATICAL MODEL OF TRANSVERSE VIBRATIONS OF MINE HOISTS, COUPLED WITH LONGITUDINAL VIBRATIONS

Summary. Complex mathematical model of transverse vibrations of mine hoisting system is described. Coupling with longitudinal vibrations, deformations of cage, vibrations of ropes, variations of stiffness of cage support (by system of elastic rollers, slide shoes and guides), as well as damping, asymmetry and unbalance of load are taken into account. Accepted assumptions, system of differential model equations (partial and ordinary), method of its solving and exemplary results of computer simmulation are presented.

1. INTRODUCTION

Scheme of analysed mine hoisting system is presented on fig.1. System is composed of elastic lifting and balance ropes, cage shoes, rollers and shaft guides. Cage is treated as rigid body, having five degrees of freedom (for transverse vibrations and one for longitudinal). It may be also divided into three parts - upper and bottom rigid heads, connected by group of elastic rods. All ropes are treated individually, or one equivalent lifting and balance rope is introduced. Formulated mathematical model of transverse vibrations takes into account also:

- internal viscous damping in ropes, guides and rollers
- clearances between shoes or rollers and guides
- cooperation of rollers and slide shoes
- transmission of vibrations between two branches of balance rope through its bottom loop
- asymmetry of guides, skip or cage
- unbalance of skip and its loading

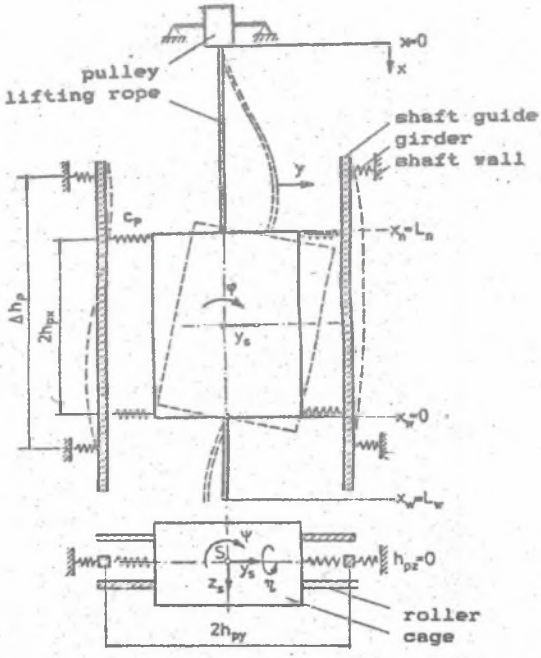

Fig. 1. Scheme of analysed hoisting system

- variations of ropes length and of stiffness of guides
- action of unscrewing moments in ropes on cage

As main cause of transverse vibrations are treated irregularities of shaft guides. Transverse vibrations are also forced by coupling with longitudinal vibrations - through variations of tensions in ropes

2. MATHEMATICAL MODEL OF TRANSVERSE VIBRATIONS

Mathematical model of transverse vibrations of hoisting system consists of equations of dynamic balance of ropes and cage or skip.

2.1. Equations of dynamic balance of rope

Most adequate mathematical model of transverse vibrations of rope can be obtained treating rope as ponderable elastic beam, having transverse stiffness EI and unitary mass ρA, loaded with variable strength S. It can be written as:

$$EI\left(1+\nu\frac{\partial}{\partial t}\right)\frac{\partial^4 y}{\partial x}-\left(1+\nu\frac{\partial}{\partial t}\right)\frac{\partial}{\partial x}\left(S\frac{\partial y}{\partial x}\right)+$$

$$+\rho A\frac{\partial^2 y}{\partial t^2}=0 \qquad (1)$$

Four boundary conditions for every rope consist of assuming : zero transverse displacements near the pulley, lack of bending moment in ends , equality of displacements and equilibria of forces and moments in two branches of bottom loop of balance rope, equality of displacements of suspension gear (cage) and end of rope, as well as relation between force F_L (regarded in equations of dynamic balance of cage) and displacement of the end of rope :

$$F_L=\left[EI\frac{\partial^3 y}{\partial x^3}-S\frac{\partial y}{\partial x}\right]_{x=L} \qquad (2)$$

