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Jerzy RESPONDEK
Politechnika Śląska

THE ANALYSIS OF CONTROLLABILITY OF A MECHANICAL SYSTEM WITH DISTRIBUTED PARAMETERS

Summary. The paper is devoted to the verification of the controllability conditions of flexible mechanical system with distributed parameters. Following this aim the functional analysis methods and results from previous papers are applied. At first presented the system state equation, in the form of the abstract differential equation, and studied the properties of the system's eigenfunctions. Next the state equation was transformed into infinite series of finite first order dimensional systems. Describing the system in this form enabled use of proper theorems on controllability without constrains and U-controllability for an examined system. Finally received results were used in examination of controllability some particular control forces.

ANALIZA STEROWALNOŚCI UKŁADU MECHANICZNEGO O PARAMETRCH ROZŁOŻONYCH

Streszczenie. W ramach pracy zbadano warunki sterowalności elastycznego układu mechanicznego o parametrach rozłożonych. Analizę przeprowadzono opierając się na metodach analizy funkcjonalnej i wynikach uzyskanych w poprzednich pracach. Na początku przedstawiono wyjściowe równanie stanu układu w postaci abstrakcyjnego równania różniczkowego i zbadano własności funkcji własnych układu. Następnie przekształcono go do postaci dwóch nieskończonych ciągów układów skończenie wymiarowych pierwszego rzędu. Przedstawienie układu w tej postaci pozwoliło na zastosowanie odpowiednich twierdzeń podających warunki sterowalności bez ograniczeń i U-sterowalności badanego układu. W końcu pracy uzyskane warunki zastosowano do zbadania sterowalności dla kilku konkretnych wymuszeń.

1. Basic Concepts

Recently in the modern mechanical constructions it can be noticed decreasing density because of appearing new light materials. Decreasing mass of the constructions caused improvement of the dynamical properties of the mechanical systems. Unfortunately simultaneously, because of the elastically deformations, the resonance vibrations was observed. So the key significance turned out the damping of that vibrations. Following this aim necessary is the mathematical model considering the distributed parameters of the investigated mechanical system.

1.1. Mathematical Model

Let us consider a mechanical system described by the following linear partial differential equation:

$$\frac{\partial^2 u(x,t)}{\partial t^2}+\frac{\partial^4 u(x,t)}{\partial x^4}+2\alpha\frac{\partial^5 u(x,t)}{\partial x^4 \partial t}-2\beta\frac{\partial^3 u(x,t)}{\partial x^2 \partial t}=\sum_{i=1}^{p} b_i(x)h_i(t) \tag{1}$$

where:

$x \in (0,l),\ t>0$

$\alpha>0,\ \beta\in[0,1)$

with initial conditions:

$u(x,0)=u_0(x),\ x\in(0,l)$

$\frac{\partial u(x,0)}{\partial t}=u_1(x),\ x\in(0,l)$

and boundary conditions:

$u(0,t)=\frac{\partial u(0,t)}{\partial x}=0,\ t>0$

$u(l,t)=\frac{\partial u(l,t)}{\partial x}=0,\ t>0$

The function u(x,t) is equal to the movement of the considered elastic beam in the Y axis direction in the time moment t>0 and in the point x (0<x<l). The firsts two terms in the equation (1) are the only terms taking into account for the ideally springy elastic beam. The remaining two terms are modelling the phenomenon of the internal friction. More detailed description of these terms and the phenomenon they are describing can be found in the papers [3, 7, 13]. The look of the system can be seen on the fig. 1.

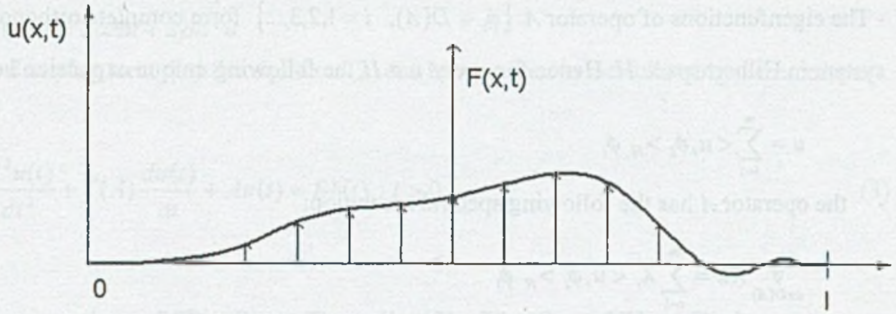

Fig. 1.
Rys. 1.

