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RELATIVE CONTROLLABILITY OF POSITIVE SYSTEMS WITH DELAYS

Summary. This article contains some definitions and basic theorems concerning the relative controllability of positive, linear, stationary, finite dimensional dynamical systems with multiple delays in control. The minimum energy control problem for these systems is also studied in the paper.

WZGLĘDNA STEROWALNOŚĆ DODATNICH UKŁADÓW LINIOWYCH Z OPÓŹNIENIAMI

Streszczenie. Niniejszy artykuł zawiera definicje oraz podstawowe twierdzenia dotyczące względnej sterowalności dodatnich, linowych, stacjonarnych, skończenie -wymiarowych układów dynamicznych ze stałymi, wielokrotnymi opóźnieniami w sterowaniu. W pracy bada się również zagadnienie sterowania z minimalną energią dla wymienionych układów.

1. Introduction

Controllability belongs to basic properties of various type dynamical systems. One of the kinds of dynamical systems with delays are linear systems with delays in control. In this paper positive, stationary, linear systems with constant, lumped delays in control will be presented. We say about positive systems, when every state variables, initial conditions, controls, reachable states and outputs are nonnegative.

Let us consider linear, stationary, finite dimensional dynamical systems with constant, multiple, lumped delays in control described by the following ordinary differential equation:

$$\dot{x}(t) = A\, x(t) + \sum_{i=0}^{M} B_i\, u(t - h_i), \qquad t \geq 0, \quad (1)$$

where

$x(t) \in \mathbf{R}^n$ is the state n-vector at the time instant t,

$u \in L^2_{loc}([0,\infty), \mathbf{R}^m)$ is the control,

A is $(n \times n)$–dimensional matrix with elements $a_{kj} \in \mathbf{R}$, $k,j = 1,2,...,n$,

B_i $(i = 0,1,2,..., M)$ are $(n \times m)$-dimensional matrices with elements $b_{ikj} \in \mathbf{R}$,

$k = 1,2,...,n$, $j = 1,2,...,m$,

$h_i \in \mathbf{R}$, $i = 0,1,2,...,M$ - constant delays in control satysfying the following inequalities:

$$0 = h_0 < h_1 < ... < h_i < ... < h_{M-1} < h_M.$$

Let $L^2([0, t], \mathbf{R}^m)$ denote Hilbert's space of square integrable functions defined in the time interval $[0, t]$ with values in the set $\mathbf{R}^m$. Any control $u \in L^2([0, t], \mathbf{R}^m)$ is called admissible control for dynamical system (1). For the given initial conditions $z(0)=\{x(0), u_0\} \in \mathbf{R}^n \times L^2([-h_M, 0], \mathbf{R}^m)$, where u_0 is the given in time interval $[-h_M, 0]$ initial function, and admissible control $u \in L^2([0, t_1], \mathbf{R}^m)$, for every $t \geq 0$ there exists a unique, absolutely continuous solution $x(t, z(0), u)$ of differential equation (1). This solution has the form [4]:

$$x(t, z(0), u) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} \sum_{i=0}^{M} B_i u(\tau - h_i)\, d\tau, \tag{2}$$

Let $\mathbf{R}^n_+$ and $\mathbf{R}^m_+$ denote positive orthants in spaces $\mathbf{R}^n$ and $\mathbf{R}^m$, respectively.

Definition 1.[1],[2],[3] The dynamical system (1) is said to be (internally) positive system if, for any given nonnegative initial conditions $z(0) \in \mathbf{R}^n_+ \times L^2([-h_M, 0], \mathbf{R}^m_+)$ and nonnegative admissible controls $u(t) \in \mathbf{R}^m_+$ the solution $x(t, z(0), u) \in \mathbf{R}^n_+$ and output $y(t) \in \mathbf{R}^p_+$, i.e. are nonnegative for every $t \geq 0$.

2. Relative controllability

For dynamical systems with delays we can formulate many various definitions of controllability, among them the most important are relative controllability and absolute controllability. In this paper we will consider the relative controllability of positive systems.

Definition 2. The positive dynamical system (1) is said to be relative controllable in the time interval $[0,t_1]$, if for any nonnegative, initial, complete state $z(0) \in \mathbf{R}^n_+ \times L^2([-h_M, 0], \mathbf{R}^m_+)$

and for any nonnegative vector $x_1 \in R^n_+$ there exist nonnegative admissible control $u \in L^2([0, t_1), R^m_+)$ such that the corresponding trajectory $x(t, z(0), u)$ of the dynamical system (1) satisfies the following condition:

$$x(t_1, z(0), u) = x_1.$$

In this section we shall formulate and prove sufficient conditions for relative controllability of dynamical system (1).

