ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2003
Seria: AUTOMATYKA z. 138 Nr kol. 1581

Jerzy RESPONDEK
Politechnika Śląska

CONTROLLABILITY OF DYNAMICAL SYSTEMS WITH CONSTRAINED CONTROLS

Summary. In the paper presented the methodology of investigation of the controllability of an infinite dimensional systems with constrained controls. To this aim presented known method of transforming an infinite dimensional system to equivalent form in an infinite series of finite dimensional dynamical systems. Next known theorem on controllability of a finite dimensional systems with constrained controls applied to case of infinite dimensional control space. Received theorem applied to investigation of controllability of given example of infinite dimensional system.

STEROWALNOŚĆ UKŁADÓW DYNAMICZNYCH Z OGRANICZONYMI STEROWANIAMI

Streszczenie. W ramach pracy przedstawiono metodykę badania sterowalności nieskończenie wymiarowych układów dynamicznych przy ograniczonych sterowaniach. W tym celu zaprezentowano znany sposób przedstawienia układów nieskończenie wymiarowych w postaci nieskończonego ciągu układów skończenie wymiarowych. Następnie uogólniono znane twierdzenie o sterowalności skończenie wymiarowych układów dynamicznych na przypadek układów nieskończenie wymiarowych. Udowodnione twierdzenie zastosowano do zbadania warunków sterowalności przykładowego układu nieskończenie wymiarowego.

1. Introduction

The dynamical system is said to be controllable, if there exist such a control which carries out the dynamical system from any given initial state to any final state. Since in industrial conditions non-controllable dynamical system cannot be used in any automatic control system, this notion has great importance.

The basic criteria for investigation of the controllability apply only to finite-dimensional, unconstrained controllability. Unfortunately this assumptions fulfils only very limited class of real systems. Therefore in recent years had been performing investigations for controllability of broader class of dynamical systems.

This article is devoted to investigation of controllability infinite dimensional systems with non-negative controls. Considered system is described by parabolic type partial differential equation.

As physical examples of infinite dimensional systems may by mentioned heat exchangers, biological reactors etc.

This article applies known criteria of constrained controllability to infinite dimensional systems. Following this aim it is at first showed how to represent an infinite dimensional system by an infinite series of finite dimensional dynamical systems. Finally as an example one particular system is investigated.

2. Basic concepts

2.1. The description of dynamical system in form of the abstract differential equation

It is given continuous, stationary, infinite dimensional system described by the following abstract differential equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad t \geq 0 \tag{1}$$

where:

$x(t) \in X$ - Hilbert state space,

$u \in L^2_{loc}([0,\infty), U)$ - U- Hilbert control space,

$B \in L(U,X)$ - linear, bounded operator $B: U \to X$,

$A: X \supset D(A) \to X$ - linear, bounded or unbounded, self-adjoined operator with discrete spectrum,

$x(0) \in X$ - given initial condition in the state space.

It is generally assumed that operator A generates an analytic semigroup of linear bounded operators:

$$S(t): X \to X, t \geq 0$$

With these assumptions there exist an unique solution of the given abstract differential equation represented by the formula:

$$x[t, x(0), u] = S(t)x(0) + \int_0^t S(t-\tau)Bu(\tau)d\tau \quad t \geq 0$$

Comments:

1. If an operator A has a compact resolwent then has discrete spectrum.
2. If the spectrum of an operator A lies on a complex plane inside a angle with vertex $(0,\alpha)$, $\alpha>0$ with measure less than 180 degrees towards left half-plane than operator A generates an analitic semigroup. Particularly self-adjoint operators generates an analityc semigroup.

2.1.1. Particular case: finite dimensional control space

In this case the control space has form.

$$U = R^p$$

Thus operator B:

$$B = [b_1 \mid b_2 \mid ... \mid b_j \mid ... \mid b_p],\ b_j \in X,\ \ j = 1..p$$

and controls:

$$\boldsymbol{u(t) = [u_1(t), u_2(t), ..., u_p(t)]^T \in R^p}$$

where $u_j(t)$ j=1,2,...,p denote scalar controls.