2.2. Mathematical model of rigid cage

The scheme of rigid cage, vibrating with five degrees of freedom - $y_s, z_o, \varphi, \psi, \eta$ - is presented on fig.2. Optional number of lifting and balance ropes N_n, N_v and of slide shoes or rollers N_{py}, N_{pz} (acting in directons of axes y, z) is regarded. Denoting distances between centre of gravity of cage and shoes as h_{px}, h_{py}, h_{pz}, forces and moments of inertia as $m_s\ddot{y}_s, m_s\ddot{z}_s, J_s\ddot{\varphi}_s, J_s\ddot{\psi}_s, J_s\ddot{\eta}_s$, strenghts in lifting and balance ropes as S_n, S_v, coefficients of unscrewing moment as K_o, forces issued by transverse vibrations of ropes as F_L, forces between cage and guides as F_p , and friction factor in guides as μ (fig.2) and using indexes: n for lifting, v for balance ropes and p for rollers - the equations of dynamic balance of rigid cage can be written as:

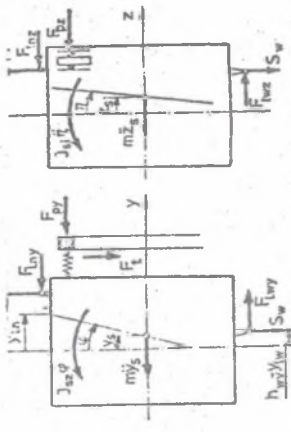

Fig. 2. Scheme of dynamic balance of cage

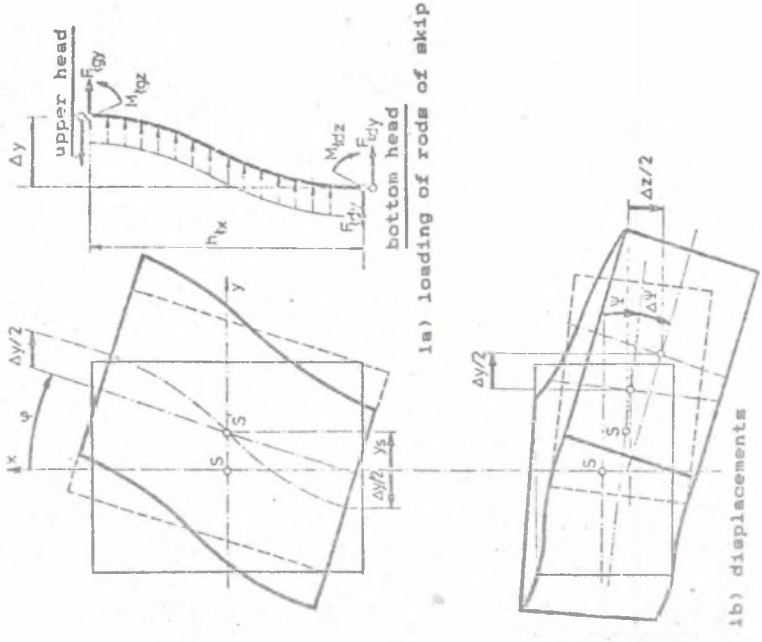

Fig. 3. Scheme of transverse vibrations of deformable cage

$$\sum_i F_{iy} = m_s \ddot{y}_s + \sum_{p=1}^{N_{py}} F_{py} + \sum_{n=1}^{N_n} F_{Lny} - \sum_{v=1}^{N_v} F_{Lvy} = 0$$