1.2. The Definition of The Differential Operator "A"

Let us define linear unbounded differential operator $A: D(A) \subset H \to H$ [18] in the following way:

$$Au(x) = \frac{\partial^4 u(x)}{\partial x^4}, \quad u \in D(A)$$

$$D(A) = \left\{ u(x) \in H^2(0,l) : \int_0^l u^2(x)dx < \infty, \; u(0) = u'(0) = 0, \; u(l) = u'(l) = 0 \right\}$$

where $H = L^2(0,l)$ is a Hilbert space of functions integrable with square in the interval [0,l].

It can be proved that the operator A has the following properties [18]:

- Operator A has only purely discrete point spectrum consisting entirely of distinct real positive eigenvalues λ_i

. each eigenvalue can be calculated by formula $\lambda_i = \eta_i^4$ where η_i is the positive root of the following equation:

$$\cosh(\eta l)\cos(\eta l) - 1 = 0$$

- $\lambda_i \neq \lambda_j \;\; for \;\; i \neq j, \;\; \lambda_i < \lambda_j \;\; for \;\; i < j, \;\; \lim_{i\to\infty} \lambda_i = \infty$

- the operator A is self-adjoin and positive defined

- there exists an inverse operator A^{-1} and it is bounded

- for each eigenvalue there exists a corresponding eigenfunction of the operator A:

$$\phi_i(x) = k\{\cos(\eta_i x) - \cosh(\eta_i x) + d_i(x)[\sin(\eta_i x) - \sinh(\eta_i x)]\}$$

$$d_i(x) = \frac{\cos(\eta_i x) - \cosh(\eta_i x)}{\sin(\eta_i x) - \sinh(\eta_i x)}$$

$$i = 1,2,3,..., \;\; x \in (0,l), \;\; k > 0$$

- The eigenfunctions of operator A $\{\phi_i \in D(A),\ \ i=1,2,3,...\}$ form complete orthonormal system in Hilbert space H. Hence for every $u \in H$ the following unique expansion holds true:

$$u=\sum_{i=1}^{\infty} < u,\phi_i >_H \phi_i$$

- the operator A has the following spectral resolution:

$$\underset{u\in D(A)}{\forall} Au=\sum_{i=1}^{\infty}\lambda_i < u,\phi_i >_H \phi_i$$

$$D(A)=\left\{u\in H:\sum_{i=1}^{\infty}\lambda_i^{\,2}\left|< u,\phi_i >_H\right|^2 <\infty\right\}$$

- The fractional power of operator A is defined as follows [15]:

$$\underset{u\in D(A^{\beta}),\beta\in(0,1)}{\forall} A^{\beta}u=\sum_{i=1}^{\infty}\lambda_i^{\,\beta} < u,\phi_i >_H \phi_i$$

$$D(A^{\beta})=\left\{u\in H:\sum_{i=1}^{\infty}\lambda_i^{\,2\beta}\left|< u,\phi_i >_H\right|^2 <\infty\right\}$$

particularly:

$$A^{\frac{1}{2}}u=-\frac{\partial^2 u}{\partial x^2}$$

1.3. State Equation

Basing on the definition and properties of the operator A partial differential equation (1) can be expressed in form of the linear abstract ordinary differential equation in the Hilbert space H:

$$\frac{d^2u(t)}{dt^2}+2\alpha A\frac{du(t)}{dt}+2\beta A^{\frac{1}{2}}\frac{du(t)}{dt}+Au(t)=Bh(t),\ t>0 \tag{2}$$

where: $\dfrac{d^2u(t)}{dt^2},\dfrac{du(t)}{dt},u(t)\in H$

and operator B:

$$B=\lfloor b_1\,|\,b_2\,|\,...\,|\,b_j\,|\,...\,|\,b_p\rfloor\quad b_j\in H,\ j=1..p$$

and controls:

$$h(t)=\left[h_1(t),h_2(t),...,h_p(t)\right]\in R^p$$

where $h_j(t)$ $j=1,2,...,p$ denote scalar controls.