With these assumptions given abstract differential equation receives the following form:

$$\dot{x}(t) = Ax(t) + \sum_{j=1}^{p} b_j u_j(t),\ \ t \geq 0 \tag{2}$$

And its solution has form:

$$x[t, x(0), u] = S(t)x(0) + \int_0^t S(t-\tau) \sum_{j=1}^{p} b_j u_j(\tau) d\tau,\ \ t \geq 0$$

2.2. Basic definitions and notions

It is necessary to introduce a few definitions of different kinds of controllability before formulating theorems on controllability conditions.

2.2.1. Definition 1

The attainable set $K_T(U)$ for abstract differential equation (1) in the time moment T from the zero initial state $x(0) \in X$ is defined as follows:

$$K_T(U) = \left\{ x : x[t, x(0), u] = \int_0^t S(t-\tau) Bu(\tau) d\tau,\ \ u \in L^2[(0,T), U] \right\}$$

2.2.2. Definition 2

Similarly is defined the attainable set $K_T(U)$ in the moment T from the zero initial state for the equation (2):

$$K_T(U)=\left\{x:x[t,x(0),u]=\int_0^t S(t-\tau)\sum_{j=1}^{p} b_j u_j(\tau)d\tau,\ \ u\in L^2[(0,T),U]\right\}$$

Also can be defined the attainable set:

$$K_\infty(U)=\bigcup_{T>0} K_T(U)$$

In the dynamical systems defined in infinite-dimensional spaces should be distinguished exact and approximate controllability.

2.2.3. Definition 3

The dynamical system (1) is said to be approximately controllable in finite time if

$$\overline{K_\infty(U)}=X$$

2.2.4. Definition 4

The dynamical system (1) is said to be exact controllable in finite time if

$$K_\infty(U)=X$$

2.2.5. Definition 5

The dynamical system (1) is said to be U-controllable to zero from given initial state in the state space, if for any initial state $x(t_0)=x_0$, there exist an admissible control $u\in L^2([0,\infty),U)$ such that the corresponding trajectory $x(t,x(t_0),u)$ of the dynamical system satisfies for some $t\ \in[t_0,\infty)$ the condition:

$$x(t_1,x(t_0),u)=0$$

2.2.6. Definition 6

The dynamical system (1) is said to be globally U-controllable to zero in finite time if it is U-controllable to zero from each initial state from state space X.

2.3. *Representation of an infinite dimensional system by an infinite series of finite dimensional dynamical systems*

It can be easy shown that the infinite dimensional dynamical system (2) is equivalent to the following infinite series of finite dimensional dynamical systems:

$$\dot{x}_i(t)=A_i x_i(t)+B_i u(t)\ \ i=1,2,3,... \tag{3}$$

Where A_i and B_i are the following matrixes:

$$A_i=diag[\lambda_i,...,\lambda_i]\ \ \dim A_i=m_i\times m_i$$

$$B_i = \begin{bmatrix} <b^1,\phi_{i1}>_X & \dots & <b^2,\phi_{i1}>_X & \dots & <b^p,\phi_{i1}>_X \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ <b^1,\phi_{ik}>_X & \dots & <b^2,\phi_{ik}>_X & \dots & <b^p,\phi_{ik}>_X \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ <b^1,\phi_{im_i}>_X & \dots & <b^2,\phi_{im_i}>_X & \dots & <b^p,\phi_{im_i}>_X \end{bmatrix}$$

where:

λ_i is the i[th] eigenvalue of the operator A,

ϕ_{ij} is the eigenfunction of the operator A, corresponding to its i[th] eigenvalue

m_i is the multiplicity of the i[th] eigenvalue, $m_i < \infty$

The vector x_i is given by:

$$x_i(t) = \left[c_{i1}(t) \quad ,..., \quad c_{ik}(t) \quad ,..., \quad c_{im_i}(t)\right]^T$$

where c_{ik} is the i[th] coefficient of the Fourier series of spectral representation for the element x in the state space X. The coefficients are explicit given by the inner product between element in the state space X and the appropriate eigenfunction $\phi_{ik}$of the operator A:

$$c_{ik} = < x, \phi_{ij} > \quad i = 1,2,3,... \quad k = 1,2,...,m_i$$

3. Basic criteria of controllability with constrained controls

3.1. Controllability of finite dimensional systems

It is given stationary finite dimensional system described by the following equations:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \ t \geq 0 \\ y(t) = Cx(t) + Du(t), \ t \geq 0 \end{cases} \tag{4}$$

where A,B,C,D are constants matrices with dimensions respectively n×n, n×m, p×n, p×m.