$$\sum_i F_{iz} = m_s \ddot{z}_s + \sum_{p=1}^{N_{pz}} F_{pz} + \sum_{n=1}^{N_n} F_{Lnz} - \sum_{v=1}^{N_v} F_{Lvz} = 0$$

$$\sum M_{ix} = J_{sx} \ddot{\psi}_s + \sum_{p=1}^{N_{pz}} F_{pz} h_{py} - \sum_{p=1}^{N_{py}} F_{py} h_{pz} + \sum_{n=1}^{N_n} \left[K_{on} S_n + F_{Lnz} h_{ny} - \right.$$

$$\left. + - F_{Lny} h_{nz} \right] - \sum_{v=1}^{N_v} \left[K_{ov} S_v + F_{Lvz} h_{vy} - F_{Lvy} h_{vz} \right] = 0$$

$$\sum_i M_{iy} = J_{sy} \ddot{\eta}_s + \sum_{p=1}^{N_{pz}} F_{pz} \left[h_{px} + \mu' h_{pz} \right] + \sum_{p=1}^{N_{py}} \mu' F_{py} h_{pz} +$$

$$+ \sum_{n=1}^{N_n} F_{Lnz} h_{nx} + \sum_{v=1}^{N_v} F_{Lvz} h_{vx} + \sum_{n=1}^{N_n} S_n \left[h_{nz} + z_{Ln} \right] +$$

$$- \sum_{v=1}^{N_v} S_v \left[h_{vz} + z_{Lv} \right] = 0$$

$$\sum_i M_{iz} = J_{sz} \ddot{\varphi}_s + \sum_{p=1}^{N_{py}} F_{py} \left[h_{px} + \mu' h_{py} \right] + \sum_{p=1}^{N_{pz}} \mu' F_{pz} h_{py} +$$

$$+ \sum_{n=1}^{N_n} F_{Lny} h_{nx} + \sum_{v=1}^{N_v} F_{Lvy} h_{vx} + \sum_{n=1}^{N_n} S_n \left[h_{ny} + y_{Ln} \right] +$$

$$- \sum_{v=1}^{N_v} S_v \left[h_{vy} + y_{Lv} \right] = 0 \qquad (3)$$

where $\mu' = \{ \mu ; -\mu \}$ for falling or lifting cage

Tensions S_n, S_v are in every time step calculated by separate part of computer program, simulating longitudinal vibrations (not presented here).

Displacements of any point of cage placed on distance h_x, h_y, h_z from gravity centre - can be described by five main coordinates as :

$$y = y_s + \varphi h_x - \psi h_z$$
$$z = z_s + \psi h_y + \eta h_x \qquad (4)$$

2.3. Mathematical model of deformable cage

Most of authors modelling vibrations of hoisting systems [1-4] treats cage as rigid body. More precise model can be obtained

regarding two rigid frames (upper and bottom heads), connected by group of elastic rods. Displacements of heads (without elongation of rods) are described by three degrees of freedom : $\Delta y, \Delta z, \Delta\psi$. Displacement of any point can be expressed by main coordinates as :

$$y = y_s + \varphi h_x - \psi h_z + \zeta/2 \, (\Delta y - \Delta\psi h_z)$$
$$z = z_s + \psi h_y + \eta h_x + \zeta/2 \, (\Delta z + \Delta\psi h_y) \qquad (5)$$

where : $\zeta = (1; -1; 0)$ - for points on upper or bottom head or in the central part of cage