Let us define so called damping term by the following formula:

$$f(A)u = 2\alpha Au + 2\beta A^{\frac{1}{2}}u$$

Using the damping term *f(A)* the equation (2) can be rewritten in the following more compact form:

$$\frac{d^2u(t)}{dt^2} + f(A)\frac{du(t)}{dt} + Au(t) = Bh(t), \quad t > 0 \tag{3}$$

2. Representation of The State Equation By The Set of First Order Equations

In [17] it was proved that the second order equation (3) is equivalent to the following set of first order differential equations:

$$\frac{d}{dt}\begin{bmatrix}\xi(t)\\ \mu(t)\end{bmatrix} = \begin{bmatrix}A^{+} & 0\\ 0 & A^{-}\end{bmatrix}\begin{bmatrix}\xi(t)\\ \mu(t)\end{bmatrix} + \begin{bmatrix}g(A)^{-1}B\\ -g(A)^{-1}B\end{bmatrix}h(t) \tag{4}$$

where the linear operators A^{+} and A^{-} are defined by the following formulas:

$$A^{\pm} = -\alpha A - \beta A^{\frac{1}{2}} \pm g(A)$$

$$A^{\pm} : H \supset D(A) \rightarrow H, D(A^{\pm}) = D(A)$$

The operator *g(A)* is defined as follows [3]:

$$\underset{u \in D(g(A))}{\forall} \; g(A)u = \sum_{i=1}^{\infty} g(\lambda_i) < u, \phi_i >_H \phi_i$$

$$D(g(A)) = \left\{ u \in H : \sum_{i=1}^{\infty} g^2(\lambda_i) |< u, \phi_i >_H|^2 < \infty \right\}$$

with domain *D(g(A))=D(A)* [13].

Appearing in the above formulas function $g(\lambda_i) : R \rightarrow C$ is defined as follows:

$$g(\lambda_i) = \sqrt{\left(\alpha\lambda_i + \beta\lambda_i^{\frac{1}{2}}\right)^2 - \lambda_i}, \quad \lambda_i > 0$$

Therefore, operator Ω of the system (4) is given by the equality:

$$\Omega = \begin{bmatrix}A^{+} & 0\\ 0 & A^{-}\end{bmatrix}$$

and moreover has purely discrete point spectrum $\sigma(\Omega)$ of the following form:

$$\sigma(\Omega)=\{s_i^{\pm}, i\in N\}\cup\left\{\frac{1}{2\alpha_0}\right\}$$

where $s_i^{\pm}$ are distinct eigenvalues of Ω given by the formula:

$$s_i^{\pm}=-\alpha_0\lambda_i-\beta\lambda_i^{\frac{1}{2}}\pm g(\lambda_i)$$

3. Representation of The Set of The Infinite Dimensional Equations By The Infinite Series of Finite Dimensional Systems

As it was showed in [12] the set of differential equations (4) can be represented by the infinite series of finite dimensional systems as well.

Taking into account the fact that the differential state equation (3) of the considered mechanical system has only single eigenvalues (m_i=1) the infinite series receives the following simple form:

$$\begin{cases}\dfrac{d\xi_i(t)}{dt}=s_i^{+}\xi_i(t)+B_i^{+}h(t)\\[2mm] \dfrac{d\mu_i(t)}{dt}=s_i^{-}\mu_i(t)+B_i^{-}h(t)\end{cases}$$

where:

$$B_i^{+}=\left[g^{-1}(\lambda_i)b_i^1\ldots g^{-1}(\lambda_i)b_i^k\ldots g^{-1}(\lambda_i)b_i^p\right]$$

$$B_i^{-}=-B_i^{+}$$

Furthermore:

$$b_i^k=<b_k,\phi_i>_H \quad i=1,2,3,\ldots,\quad k=1,2,\ldots,p$$

and $\xi_i(t)$, $\mu_i(t)$ denotes the i^{th} coefficient of the Fourier series of spectral representation for the element x in the state space H. The coefficients are explicitly given by the inner product of elements $\xi(t)$ and $\mu(t)$ in the state space H and the appropriate eigenfunction ϕ_i of the operator A:

$$\xi_i(t)=<\xi(t),\phi_i>_H,\qquad \mu_i(t)=<\mu(t),\phi_i>_H,\ i=1,2,3,\ldots$$

4. The Analysis of Controllability

4.1. Basic Concepts

It is given a stationary, finite dimensional system, described by the following equations:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + B_0 u(t), & t \geq 0 \\ y(t) = C_0 x(t) + D_0 u(t), & t \geq 0 \end{cases} \tag{5}$$

where A_0, B_0, C_0, D_0 are constants matrices with dimensions $n \times n$, $n \times m$, $p \times n$, $p \times m$, respectively.