3.1.1. Theorem 1 [5]

The dynamical system (4) is globally U-controllable to zero if and only if the following conditions are satisfied simultaneously :

(1) There exists a $w \in U$ such that $Bw=0$

(2) The convex hull CH(U) has a nonempty interior in the space R^p.

(3) rank$[B|AB|A^2B,...,A^{n-1}B]$=n

(4) There is no real eigenvector $v \in R^n$ of matrix A^T satisfying $v^T Bw \leq 0$ for all $w \in U$

(5) No eigenvalue of matrix A has a positive real part

3.2. Controllability of infinite dimensional systems

Since an infinite dimensional system can be rewritten in equivalent form by an infinite series of finite dimensional dynamical systems, for each subsystem can be applied theorem (1), which gives conditions for the controllability of finite dimensional system with constrained controls. In this way we can obtain conditions for controllability of infinite dimensional dynamic systems with constrained controls.

Using the theorem (1) for the series of finite dimensional dynamical systems (3) yields the following theorem:

3.2.1. Theorem 2

The infinite dimensional system (1) is globally approximately controllable to zero if and only if the following conditions are satisfied simultaneously:

(1) There exists a $w \in U$ such that $B_i w=0$ for every i=1,2,3,...

(2) The convex hull CH(U) has a nonempty interior in the space R^p.

(3) rank[B_i]=m_i , for every i=1,2,3,...

(4) There is no real eigenvector $v_i \in R^{m}$ of matrix A_i^T satisfying $v_i^T B_i w \leq 0$ for all $w \in U$, for every i=1,2,3,...

(5) No eigenvalue of A_i has a positive real part, for every i=1,2,3,..

3.2.2. Proof

The proof bases on applying theorem1 to every subsystem of the infinite series (3).

- The condition (2) can be rewritten in the same form, because the control space of each subsystem remains the same set by assumption

- The conditions (1), (4), (5) follows immediately from applying the theorem 1 for every of finite dimensional subsystems in the form (3)

- The condition (3) of the theorem (1) after applying to the ith subsystem in the form (3) receives form:

$$rank[B_i \mid A_i B_i \mid A_i^2 B_i \mid ... \mid A_i^{m-1} B_i] = m_i \quad i = 1,2,3,... \tag{5}$$

Let's notice that the lth $l \in Z_+$ power of the matrix A_i has form:

$$A_i^l = diag[\lambda^l{}_1,...,\lambda^l{}_i] \quad \dim A^l{}_i = m_i \times m_i \quad i = 1,2,3,...$$

Now let's calculate the product $A_i^l B_i$:

$$A_i^l B_i = \begin{bmatrix} \lambda_i^l < b^1, \phi_{i1} >_X & \cdots & \lambda_i^l < b^2, \phi_{i1} >_X & \cdots & \lambda_i^l < b^p, \phi_{i1} >_X \\ \vdots & \ddots & \vdots & & \vdots \\ \lambda_i^l < b^1, \phi_{ik} >_X & \cdots & \lambda_i^l < b^2, \phi_{ik} >_X & \cdots & \lambda_i^l < b^p, \phi_{ik} >_X \\ \vdots & & \ddots \vdots & \ddots & \vdots \\ \lambda_i^l < b^1, \phi_{im_i} >_X & \cdots & \lambda_i^l < b^2, \phi_{im_i} >_X & \cdots & \lambda_i^l < b^p, \phi_{im_i} >_X \end{bmatrix} = \lambda_i^l B_i \quad i=1,2,3,...$$

Using above equality the equation (5) can be expressed in the following form:

$$rank[B_i \mid \lambda_i B_i \mid \lambda_i^2 B_i \mid ... \mid \lambda_i^{m_i-1} B_i] = m_i \quad i = 1,2,3,... \tag{6}$$

It can be noticed that in the block matrix $[B_i \mid \lambda_i B_i \mid \lambda_i^2 B_i \mid ... \mid \lambda_i^{m_i-1} B_i]$ every kth column is linearly dependent on the column number (k+pl) for every $l \in Z$, such as $1 \leq k + pl \leq pm_i$. Taking into account this fact according to the definition of the rank of the matrix the condition (6) is equivalent to:

$$rank[B_i] = m_i \, , \; i=1,2,3,...$$

Q.E.D.