Taking into account relations between displacements Δy (Δz) of ends of rods and forces in them (obtained from analysis of their bending), and using indexes g and d for upper and bottom heads and l for connecting rods - the equations of dynamic balance of upper head of cage (added to system (3)) can be written as :

$$\sum_i F^g_{iy} = m_g \left(\ddot{y}_s + \Delta\ddot{y}/2 \right) + \sum_{p=1}^{N_{pgy}} F_{py} + \sum_{n=1}^{N_n} F_{Lny} +$$
$$+ (m_s - m_g - m_d) \ddot{y}_s/2 + 12 \, (EI/h_x)_l \Delta y = 0$$
$$\sum_i F^g_{iz} = m_g \left(\ddot{z}_s + \Delta\ddot{z}/2 \right) + \sum_{p=1}^{N_{pgz}} F_{pz} + \sum_{n=1}^{N_n} F_{Lnz} +$$
$$+ (m_s - m_g - m_d) \ddot{z}_s/2 + 12 \, (EI/h_x)_l \Delta z = 0$$
$$\sum_i M^g_{ix} = J_{gx} \left(\ddot{\psi} + \Delta\ddot{\psi}/2 \right) + \left(J_{sx} - J_{gx} - J_{dx} \right) \ddot{\psi}/2 +$$
$$+ 12 \left(EI * R^2/h_x^3 \right)_l \Delta\psi + \sum_{p=1}^{N_{pgz}} F_{pz} h_{py} - \sum_{p=1}^{N_{pgy}} F_{py} h_{pz} +$$
$$+ \sum_{n=1}^{N_n} \left[K_{on} S_n + F_{Lnz} h_{ny} - F_{Lny} h_{nz} \right] = 0 \qquad (6)$$

To other equations of system (3) must be added terms representing movement of heads:

$$\Delta \sum F_{iy} = (m_g - m_d) \Delta\ddot{y} / 2$$
$$\Delta \sum F_{iz} = (m_g - m_d) \Delta\ddot{z} / 2$$
$$\Delta \sum M_{ix} = (J_{gx} - J_{dx}) \Delta\ddot{\psi} / 2$$
$$\Delta \sum M_{iy} = (m_g h_{gx} - m_d h_{dx}) \Delta\ddot{z} / 2$$
$$\Delta \sum M_{iz} = (m_g h_{gx} - m_d h_{dx}) \Delta\ddot{y} / 2 \qquad (7)$$

3. FORCES BETWEEN SHOES AND GUIDES

Cage contacts with shaft guides through rollers and slide shoes (in moments of greater transverse displacements). Scheme of cage movement is presented on fig 4.

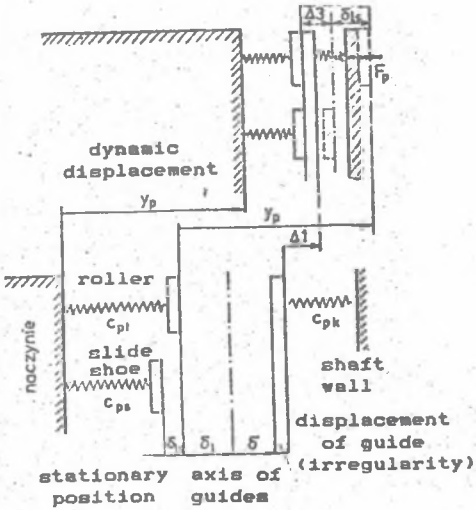

Fig. 4. Dynamic displacements of roller

Denoting by δ_l, δ_s nominal clearances between roller and guide or slide shoe, by $\delta(x)$ actual irregularity of guide, by y_p - dynamic displacement of cage (base of roller) and by c_{pl}, c_{ps}, c_{pk} coefficients of stiffness of roller, slide shoe and guide - force F_p between cage and guide can be described as :

$$F_p = \begin{cases} 0 & \text{- when roller doesn't contact with guide} \\ \dfrac{\left(1 + \nu \dfrac{\partial}{\partial t}\right) * \left(y_p - \delta_l - \delta(x)\right)}{1 / c_{pl} + 1 / c_{pk}} & \text{- when contacts only roller} \\ \dfrac{\left(1 + \nu \dfrac{\partial}{\partial t}\right) * \left(y_p - \delta_l - \delta(x) - \delta_s (1 + c_{pl}/c_{pk})\right)}{1 / (c_{pl} + c_{ps}) + 1 / c_{pk}} & \text{- when contacts roller and slide shoe} \end{cases} \tag{8}$$