Let us fix final time $t_1 > 0$. Without loss of the generality, for the simplicity of notation, we may assume that there exists index $k \le M$, such that $t_1 - h_k = 0$. If such k does not exist, then we introduce additional delay h_k with control matrix $B_k = 0$. Then the solution (2) of dynamical system (1) in time t_1 has the following form [4],[6]

$$x(t_1, z(0), u) = e^{At_1} x(0) + \int_0^{t_1} e^{A(t_1-\tau)} \sum_{i=0}^{M} B_i u(\tau - h_i)\, d\tau = e^{At_1} x(0) + \sum_{i=0}^{M} \int_0^{t_1} e^{A(t_1-\tau)} B_i u(\tau - h_i)\, d\tau =$$

$$= e^{At_1} x(0) + \sum_{i=0}^{M} \int_{-h_i}^{t_1-h_i} e^{A(t_1-\tau-h_i)} B_i u(\tau)\, d\tau = e^{At_1} x(0) + \sum_{i=0}^{k} \int_{-h_i}^{0} e^{A(t_1-\tau-h_i)} B_i u_0(\tau)\, d\tau +$$

$$+ \sum_{i=k+1}^{M} \int_{-h_i}^{t_1-h_i} e^{A(t_1-\tau-h_i)} B_i u_0(\tau)\, d\tau + \sum_{i=0}^{k} \int_{0}^{t_1-h_i} e^{A(t_1-\tau-h_i)} B_i u(\tau)\, d\tau =$$

$$= e^{At_1} \Big(x(0) + \sum_{i=0}^{k} \int_{-h_i}^{0} e^{A(-\tau-h_i)} B_i u_0(\tau)\, d\tau + \sum_{i=k+1}^{M} \int_{-h_i}^{t_1-h_i} e^{A(-\tau-h_i)} B_i u_0(\tau)\, d\tau \Big) +$$

$$+ \sum_{i=0}^{k} \int_{0}^{t_1-h_i} e^{A(t_1-\tau-h_i)} B_i u(\tau)\, d\tau.$$

For reduction and simplification of the notation we introduce the following notations:

$$q(z(0)) = e^{At_1} \Big(x(0) + \sum_{i=0}^{k} \int_{-h_i}^{0} e^{A(-\tau-h_i)} B_i u_0(\tau)\, d\tau + \sum_{i=k+1}^{M} \int_{-h_i}^{t_1-h_i} e^{A(-\tau-h_i)} B_i u_0(\tau)\, d\tau \Big)$$

and

$$B_{t_1}(t) = \sum_{j=0}^{i} e^{A(-t-h_j)} B_j,\ t \in [t_1 - h_{i+1}, t_1 - h_i),\ i = 0, 1, \ldots, k-1. \tag{3}$$

Hence, $q(z(0)) \in \mathbf{R}^n$ is a constant vector depending on initial complete state $z(0)$ and a dynamical system's parameters only, and $B_{t_1}(t)$ is $(n \times m)$-dimensional matrix defined in $[0, t_1]$.

Lemma 1. [4] Let

$$\dot{y}(t) = A\, y(t) + B_{t_1}(t) u(t),\ t \in [0, t_1]$$

be the linear dynamical system without delays in control. Then

$$x(t, z(0), u) = y(t, q(z(0)), u),\ t \in [0, t_1],$$

where $x(t, z(0), u)$ is defined by formula (2).

Based on Lemma 1 we receive the following form of the solution of dynamical system (1) in time $t_1 > 0$

$$x(t_1, z(0), u) = x_1 = q(z(0)) + \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau)u(\tau)d\tau. \quad (4)$$

In order to investigate controllability of positive dynamical systems it is necessary to introduce the notion of a Metzler matrix [1],[2].

The matrix $A = [a_{ij}]$, $i,j = 1,2, \ldots ,n$ is called the Metzler matrix if $a_{ij} \geq 0$ for $i \neq j$. It is known [5], that $e^{At} \in \mathbf{R}_+^{n\times n}$, for every $t \geq 0$, if and only if A is a Metzler matrix. Moreover, [1],[2],[3] dynamical system without delays

$$\dot{x}(t) = A\,x(t) + B\,u(t),\ t \geq 0,$$

is positive if and only if A is Metzler matrix and $B \in \mathbf{R}_+^{n\times m}$. In that case, based on Lemma 1, we can ascertain that the dynamical system with delays of the form (1) is positive if A is a Metzler matrix and, for every $t_1 > 0$, $B_{t_1}(t) \in \mathbf{R}_+^{n\times m}$, for $t \in [0,t_1]$.