4. Example

4.1. Problem Statement

Let us consider an infinite dimensional dynamical system, given by the following state linear partial differential equation:

$$\frac{\partial x(z,t)}{\partial t} = \frac{\partial^2 x(z,t)}{\partial z^2} + b_1(z)u_1(t) + b_2(z)u_2(t) + b_3(z)u_3(t) \tag{7}$$

where:

z - the spatial variable

$$b_i(z) = C_i z^n, \; n \in Z_+, \; i = 1,2,3$$

Let the spatial domain of the equation be a following segment:

$$D = [0, a]$$

Let the time domain be the non-negative half line:

$$0 \leq t \leq \infty$$

Let us assume zero initial conditions Dirichlet type:

$$x(z,t) = 0\big|_{z\in\Gamma}$$

where Γ is the boundary of the D domain.

Additionally let us assume non-negative controls:

$$u_i(t) \geq 0 \quad i = 1,2,3$$

4.2. Problem analysis

First of all let us transform given partial differential equation to abstract differential equation of form (1). To achieve this aim will be necessary the following operators:

- operator A

Let the operator A would be defined by the following formula:

$$Ax(z) = \frac{\partial^2 x(z,t)}{\partial z^2}, \quad x(z) \in D(A)$$

Domain of the operator A:

$$D(A) = \left\{ x(z) \in L^2(D) : Ax(z) \in L^2(D),\ x(z,t) = 0\big|_{z\in\Gamma} \right\}$$

- operator B

Respectively the definition formula and the domain of the operator B has form:

$$B = [b_1(z)\, b_2(z)\, b_3(z)]$$

$$B : R^3 \to L^2(D)$$

In can be shown [1] that the eigenvalues of the operator A have form:

$$\lambda_i = -\frac{\pi^2 i^2}{a^2} \quad i = 1,2,3,\ldots$$

and eigenfunctions:

$$\phi_i(z) = C \sin\frac{\pi i z}{a} \quad i = 1,2,3,\ldots$$

Using defined above operators A and B given partial differential equation (7) can be rewritten in the form (1) as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad t \geq 0 \tag{8}$$

Abstract differential equation (8) can be transformed into infinite series of finite dimensional dynamical systems of the form (3), by defining the appropriate matrices.

Since operator A has single eigenvalues, the state matrix in considered example is a scalar:

$$A_i = \lambda_i \quad i = 1,2,3,\ldots$$

The input vector has form p=3, m_i=1:

$$B_i = [< C_1 z^n, \phi_i > \ < C_2 z^n, \phi_i > < C_3 z^n, \phi_i >]$$

Using shown above state and input matrix abstract differential (8) equation can be rewritten in the following form of infinite series of finite dimensional dynamical systems:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u(t) \quad i = 1,2,3,...$$

Now the investigation of the controllability with constrains of given dynamical system can by done by using theorem (2).

Let's check in sequence conditions of theorem (2).

- Condition 1

Lets substitute $w = [0\ 0\ 0]^T$. Then $\underset{i=1,2,3,...}{\wedge} B_i w = 0$ and condition is fulfilled.

- Condition 2

Since $u_i(t) \geq 0 \ \ for \ \ i = 1,2,3$ the convex hull has a nonempty interior in R^3 and condition holds true.

- Condition 3

Checking this condition involves the following lemma:

- Lemma 1

The integral:

$$\int_0^{i\pi} z^n \sin z dz, \ i \in Z^+$$

for $n \in N$ vanishes only when n is equal zero; N- set of natural numbers.

Proof

Case A : i is an even number.