4. METHOD OF SOLVING OF MODEL EQUATIONS

4.1. Equations describing vibration of ropes

Fast variational method is used for solving partial differential equations (1) describing transverse vibrations of ropes. Process of vibration is approximated by m-element series:

$$y(x,t) = \sum_{i=1}^{m} Y_i(x) * T_i(t) \tag{9}$$

Functions $Y_i(x)$ are obtained analytically, as solutions of little simplified model of free vibrations of system. Ignoring damping and variations of tension S in ropes and performing Fourrier transform - the equation (1) can be written as :

$$EI \frac{d^4 Y}{d x} - S_o \frac{d Y}{d x} + \rho A \omega^2 Y = 0 ; \qquad Y(\omega, x) = \mathcal{F}\left[y(x,t) \right] \tag{10}$$

Its solution has the form :

$$Y_i(x) = \sum_{i=1}^{4} C_{il} * f_{il}(x) ; \qquad i = 1 - m$$

$$f_{i1}(x) = \sin(\lambda_{i1} x) ; \qquad f_{i2}(x) = \cos(\lambda_{i2} x) ;$$

$$f_{i3}(x) = e^{\lambda_{i3} x} / e^{\lambda_{i3} L} ; \qquad f_{i4}(x) = e^{-\lambda_{i4} x}$$

$$\lambda_{il} = \sqrt{\left[\frac{S_o}{2\,EI} \right]^2 + \frac{\rho A \omega_i^2}{EI} + \frac{S_o}{2\,EI}} \tag{11}$$

Frequencies ω_i of free vibrations and values C_{il} ($C_{i1}=1$) are obtained by numerical solving of system of algebraic linear equations, resulting from boundary conditions, little simplified in comparison to presented in par 2.1 (with relations between forces and displacements of end of rope near cage obtained from simplyfied analysis of plane motion of rigid cage, supported by elastic shoes). Expression (9) fulfills partial differential equation (1) with error, having dimension of derivative of force: $\partial F_{bl} / \partial x$. Force F_L in equations of dynamic balance of cage (3,6,7) satisfies boundary condition (2) with error ΔF_{bl}. Other boundary conditions are fulfilled exactly. According to principles of variational method - as condition of best

approximation of equation (1) with boundary condition (2) by series (9) - is treated zeroing of expressions :

$$\int_0^L \frac{\partial F_{bl}}{\partial x} * Y_j(x)\, dx + \Delta F_{bl} * Y_j(L) = 0 \qquad (12)$$

for all j elements of approximating series ($j = 1 - m$)

Regarding effect of dead weight on tension S :

$$S = S_{o1} + q\, x \qquad (13)$$

where:

$S_{o1} = S_L - q\, L$; $q = -\rho\, A\, g$ for lifting rope;

$S_{o1} = 0$; $q = S_L / L$ for balance rope;

after rather complicated mathematical operations the system of $m + 1$ ordinary differential equations, relating approximating functions T_i, force F_L and displacement y_L can be obtained :

$$\int_0^L \sum_{k=1}^{4} C_{jk} f_{jk}(x) * \left\{ \sum_{i=1}^{m} \left[T_i + \nu \frac{d\,T_i}{d\,t} \right] * \left[\sum_{l=1}^{4} C_{il} * \right.\right.$$

$$* \left[EI * \lambda_{il}^4 - S_{o1} l_1 \lambda_{il}^2 - q\, l_4 \lambda_{il} - q\, l_2 \lambda_{il}^2 x \right] f_{il}(x) \Big] +$$

$$+ \frac{d^2 T_i}{dt^2} * \left[\sum_{l=1}^{4} - C_{il} \rho\, A\, f_{il}(x) \right] \Big\} * dx + \sum_{k=1}^{4} C_{jk} f_{jk}(x) *$$