A matrix with nonnegative elements is called generalised permutation matrix, shortly GPM, if in each row and in each column only one element is positive and the remaining entries are zero [1],[2],[3]. Let us recall that in a permutation matrix all these elements are equal one.

Theorem 1. The positive dynamical system (1) is relatively controllable in $[0,t_1]$, $t_1 > 0$, if the matrix R_{t_1} of the form

$$R_{t_1} = \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau) B_{t_1}^T(\tau) e^{A^T(t_1-\tau)} d\tau, \quad (5)$$

where matrix $B_{t_1}(t)$ is given by the formula (3), is GPM and $[x_1 - q(z(0))] \in \mathbf{R}_+^n$. Moreover, admissible control vector $u(t)$, which steers the positive dynamical system (1) from initial complete state $z(0) = \{x(0), u_0\}$ to state $x_1 \in \mathbf{R}_+^n$ has the form:

$$u(t) = B_{t_1}^T(t) e^{A^T(t_1-t)} R_{t_1}^{-1} [x_1 - q(z(0))],\ t \in [0, t_1]. \quad (6)$$

P r o o f: First of all, let us observe, that if R_{t_1} is a GPM than $R_{t_1}^{-1} \in \mathbf{R}_+^{n\times n}$ [1],[2]. Since by assumption, the dynamical system (1) is positive, so A is a Metzler matrix and $B_j \in \mathbf{R}_+^{n\times m}$.

Therefore, $e^{At} \in \mathbf{R}_+^{n\times n}$ for $t \geq 0$ and $B_{t_1}(t) \in \mathbf{R}_+^{n\times m}$ for $t \geq 0$. Moreover, we assumed that $[x_1 - q(z(0))] \in \mathbf{R}_+^n$. Consequently, based on formula (6) we have $u(t) \in \mathbf{R}_+^m$, for $t \in [0, t_1]$.

Now, we will show that positive admissible control (6) steers positive dynamical system (1) from initial complete state $z(0) = \{x(0), u_0\}$ to the final state x_1 in time t_1. Substituting (6) into (4) we obtain

$$x(t_1, z(0), u) = q(z(0)) + \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau) B_{t_1}^T(\tau) e^{A^T(t_1-\tau)} R_{t_1}^{-1}[x_1 - q(z(0))]d\tau =$$

$$= [\int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau) B_{t_1}^T(\tau) e^{A^T(t_1-\tau)} d\tau] R_{t_1}^{-1}[x_1 - q(z(0))] =$$

$$= q(z(0)) + R_{t_1} R_{t_1}^{-1}[x_1 - q(z(0))] = x_1.$$

■

Corollary 1. Let M =1. The positive dynamical system (1) is relatively controllable in $[0,t_1]$, $t_1 > 0$, if the matrix R_{t_1} of the form

$$R_{t_1} = \int_0^{t_1} e^{A(t_1-2\tau)} B_0 B_0^T e^{A^T(t_1-2\tau)} d\tau, \tag{7}$$

is GPM. Then the control vector u(t), which steers positive dynamical system (1) from initial complete state $z(0) = \{x(0), u_0\}$ to final state $x_1 \in \mathbf{R}_+^n$ has the following form

$$u(t) = B_0^T(t) e^{A^T(t_1-2t)} R_{t_1}^{-1}[x_1 - q(z(0))],\ t \in [0, t_1]. \tag{8}$$

P r o o f: In the case when M = 1, based on the formula (3) the matrix $B_{t_1}(t) = e^{-At}B_0$. Hence our corollary follows.

■

Corollary 2. The positive dynamical system (1) is relatively controllable in $[0,t_1]$, $t_1 > 0$, if there occurs the following equality holds

$$\operatorname{rank} \sum_{i=0}^{k-1} \int_{t_1-h_{i+1}}^{t_1-h_i} [\sum_{j=0}^{i} e^{A(t_1-h_j-t)} B_j][\sum_{j=0}^{i} e^{A(t_1-h_j-t)} B_j]^T dt = n.$$

P r o o f: This corollary follows directly from Corollary 4.6.2 published in the monograph [4].

