ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 2006
Seria: AUTOMATYKA z. 144 Nr kol. 1727

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ATTEMPT AT DETERMINATION OF AN OPTIMUM DISASSEMBLY SEQUENCE FOR ELECTRO-MECHANICAL PRODUCTS

Summary. In this paper, the author presents a procedure for determination a sequence of an economic disassembly operation taking into account the disassembly costs, course and degree. The tools used for solving disassembly planning problems were elements of the graph theory, Petri Nets theory and linear programming for finding the optimum dendrite which describes the best disassembly method.

PRÓBA WYZNACZENIA OPTYMALNEJ SEKWENCJI DEMONTAŻU DLA WYROBÓW ELEKTROMECHANICZNYCH

Streszczenie. W artykule przedstawiona została procedura wyznaczenia sekwencji operacji demontażowej optymalnej ze względu na koszt, przebieg i stopień demontażu. Narzędziami w rozwiązywaniu problemów planowania demontażu były elementy teorii grafów, teoria sieci Petri oraz programowanie liniowe znajdujące optymalny dendryt opisujący najlepszy sposób demontażu.

1. Introduction

The disassembly operation sequence planning, and in particular that for utility purposes, belongs to the issues which require extensive studies. The majority of scare publications on the subject concern the assembly process planning, e.g. [3, 12]. Fewer studies are dedicated to the generation of disassembly sequences. For example: typically economic approaches are proposed by K.D. Penev [6] and M.R. Johnson [1]. Special attention should be paid to the work by H. Srinivasan [9] who provided details for the principle of *design for disassembly*. K.N. Taleb and S.M. Gupta [10] extended their studies to scheduling of disassembly processes. Also, the problem of separation of a single element or subassembly to repair it is studied by Bourjault *et al.* [7].

The author of this paper contributed to the disassembly process modelling as well. In [4, 5], a procedure was presented to allow for the selection of an operation sequence with regard to disassembly costs, course and degree. Using dynamic programming, it is possible to establish the process interruption moment, which is the moment in which it is not further possible to expect obtaining of additional benefits from continuation of the disassembly process.

More and more often, Petri Nets tools are used, e.g. [2, 8, 12]. Those are universal tools which are very effective at each stage of problem decomposition. The Petri Net properties naturally allow for a dynamic description of the events which are asynchronic in a system. They also allow for system descriptions in the terms of occurrence of interrelated phenomena and processes.

In this paper, the author will present a Petri Net for modelling of disassembly processes regarding electro-mechanical products that are designated for disposal and constitute exclusively sources of recycled materials.

2. Disassembly Processes Recorded in a Petri Net

Let's consider a general disassembly process of a certain product X. Each subassembly X_i of product X, just like the whole product, may be characterised by a set of its parts. The process status is the configuration of product subassemblies composed of one or more parts each at any stage of the disassembly process. The process status may be determined by describing the subdivision of the set of all the parts which constitute the whole ready-made product.

In this paper, we propose that the set of all possible disassembly operation sequences should be expressed through Petri Nets of the place/transition (PN) type.

A classical Petri Net of the place/transition type is and ordered six of the form:

$$PN = (P, T, E, K, W, M_0), \tag{1}$$

where: P, T – non-empty, finite sets of places and transitions,

$P \cup T \neq \varnothing, P \cap T = \varnothing$.

E – incidence relationship fulfilling the condition $\mathrm{dom}(E) \cup \mathrm{cod}(E) = P \cup T$, where $\mathrm{dom}(E)$ and $\mathrm{cod}(E)$ mean the domain and counter-domain of relationship E,

K – place capacity function,

W – arc multiplication factor function,

M_0 – initial marking function, fulfilling the condition: $(\forall p \in P)(M_0(p) \leq K(p))$.

In the graphic representation of the network, place is symbolised by circles, transitions by short bold lines and incidence relationships by arcs with multiplication factor function W. Initial marking is executed by markers (dots) related to places in the network.

In the Petri Net which models the disassembly process, places represent subsets of the parts that either make up subassemblies or are equivalent to single parts, while transitions correspond to geometrically and mechanically feasible operations. Each transition is an order pair in which the first element is the vertex corresponding to subassembly X_k, while the second one is a set of two vertexes $\{X_i, X_j\}$ such that $X_i \cup X_j = X_k$ and thus the disassembly task described by X_i and X_j is possible. Each transition is connected with the decomposition of the subassembly determined by the first vertex (place), the one characterised by part subassemblies and the corresponding subset of relationships which do not occur in the second transition vertex. That set of relationships connected with the transition corresponds to the cut of the graph of connections [3] between the subassembly and the first transition element.

The cuts in the presented disassembly planning system are determined by the use of a very effective algorithm. In the first step, we search for all coherent subgraphs which have half of the number of vertexes of the whole graph or less. Next, for each of them, we search for such edges in the set of graph edges which have only one end in the given subgraph. When removal of those edges leads to the occurrence of exactly two disjoint subgraphs (product subassemblies), such edges determine cuts. Then, based on the cuts, we build the *PN* Net. The subdivision process is repeated until all the subassemblies are composed of single parts.

In the disassembly process, the place capacity function is identical for all vertexes and is equivalent to 1, similar to the arc multiple factor function which is equivalent to 1 for all $e_k \in E$. Initial marking assumes the value of 0 for place p_1 which models the complex assembly (p_1), while for the remaining network places $M_0(p_i) = 0$.