$$* \left\{ \sum_{i=1}^{m} \left[T_i + \nu \frac{d\,T_i}{d\,t} \right] * \sum_{l=1}^{4} C_{il} \cdot \left[EI\, l_3 \lambda_{il}^2 - S_{o1} l_2 \right] * \right.$$

$$\left. * \lambda_{il} f_{il}(L) \right\} - F_L = 0 \qquad \text{for } j = 1 - m ;$$

$$y_L = \sum_{i=1}^{4} T_i * \sum_{l=1}^{4} C_{il} f_{il}(L) \qquad (14)$$

where $l_r = (1 ; -1)$,depending on indexes l and r

Of course - analogous system describes vibrations in direction z. Functions of variable x are integrated analytically. Computer calculates coefficients of system (14) only after greater changes of tension or length of ropes (not in every time step - saving computation time). System is solved numerically - succesive steps are presented below.

4.2. Equations describing transverse vibrations of cage

Systems of ordinary differential equations (3,6,7,14), describing vibrations of cage and ropes are solved numerically, using fast method, based on parabolic approximation of variation of every dynamic variable during the time step Δt (with continuity of variable and velocity of its changes). Expressions determining values of every variable at the end of time step Δt are obtained by introducing above mentioned parabolic approximations into differential equations and zeroing integrals of these equations in range Δt.

At first - relation between $\int_{t^o}^{t'} F_L\, dt$ and $\int_{t^o}^{t'} y_L\, dt$, main coordinates of cage movement $y'_s, z'_s, \ldots$ at the end of time step $t'=t^o+\Delta t$ and variables at its beginning t^o $(y^o_s, z^o_s, \ldots, T^o_i, T^o_i)$ - is obtained from system (14). Afterwards - differential equations (3,6,7) are transformed, using value $\int F_L dt$ and parabolic approximations of $y_s, z_s, \ldots$ Obtained system of linear algebraic equations is solved, using Gauss method ,and values $y'_s, z'_s, \ldots$ are determined. Then velocities $\dot{y}'_s, \dot{z}'_s, \ldots$, accelerations $\ddot{y}'_s, \ddot{z}'_s, \ldots$, forces F'_p, F'_L and functions T'_i, $\dot{T}'_i$ are calculated.

Necessity of regarding clearances between guides and shoes and cooperation of slide shoes and rollers causes great complications in numerical calculations. Every change in condition of contact between one of rollers or shoes and guide - changes its effective stiffness and coefficients in equation (8). Computer calculates moment of time (during time step Δt), when form of contact changes, and values of all dynamic variables in this moment (based on their parabolic approximations). Then - next time step begins.

5. RESULTS OF COMPUTER SIMMULATION

Complex computer program, based on presented mathematical model was elaborated. For mine hoisting system, having determined structural and process parameters computer calculates transverse dynamic displacements and accelerations of cage and forces in rollers and ropes. Simultaneously - separate part of program simmulates longitudinal vibrations, determining longitudinal displacements and accelerations of cage and stresses in ropes (with regarding influence of transverse vibration on longitudinal through variations of frictional forces between cage and guides). On fig 5. exemplary courses of transverse accelerations of cage and forces in rollers and are presented.