■

Corollary 3. Let M = 1. The positive dynamical system (1) is relatively controllable in $[0, t_1]$, $t_1 > 0$, if the following equality holds

$$\operatorname{rank} \int_{t_1-h_1}^{t_1} [e^{A(t_1-t)} B_0][e^{A(t_1-t)} B_0]^T dt = n.$$

P r o o f: This corollary follows directly from Corollary 2 and the formula (3).

■

Example 1. Let us consider dynamical system with delays in control (1) with

$$A=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, B_0=\begin{bmatrix}0 & 2\\ 1 & 0\end{bmatrix}, B_1=\begin{bmatrix}-1 & -2\\ 0 & 2\end{bmatrix}, B_2=\begin{bmatrix}3 & 2\\ -1 & 1\end{bmatrix}, h_1=1, h_2=2.$$

We shall test the relative controllability of the system in the interval [0, 1], for zero initial conditions $z(0)=\{0,0\}$.

First we calculate that $e^{At}=\begin{bmatrix}e^{t} & 0\\ 0 & e^{-t}\end{bmatrix}$ for $t\geq 0$ and, using the formula (3), we compute the matrix $B_{t_1}(t)$ for $t_1=1$:

$$B_{t_1}(t)=e^{-At}B_0=\begin{bmatrix}0 & 2e^{-t}\\ e^{t} & 0\end{bmatrix}.$$

Since A is a Metzler matrix and $B_{t_1}(t)\in \mathbf{R}_+^{n\times m}$ then the system is positive. Now, we shall check if matrix R_{t_1} given by formula (5) is GPM. From (5) it follows that

$$R_{t_1}=\int_0^1\begin{bmatrix}e^{1-\tau} & 0\\ 0 & e^{\tau-1}\end{bmatrix}\begin{bmatrix}0 & 2e^{-\tau}\\ e^{\tau} & 0\end{bmatrix}\begin{bmatrix}0 & e^{\tau}\\ 2e^{-\tau} & 0\end{bmatrix}\begin{bmatrix}e^{1-\tau} & 0\\ 0 & e^{\tau-1}\end{bmatrix}d\tau=\begin{bmatrix}e^2-e^{-2} & 0\\ 0 & \frac{1}{4}(e^2-e^{-2})\end{bmatrix}.$$

Thus R_{t_1} is a generalised permutation matrix (GPM). Therefore, using Theorem 1 we conclude that the system with delays in control is relatively controllable in time interval [0, 1].

3. Minimum energy control

If positive dynamical system (1) is relatively controllable in $[0,t_1]$, $t_1>0$, then among admissible, nonnegative controls $u\in L^2([0,t_1),\mathbf{R}_+^m)$, which steer this system from initial complete state $z(0)\in \mathbf{R}_+^n\times L^2([-h_M,0],\mathbf{R}_+^m)$ to final state $x_1\in\mathbf{R}_+^n$ we shall look for optimal control which minimizes a performance index J. Let us assume, that the performance index has the form:

$$J(u)=\int_0^{t_1}u^T(\tau)Qu(\tau)d\tau, \tag{9}$$

where $Q\in\mathbf{R}_+^{m\times m}$ is a constant, symmetric, positive defined matrix [1],[2],[3].

We will formulate the theorem, which gives the formula of optimal control steering positive dynamical system (1) from state z(0) to state x_1 and minimises performance index (9) We will give the formula of minimal value of performance index (9) corresponding with optimal control, too. To this end let's defined the matrix $W_Q=W_Q(t_1,Q)$ by the following formula

$$W_Q = W_Q(t_1, Q) = \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau) Q^{-1} B_{t_1}^T(\tau) e^{A^T(t_1-\tau)} d\tau. \qquad (10)$$

From Theorem 1 immediately follows that, if the dynamical system (1) is relatively controllable in $[0, t_1]$, then the matrix (10) is nonsingular.

Theorem 2. Let the positive system (1) be relatively controllable in $[0, t_1]$, $t_1 > 0$ and let $\tilde{u} \in L^2([0, t_1), \mathbf{R}_+^m)$ be any admissible control steering the dynamical system (1) from a given initial complete state $z(0) \in \mathbf{R}_+^n \times L^2([-h_M, 0], \mathbf{R}_+^m)$ to a given final state $x_1 \in \mathbf{R}_+^n$ in time t_1. If $Q^{-1} \in \mathbf{R}_+^{m\times m}$, $W_Q \in \mathbf{R}_+^{n\times n}$ and $[x_1 - q(z(0))] \in \mathbf{R}_+^n$, then the control $\hat{u} \in L^2([0, t_1), \mathbf{R}_+^m)$ given by formula

$$\hat{u}(t) = Q^{-1} B_{t_1}^T(t) e^{A^T(t_1-t)} W_Q^{-1}[x_1 - q(z(0))], \; t \in [0, t_1] \qquad (11)$$

is admissible control. The control $\hat{u}$ steers the positive dynamical system (1) from the initial complete state $z(0) \in \mathbf{R}_+^n \times L^2([-h_M, 0], \mathbf{R}_+^m)$ to the given final state $x_1 \in \mathbf{R}_+^n$ in time t_1 and minimises performance index (9), i.e.