The considered integral can be rewritten in form:

$$\int_0^{i\pi} z^n \sin z dz = \sum_{j=0}^{\frac{i}{2}-1} \int_{2j\pi}^{(2j+2)\pi} z^n \sin z dz \tag{9}$$

The integral inside the sum can be rewritten by appropriate substitutions in form:

$$\int_{2j\pi}^{(2j+2)\pi} z^n \sin z dz = \int_{2j\pi}^{(2j+1)\pi} [z^n - (z+\pi)^n] \sin z dz$$

The following inequality is obviously satisfied:

$$\underset{i,j\geq 0}{\wedge} z^n + \pi^n \leq (z+\pi)^n \Rightarrow \underset{i,j\geq 0}{\wedge} z^n - (z+\pi)^n \leq -\pi^n$$

furthermore, in the range $[2j\pi,(2j+1)\pi],\ j \in Z$ the function sin(z) is non-negative. So the following inequality is also satisfied:

$$\int_{2j\pi}^{(2j+2)\pi} z^n \sin z dz \le -\pi^n \int_{2j\pi}^{(2j+1)\pi} z^n \sin z dz = -2\pi^n < 0$$

So every integral in sum (9) is negative, thus the considered integral is not equal zero.

Case B : i is an odd number.

The integral can be similarly rewritten as:

$$\int_0^{i\pi} z^n \sin z dz = \int_0^{\pi} z^n \sin z dz + \sum_{j=1}^{\frac{i}{2}-1} \int_{(2j-1)\pi}^{(2j+1)\pi} z^n \sin z dz \qquad (10)$$

The first integral is obviously positive. After similar transformations the following inequality it can be easily proofed:

$$\int_{(2j-1)\pi}^{(2j+1)\pi} z^n \sin z dz \ge 2\pi^n > 0$$

So the sum (10) is positive and not equal zero.

Q.E.D.

Now let's return to verification of the third condition of the theorem (2). The l[th] element of the matrix B_i can be expanded using the form of the inner product in the infinite dimensional Hilbert state space X as follows (l=1,2,3):

$$< C_l z^n, \phi_i >= CC_1 \int_0^a z^n \sin \frac{\pi i z}{a} dz$$

The integral it the last formula by performing the substitution $z = \frac{a}{\pi i} t$ can be expressed by the following formula:

$$< C_l z^n, \phi_i >= CC_1 \left(\frac{a}{\pi i} \right)^{n+1} \int_0^{\pi i} t^n \sin t dt$$

Received integral is, by the lemma (1), not equal zero for n>0. So it can be stated that:

$$\left\langle C_l z^n, \phi_i \right\rangle \ne 0 \quad for\ n > 0,\ \ l = 1,2,3$$

and condition (3) of the theorem (2) is satisfied for n>0.

- Condition 4

First let's calculate the matrix A_l. Considering that $\dim A_i = m_i \times m_i$, in this example the matrix A_i is a scalar equal:

$$A_i = \lambda_i$$

Taking into account the fact that the eigenvectors are determined with the accuracy to the direction and considered matrix A_i in this example is degenerated to scalar, as its eigenvector can be taken any real number except for zero.

Additionally considering that the controls are non-negative this condition reduces to requirement so that the expression $B_i w$ (i=1,2,3,...) in the admissible control space had values of both signs, because only in this case does not exist eigenvector v_i of the matrix A_i fulfilling condition:

$$\bigwedge_{w\in U,\, i=1,2,3,\dots} v_i^T B_i w \le 0$$

Now let's calculate the expression $B_i w$. At first let's notice that the inner products in the input matrix B_i cannot be expressed explicitly by finite combination of elementary functions, but considering the form of the integrand it can be expressed in the following way:

$$< C_l z^n, \phi_i >= C_l f(a,i,n), \quad l = 1,2,3 \quad i = 1,2,3,\dots$$

Using last formula the term $B_i w$ can be expressed as follows:

$$B_i w = f(a,i,n)(C_1 u_1 + C_2 u_2 + C_3 u_3)$$

Now let's check when the last term has values of both signs in the admissible control space. Considering that the controls are non-negative it comes true if and only if there exist a pair of constants C_q , C_r , such that:

$$\bigvee_{\substack{q,r\in\{1,2,3\}\\ q\neq r}} C_q C_r < 0$$

- Condition 5

Condition 5 is satisfied, considering the formula for the eigenvalues of the operator A:

$$\lambda_i = -\frac{\pi^2 i^2}{a^2} \quad i = 1,2,3,\dots$$

- Outcome of the investigation.