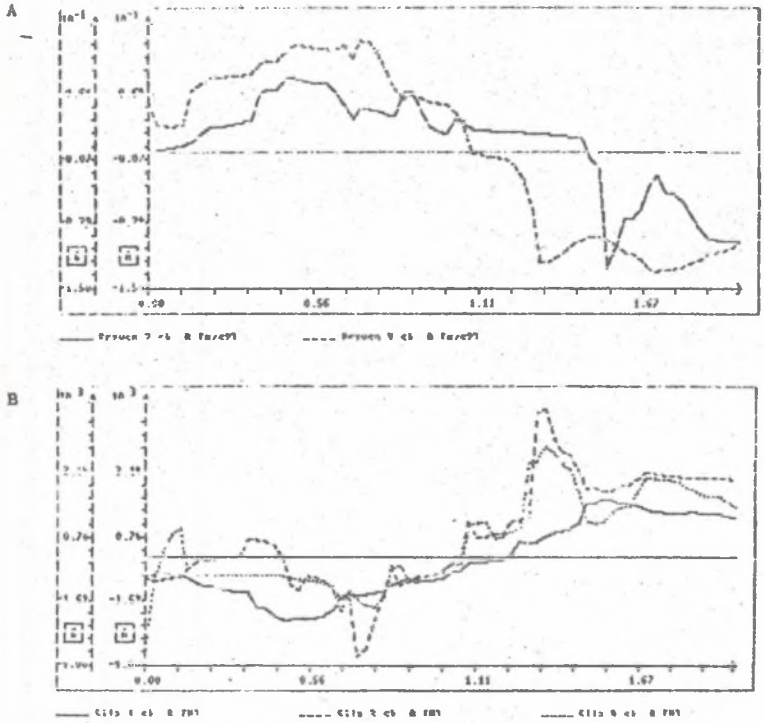

Fig. 5. Exemplary results of simmulation of transverse accelerations of cage (A) and forces between cage and shaft guides (B)

LIST OF PUBLICATIONS

1 Дворников В.И.,Ситов А.В.: Анализ решения уравнения динамики многоканатных шахтных установок.Вопросы експлуатации шахтных стационарных установок. Донецк 1985, 10-28.
2 A.Karge : Nowoczesne urządzenia wyciągowe. Katowice 1977
3 S.Kawulok : Wpływ podatności urządzenia wyciągowego na dynamikę jego prowadzenia w szybie. Przegląd Górniczy 1987,2
4 J.Vondrak i in.: Matematičke modelovani jizdy dopravni nadoby teżni jamou. Uhli 34,1986
5 D.Tejszerska,J.Wojnarowski : Modelowanie drgań naczynia wyciągowego podczas jego jazdy w szybie. Pr.IM i PKM Pol.śl. NB -385/RMT-4/87/88 - Cz 1,2. Gliwice 1987,1988

MODELOWANIE DRGAŃ POPRZECZNYCH UKŁADU WYCIĄGOWEGO SPRZĘŻONYCH Z DRGANIAMI WZDŁUŻNYMI

S t r e s z c z e n i e

W artykule przedstawiono rozbudowany model matematyczny drgań poprzecznych górniczych układów wyciągowych. W modelu uwzględniono sprzężenie drgań wzdłużnych i poprzecznych, podatność naczynia wydobywczego, powiązanie drgań naczynia i lin, zmiany sztywności podparcia naczynia (w układzie sprężystych prowadników i prowadnic tocznych i ślizgowych), a także tłumienie, niewyważenie i asymetrię układu. Omówiono przyjęte założenia, podano podstawowy układ równań różniczkowych (cząstkowych i zwyczajnych), przedstawiono metodę jego rozwiązywania oraz przykładowy rezultat symulacji numerycznej

МОДЕЛИРОВАНИЕ ПОПЕРЕЧНЫХ КОЛЕБАНИЙ В СВЯЗИ С ПРОДОЛЬНЫМИ В ШАХТНОЙ ПОДЕМНОЙ УСТАНОВКЕ

Р е з ю м е

В статье представлено сложную математическую модель поперечных колебаний шахтной подьемной установки. В модели учитывается связь продольхых и поперечных колебаний, деформации клети, колебания канатов, смены жесткости опоры клети на проводниках, а также демпфирование, асимметрия и неравновесие клети. Представлено принятые предположения, систему дифференциальных уравнений модели (частных и обыкновенных), метод ее решения и примерные результаты вычислений на ЭВМ.