$$J(\hat{u}) = \int_0^{t_1} \hat{u}^T(\tau) Q \hat{u}(\tau) d\tau \le \int_0^{t_1} (\tilde{u})^T(\tau) Q \tilde{u}(\tau) d\tau = J(\tilde{u}). \qquad (12)$$

Moreover, the minimal value of the performance index $J(\hat{u})$ is given by the formula:

$$J(\hat{u}) = [x_1 - q(z(0))]^T W_Q^{-1} [x_1 - q(z(0))]. \qquad (13)$$

P r o o f: Taking into account the assumption that $Q^{-1} \in \mathbf{R}_+^{m\times m}$, $W_Q \in \mathbf{R}_+^{n\times n}$ and $[x_1 - q(z(0))] \in \mathbf{R}_+^n$, by (11), we obtain positive control, so $\hat{u}(t) \in \mathbf{R}_+^m$ for every $t \in [0, t_1]$. We shall show that this admissible control steers the positive dynamical system (1) from the initial complete state $z(0) \in \mathbf{R}_+^n \times L^2([-h_M, 0], \mathbf{R}_+^m)$ to the given final state $x_1 \in \mathbf{R}_+^n$ in time t_1

Substituting (11) into (4) we get

$$x(t_1, 0, u) = x_1 = q(z(0)) + \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau) \hat{u}(\tau) d\tau =$$

$$= q(z(0)) + \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau) Q^{-1} B_{t_1}^T(\tau) e^{A^T(t_1-\tau)} W_Q^{-1}[x_1 - q(z(0))] d\tau =$$

$$= q(z(0)) + \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau) Q^{-1} B_{t_1}^T(\tau) e^{A^T(t_1-\tau)} d\tau W_Q^{-1}[x_1 - q(z(0))] = x_1$$

Since by the assumption, the control $\tilde{u}$ steers the positive dynamical system (1) from the initial complete state $z(0) \in \mathbf{R}_+^n \times L^2([-h_M, 0], \mathbf{R}_+^m)$ to the given final state $x_1 \in \mathbf{R}_+^n$ in time t_1 as well as the control $\hat{u}$, we obtain

$$x_1 = q(z(0)) + \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau)\tilde{u}(\tau)d\tau = q(z(0)) + \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau)\hat{u}(\tau)d\tau,$$

and

$$\int_0^{t_1} e^{A(t_1-\tau)} B_{t_1}(\tau)[\tilde{u}(\tau) - \hat{u}(\tau)]d\tau = 0.$$

From the last equality it follows that

$$\int_0^{t_1} [\tilde{u}(\tau) - \hat{u}(\tau)]^T B_{t_1}^T e^{A^T(t_1-\tau)} d\tau \; W_Q^{-1} [x_1 - q(z(0))] = 0,$$

and, after substituting the equality (11) we have

$$\int_0^{t_1} [\tilde{u}(\tau) - \hat{u}(\tau)]^T Q\hat{u}(\tau)d\tau = 0.$$

Now, it is easy to check, that

$$\int_0^{t_1} (\tilde{u})^T(\tau)Q\tilde{u}(\tau)d\tau = \int_0^{t_1} \hat{u}^T(\tau)Q\hat{u}(\tau)d\tau + \int_0^{t_1} [(\tilde{u})^T(\tau) - \hat{u}(\tau)]^T Q[\tilde{u}(\tau) - \hat{u}(\tau)]d\tau.$$

Since the second term of the sum on right hand side of the above equality is nonnegative, we directly obtain that

$$\int_0^{t_1} \hat{u}^T(\tau)Q\hat{u}(\tau)d\tau \le \int_0^{t_1} (u')^T(\tau)Qu'(\tau)d\tau,$$

and therefore we get the inequality (12).