Considered dynamical system (7) is globally controllable to zero if and only if when the following conditions are satisfied simultaneously:

$$\bigvee_{\substack{q,r\in\{1,2,3\}\\ q\neq r}} C_q C_r < 0$$

$$n > 0$$

5. Summary

The paper presents the methodology for determining the controllability conditions of infinite dimensional dynamical systems with constrained controls. At first are presented basic definitions and theorems on constrained controllability. Next is presented the methodology of transforming the infinite dimensional system into infinite series of finite dimensional systems, then known theorem on constrained controllability is adapted to this case. Finally received theorem on controllability of infinite dimensional systems is applied to given example system, in the form of partial differential equation parabolic-like type. On this example are showed all the stages of determining the controllability conditions of such system. At first is shown the selection of the proper operators. Then the transformation of the given system into the infinite series of finite dimensional systems is shown. Next are testified the conditions of the proper theorem on constrained controllability. Received outcome, for considered example, has very compact form.

REFERENCES

1. Butkowskij A. G.: Charakteristiki sistiem s raspriedieliennymy parametrami. sprawoczno-je posobie, „Nauka". Gławnaja Redakcja fizyko-matematiczeskoj literatury, Moskwa 1979.
2. Fichtencholtz M.: Kurs differiencialnowo i integralnowo iscislenia., "Nauka". Gławnaja Redakcja fizyko-matematiczeskoj literatury, Moskwa 1979.
3. Kaczorek T.: Teoria sterowania. PWN, Warszawa 1977.
4. Kaczorek T.: Wektory i macierze w automatyce i elektrotechnice. WNT, Warszawa 1998.
5. Klamka J.: Controllability of dynamical systems. Kluwer, Dordrecht 1991.
6. Kudrewicz J.: Analiza funkcjonalna dla automatyków i elektroników. PWN, Warszawa 1976.
7. Mitkowski W.: Stabilizacja systemów dynamicznych. WNT, Warszawa 1991.
8. Mostowski A., Stark M.: Elementy algebry wyższej. PWN, Warszawa 1972.
9. Sneddon N.: Równania różniczkowe cząstkowe. PWN, Warszawa 1962.
10. Tanabe H.: Equations of evolution. Pitman, London 1979.

Recenzent: Prof.dr hab.inż. Wojciech MITKOWSKI

Wpłynęło do Redakcji dnia 30 lipca 2002 r.

Streszczenie

Artykuł prezentuje analizę sterowalności nieskończenie wymiarowych układów dynamicznych przy uwzględnieniu ograniczeń na sterowania w postaci nieujemnych sterowań. Zaproponowane podejście bazuje na metodach analizy funkcjonalnej.

Na początku przedstawiono ogólny model układów dynamicznych w postaci abstrakcyjnego równania różniczkowego i wprowadzono podstawowe definicje sterowalności. Następnie przedstawiono metodę zamiany układu nieskończenie wymiarowego na nieskończony ciąg układów skończenie wymiarowych, a dalej pokazano metody badania sterowalności układów dynamicznych z ograniczeniami. W tym celu najpierw przedstawiono znane twierdzenie (Klamka [1]) podające warunki konieczne i wystarczające globalnej sterowalności do zera stacjonarnego, liniowego i skończenie wymiarowego układu dynamicznego. Na podstawie tego twierdzenia sformułowano twierdzenie podające warunki konieczne i wystarczające globalnej aproksymacyjnej sterowalności układów nieskończenie wymiarowych o samosprzężonym operatorze stanu. Dowód twierdzenia wykorzystuje twierdzenie o sterowalności skończenie wymiarowego układu dynamicznego zastosowanego do układu nieskończenie wymiarowego przedstawionego w równoważnej postaci nieskończonego ciągu układów skończenie wymiarowych.

W końcu jest podany przykład zastosowania uzyskanego twierdzenia do badania globalnej U-sterowalności przykładowego układu danego liniowym równaniem cząstkowym typu parabolicznego. Rozwiązanie zaczyna się od doboru odpowiednich operatorów, sprowadzających dane równanie do abstrakcyjnego równania różniczkowego. Następnie po jego zamianie na nieskończony ciąg układów skończenie wymiarowych badane są kolejno warunki twierdzenia o U-sterowalności układów nieskończenie wymiarowych.