4.1.1. Controllability Without Constrains

Theorem 1 [8]

The dynamical system (5) is controllable if and only if:

$$rank[B_0|A_0B_0|A_0^2B_0|...|A_0^{n-1}B_0]=n$$

4.1.2. Controllability With Constrained Controls

Theorem 2 [12]

The infinite dimensional distributed parameter dynamical system (1) is globally approximately U-controllable to zero if and only if the following conditions are satisfied simultaneously:

(1) There exists a $w \in U$ such that $B_i^+ w=0$ for every $i=1,2,3,...$

(2) The convex hull CH(U) has a nonempty interior in the space R^p.

(3) rank[B_i'']=m_i, for every i=1,2,3,...

(4) There is no real eigenvector $v_i \in R^{m_i}$ of matrixes A_i^+, A_i^- satisfying $v_i^T B_i^+ w \leq 0$ for all $w \in U$, for every $i=1,2,3,...$

where:

$$B_i'' = \begin{bmatrix} b_{i1}^1 & \cdots & b_{i1}^k & \cdots & b_{i1}^p \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{il}^1 & \cdots & b_{il}^k & \cdots & b_{il}^p \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{im_i}^1 & \cdots & b_{im_i}^k & \cdots & b_{im_i}^p \end{bmatrix}$$

4.2. Application of Controllability Conditions to The Considered System

4.2.1. Conditions of Controllability Without Constrains

Basing on the theorem 1 the controllability condition has the form:

$$rank\left[B_i^+\right] = rank\left[g^{-1}(\lambda_i)b_i^1 \dots g^{-1}(\lambda_i)b_i^k \dots g^{-1}(\lambda_i)b_i^p\right] = 1 \quad i = 1,2,3,\dots \tag{6}$$

Considering that $g^{-1}(\lambda_i) \neq 0$ condition (6) can be rewritten in the form:

$$\langle b_1, \phi_i \rangle_H^2 + \dots + \langle b_k, \phi_i \rangle_H^2 + \dots + \langle b_p, \phi_i \rangle_H^2 \neq 0 \quad i = 1,2,3,\dots$$

4.2.2. Conditions of Controllability With Constrains

Now, let us assume that the controls are nonnegative. So conditions (1) and (2) of the theorem 2 are fulfilled. In this case let us observe that:

- The condition (3) receives the form:

$$rank\left[b_i^1, \dots, b_i^k, \dots, b_i^p\right] = 1$$

-The condition (4).

First of all, let us calculate the $B_i'w$ factor:

$$B_i^+ w = g^{-1}(\lambda_i)\left[b_i^1, \dots, b_i^k, \dots, b_i^p\right]\left[u_1, \dots, u_k, \dots, u_p\right]$$

where $u_i \in U$.

Considering that $g^{-1}(\lambda_i) \neq 0$ and u_i is non-negative the condition (4) reduces to requirement so that in the vector $\left[b_i^1, \dots, b_i^k, \dots, b_i^p\right]$ there exist elements of both signs.

5. Properties of The Eigenfunction of The Operator A

In the further calculations the knowledge of the properties of the eigenfunctions of the operator A appears necessary. Following this aim at first let us prove the following lemmas:

5.1. Lemma 1

For every $x \in R_+$ the function $f(x)=\sin x-\sinh x$ is negative and decreasing.

5.1.1. Proof

The proof is based on the following Taylor's expansion:

$$\sin x - \sinh x = \sin x - \frac{e^x - e^{-x}}{2} = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)!} x^{2i+1} - \frac{1}{2}\left[\sum_{i=0}^{\infty} \frac{x^n}{n!} - \sum_{i=0}^{\infty} \frac{(-x)^n}{n!}\right] =$$

$$=\sum_{i=0}^{\infty}\frac{(-1)^i}{(2i+1)!}x^{2i+1}-\sum_{i=0}^{\infty}\frac{1}{(2i+1)!}x^{2i+1}=-\sum_{i=0}^{\infty}\frac{1}{(4i+3)!}x^{4i+3}<0$$

Q.E.D.

5.2. Lemma 2

For every $x \in R_+$ the function *f(x)=cosx-coshx* is negative and decreasing.

5.2.1. Proof

The proof is similar as in Lemma 1.

$$\cos x-\cosh x=\cos x-\frac{e^x-e^{-x}}{2}=\sum_{i=0}^{\infty}\frac{(-1)^i}{(2i)!}x^{2i}-\frac{1}{2}\left[\sum_{i=0}^{\infty}\frac{x^n}{n!}-\sum_{i=0}^{\infty}\frac{(-x)^n}{n!}\right]=$$

$$=\sum_{i=0}^{\infty}\frac{(-1)^i}{(2i)!}x^{2i}-\sum_{i=0}^{\infty}\frac{1}{(2i)!}x^{2i}=-2\sum_{i=0}^{\infty}\frac{1}{(4i+2)!}x^{4i+2}<0$$

Q.E.D.