In order to find the minimal value of the performance index (9) we substitute (11) into (9). Then we obtain

$$J(\hat{u}) = \int_0^{t_1} \hat{u}^T(\tau)Q\hat{u}(\tau)d\tau = [x_1 - q(z(0))]^T W_Q^{-1} \int_0^{t_1} e^{A(t_1-\tau)} B_{t_1} Q^{-1} B_{t_1}^T e^{A^T(t_1-\tau)} d\tau W_Q^{-1}[x_1 - q(z(0))] =$$
$$= [x_1 - q(z(0))]^T W_Q^{-1}[x_1 - q(z(0))],$$

and we get the formula (13).

■

Example 2. The positive dynamical system with delays in control with the matrices A, B_0, B_1, B_2 and the delays h_1 i h_2 as in Example 1, and performance index (9) with the matrix $Q = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ defined in the time interval [0, 1] are given.

In the Example 1 it was shown that this dynamical system is relatively controllable in time interval [0, 1], We shall find optimal control for this system, which steers the system from zero state $z(0) = \{0, 0\}$, so $q(z(0)) = 0$, to the finale state $x_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and minimises the performance index.

Since $e^{At} = \begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix}$ and $Q^{-1} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$ from (10) we have

$$W_Q = \int_0^1 \begin{bmatrix} e^{1-\tau} & 0 \\ 0 & e^{\tau-1} \end{bmatrix} \begin{bmatrix} 0 & 2e^{-\tau} \\ e^{\tau} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & e^{\tau} \\ 2e^{-\tau} & 0 \end{bmatrix} \begin{bmatrix} e^{1-\tau} & 0 \\ 0 & e^{\tau-1} \end{bmatrix} d\tau = \int_0^1 \begin{bmatrix} \frac{4}{3} e^{2-4\tau} & 0 \\ 0 & \frac{1}{3} e^{4\tau-2} \end{bmatrix} d\tau =$$

$$= \begin{bmatrix} \frac{1}{3}(e^2 - e^{-2}) & 0 \\ 0 & \frac{1}{12}(e^2 - e^{-2}) \end{bmatrix}.$$

We compute

$$Q^{-1} = \begin{bmatrix} [\frac{1}{3}(e^2 - e^{-2})]^{-1} & 0 \\ 0 & [\frac{1}{12}(e^2 - e^{-2})]^{-1} \end{bmatrix}$$

and from the formula (11)

$$\hat{u}(t) = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & e^t \\ 2e^{-t} & 0 \end{bmatrix} \begin{bmatrix} e^{1-t} & 0 \\ 0 & e^{t-1} \end{bmatrix} \begin{bmatrix} [\frac{1}{3}(e^2 - e^{-2})]^{-1} & 0 \\ 0 & [\frac{1}{12}(e^2 - e^{-2})]^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 12e^{2t-1}(e^2 - e^{-2})^{-1} \\ 2e^{1-2t}(e^2 - e^{-2})^{-1} \end{bmatrix}$$

The admissible control $\hat{u}$ is the optimal control. The minimal value of the performance index, in this case, is equal:

$$J(\hat{u}) = x_1^T W_Q^{-1} x_1 = [1 \quad 3] \begin{bmatrix} [\frac{1}{3}(e^2 - e^{-2})]^{-1} & 0 \\ 0 & [\frac{1}{12}(e^2 - e^{-2})]^{-1} \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \frac{111}{e^2 - e^{-2}}.$$

4. Summary

In this paper, the positive stationary dynamical systems with multiple delays in control of the form (1) have been considered. The definition of relative controllability of positive systems with delays is given and the criterions of relative controllability in the time interval $[0, t_1]$ for these systems have been established. Moreover, the minimum energy control problem for the system (1) has been formulated and solved. The results obtained in this paper are an extension for positive systems with delays in control published in [1], [2] and [3] for the positive dynamical systems without delays in control.

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Streszczenie

W artykule rozpatrzono dodatnie, liniowe, stacjonarne, skończenie-wymiarowe układy dynamiczne z opóźnieniami w sterowaniu opisane różniczkowym równaniem stanu postaci (1). Podano definicje względnej sterowalności przy ograniczeniach na sterowanie układu dodatniego (1). Sformułowano kryteria badania względnej sterowalności układów dodatnich z opóźnieniami w sterowaniu. Ponadto, rozwiązano problem sterowalności z minimalną energią dodatnich układów dynamicznych z opóźnieniami opisanych równaniem różniczkowym postaci (1). Sformułowano twierdzenie, które podaje wzór na sterowanie optymalne oraz minimalną wartość przyjętego wskaźnika jakości. W przedstawionych przykładach zilustrowano podane twierdzenia.