5.3. Corollary 1

Now it can be easily seen that:

$$d_i(x)=\frac{\cos(\eta_i x)-\cosh(\eta_i x)}{\sin(\eta_i x)-\sinh(\eta_i x)}>0$$

$i=1,2,3,\ldots,\ x\in(0,l),\ k>0$

5.4. Corollary 2

Taking into account the form of the eigenfunctions:

$$\phi_i(x)=k\{\cos(\eta_i x)-\cosh(\eta_i x)+d_i(x)[\sin(\eta_i x)-\sinh(\eta_i x)]\}$$

it can be seen that they are negative and decreasing.

6. Conditions of Controllability For Mechanical Systems With Particular Control Forces

6.1. Forces With Fixed Sign In Considered Space Domain

For example $b_i(x)=C_i,\ C_i x^n,\ C_i m^x$ belongs to this class. Let us remind the common condition for constrained and unconstrained controllability:

$$rank\left[b_i^1,\ldots,b_i^k,\ldots,b_i^p\right]=1$$

It means that at least one element of proper matrix must not be equal 0:

$$b_i^k = < b_k, \phi_i >_H \neq 0 \tag{7}$$

Let's rewrite scalar product appearing in equation (7) in form:

$$b_i^k = < b_k, \phi_i >_H = \int_0^l b_k(x)\phi_i(x)dx$$

As proved in the section 5.3. the eigenfunctions $\phi_i(x)$ are negative. Respecting that in this section we are considering forces with fixed sign, i.e.:

$$\underset{x\in(0,l)}{\forall}\ b_k(x) > 0 \vee \underset{x\in(0,l)}{\forall}\ b_k(x) < 0$$

it can be stated that:

$$\int_0^l b_k(x)\phi_i(x)dx = b_i^k \neq 0$$

and the condition:

$$rank\left[b_i^1, ..., b_i^k, ..., b_i^p\right] = 1$$

is fulfilled.

6.1.1. Corollary 3

Systems (1) with fixed sign forces are controllable.

6.1.2. Conditions of Controllability With Constrains

As we stated in [11] for the non-negative controls the condition (4) reduces to the requirement so that in the vector $\left[b_i^1, ..., b_i^k, ..., b_i^p\right]$ there exist elements of both signs.

6.1.3. Corollary 4

The necessary and sufficient condition for *U*-controllability of the dynamical system (1) with fixed sign controls is:

$$\underset{\substack{i=1,2,3,...\\ p\geq 2}}{\forall}\ \underset{q,r\in\{1,2,...,p\},q\neq r}{\exists}\ b_i^q b_i^r < 0$$

6.2. Sine-like Force

Now let us assume that $b_i(x) = C_i \sin\frac{k\pi}{l}x,\ k \in Z_+$. Similarly the necessary condition for both controllability without constrains and U-controllability has the form:

$$b_i^k = < b_k, \phi_i >_H \neq 0 \tag{8}$$

Let us calculate the scalar product b_i^k:

$$b_i^k = < b_k, \phi_i >_H = C_i \int_0^l \phi_i(x) \sin\frac{k\pi}{l} x dx$$

By performing the substitution $x = \frac{l}{k\pi} t$ the number b_i^k can be expressed by the following formula:

$$b_i^k = C_i \frac{l}{k\pi} \int_0^{k\pi} \phi_i(\frac{l}{k\pi} t) \sin t dt$$

In order to prove the inequality (8) two cases are distinguished:

6.2.1. Case A : *k* Is an Even Number

In this case the considered integral can be rewritten in the form:

$$\int_0^{k\pi} \phi_i(\frac{l}{k\pi} t) \sin t dt = \sum_{j=0}^{\frac{k}{2}-1} \int_{2j\pi}^{(2j+2)\pi} \phi_i(\frac{l}{k\pi} t) \sin t dt = \sum_{j=0}^{\frac{k}{2}-1} \int_{2j\pi}^{(2j+1)\pi} \left[\phi_i(\frac{l}{k\pi} t) - \phi_i(\frac{l}{k\pi} t + \frac{l}{k}) \right] \sin t dt \tag{9}$$

As we proved in the section 5 the eigenfunctions $\Phi_i(x)$ are negative and decreasing, so:

$$\phi_i(\frac{l}{k\pi} t) - \phi_i(\frac{l}{k\pi} t + \frac{l}{k}) > 0$$

Additionally in the range $[2j\pi, (2j+1)\pi]$, $j \in Z$ the function sin(t) is non-negative, so every integral in sum (9) is positive, thus the considered integral is not equal zero and condition (8) is fulfilled.

6.2.2. Case B : *k* is an Odd Number

The integral after similar substitutions can be rewritten as:

$$\int_0^{k\pi} \phi_i(\frac{l}{k\pi} t) \sin t dt = \int_0^{\pi} \phi_i(\frac{l}{k\pi} t) \sin t dt + \sum_{j=0}^{\frac{k}{2}-1} \int_{2j\pi}^{(2j+1)\pi} \left[\phi_i(\frac{l}{k\pi} t) - \phi_i(\frac{l}{k\pi} t - \frac{l}{k}) \right] \sin t dt \tag{10}$$

The first integral is obviously negative. So in the case *B*:

$$\phi_i(\frac{l}{k\pi} t) - \phi_i(\frac{l}{k\pi} t - \frac{l}{k}) < 0$$

so every integral in sum (10) is negative, thus the considered integral is not equal zero and condition (8) is fulfilled and:

$$rank\left[b_i^1, ..., b_i^k, ..., b_i^p \right] = 1$$

6.2.3. Corollary 5

The system (1) with the forces:

$$b_i(x) = C_i \sin\frac{k\pi}{l}x,\ \ k \in Z_+$$

is controllable.

6.2.4. Corollary 6

The necessary and sufficient condition for U-controllability of the system (1) with the forces:

$$b_i(x) = C_i \sin\frac{k\pi}{l}x,\ \ k \in Z_+$$

is the same as for the fixed sign forces.

Concluding Remarks

This paper is devoted to the application of known representation of partial differential equations with damping term, by linear abstract differential equation, to the investigation of the controllability of elastic mechanical system with distributed parameters. First of all the selection of proper differential operator is presented and its properties are reminded. Next performed the investigations of the properties of the eigenfunctions of the dynamical system, which used in the further investigations. Next given partial differential equation was rewritten in the form of infinite series of finite dimensional dynamical system. To this form known theorems on constrained and unconstrained controllability were applied. Finally using obtained conditions controllability for some particular control forces were verified.

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Recenzent: Prof.dr hab.inż. Wojciech MITKOWSKI

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Streszczenie

W artykule przedstawiono analizę sterowalności bez ograniczeń i z ograniczeniami elastycznego układu mechanicznego o parametrach rozłożonych. Do tego celu wykorzystano metody analizy funkcjonalnej, w szczególności spektralna teoria liniowych operatorów nieograniczonych, i wyniki uzyskane w poprzednich pracach.

Najpierw zostało podane wyjściowe różniczkowe cząstkowe równanie stanu, jego warunki brzegowe i początkowe. Następnie zdefiniowano odpowiedni operator różniczkowy i podano jego własności. Opierając się na nim dane równanie stanu przedstawiono w postaci abstrakcyjnego równania różniczkowego z czynnikiem tłumiącym. Ta postać pozwoliła na przedstawienie równania stanu w postaci układu dwóch abstrakcyjnych równań różniczkowych pierwszego rzędu. Kolejnym krokiem było pokazanie, jak z użyciem odpowiednich podstawień układ dwóch abstrakcyjnych równań różniczkowych pierwszego rzędu można przedstawić, na podstawie odpowiedniego twierdzenia, w postaci układu dwóch nieskończonych ciągów układów skończenie wymiarowych. Uzyskane równoważne reprezentacje równania stanu pozwalają na zastosowanie znanych kryteriów sterowalności bez ograniczeń i z ograniczeniami dla układów z czynnikiem tłumiącym. Zacytowano odpowiednie twierdzenia i opierając się na nich sformułowano konieczne i wystarczające warunki, dla przypadku dowolnej siły wymuszającej. W celu zbadania prawdziwości tych warunków dla konkretnych funkcji wymuszających konieczna okazała się analiza przebiegu funkcji własnych operatora układu. W tym celu metodą rozwinięcia w szereg Taylora wykazano iż są one ujemne i malejące w dziedzinie równania. Zgodnie z tym wnioskiem podano szczegółowe warunki sterowalności bez- i z ograniczeniami dla przykładowych funkcji: o stałym znaku i sinusoidalnej.