Figures 1 and 2 present a simple four-element product and its connection graph. By connection (s), we understand a contact or joint towards a specific direction of selected parts. The Petri Net of disassembly sequences of the product from Fig. 1 is shown in Fig. 3.

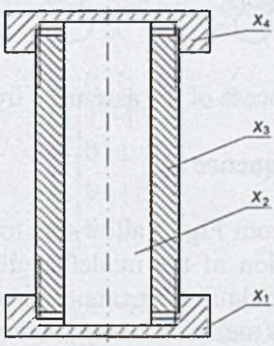

Fig. 1. Composition drawing of a model product

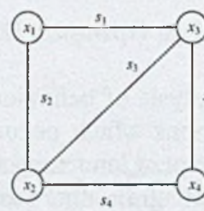

Fig. 2. Product connection graph

To describe the dynamics of the modelling network system of the disassembly process, we may use the transition function determined for pairs (M, t), fulfilling the following conditions of transition preparation:

a) $(\forall p \in {}^*t)\ (M(p) = W(p, t))$, (2)

b) $(\forall p \in t^*)\ (M(p) = K(p) - W(p, t))$, (3)

where: ${}^*t = \{p: (p, t) \in E\}$ – set of input places of transitions t,

$t^* = \{p: (t, p) \in E\}$ – set of output places of transitions t.

The relationship between the current status M and the status which directly results from it M' is determined by the equation:

$$M' = \begin{cases} M(p) - 1, & \text{if } p \in {}^*t, \\ M(p) + 1, & \text{if } p \in t^*, \\ M(p), & \text{in other cases.} \end{cases} \quad (4)$$

The final marking, or marking of the network which presents the disassembled product, has the form of $M(p) = 1$ only for the places which represent single parts and subassemblies that will not be subjected to further disassembly. That set of parts and subassemblies is marked by L. For the remaining places, $M(p) = 0$.

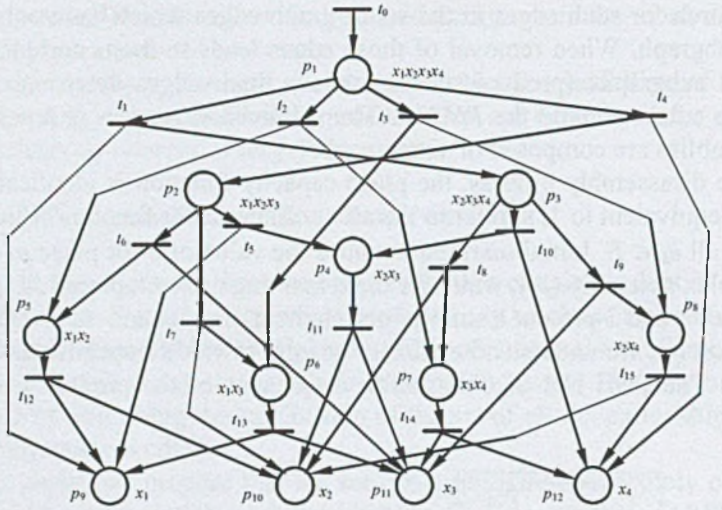

Fig. 3. Petri Net of the disassembly process of the assembly from Fig. 1

3. Selection of an Optimum Disassembly Sequence

The analysis of behaviour of the net from Fig. 3 allows us to discover typical conflict situations which occur in the operation of the modelled disassembly. Even when we attempt at launching transition t_1, ambiguity appears: four transitions are capable of launching. In other words, we have to make in place p_1 a choice between four options of disassembly operation execution, marked by t_1, t_2, t_3, t_4, and complete only one of them. It is necessary to propose such a control for the whole disassembly process (network control) which will uniquely conduct the process from the status described by the assembled product to the status in which further continuation of disassembly becomes unprofitable. The disassembly operation sequence is uniquely described by the transition launch sequence.

The best selection criterion of subsequent transitions will be maximisation of the generally understood profit obtained as a result of the repeated application of the disassembled single parts or subassemblies.

Consequently, it is proposed to expand the set of values and concepts which define the Petri Net as follows:

a) Each place p should be assigned the value $c(p) \in C$, expressing the actual value of a product, a subassembly or a single part. The value $c(p)$ is a positive number when the given subassembly can be sold, reused or designed for recycling. If a part (or subassembly) is sent to scrap, the cost of disposal and storage will be involved, and $c(p)$ assumes a negative value.
b) Each transition should be assigned the value $kd(t) \in KD$, or the cost of disassembly operation represented by the given transition.

The Petri Net structure is recorded in the incidence matrix $A = \{a_{ij}\}$, whose lines represent the places p_i of the network (a complex product, subassemblies and single parts), while the columns represent the transitions t_j (disassembly operations). The element a_{ij} of the matrix is equivalent to 1 if there is the arc (t_j, a_{ij}) between the transition t_j ant the place p_i, while $a_{ij} = -1$ in the case of existence of arc $(a_{ij}, t_j,)$. In other cases, $a_{ij} = 0$.

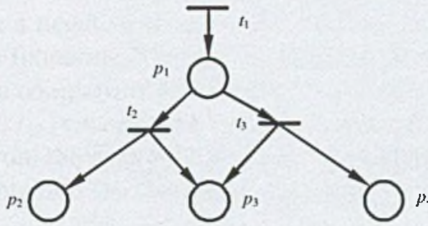

Fig. 4. Example of a conflict in the execution of operations t_2, t_3

Matrix A of the Petri Net presented on Fig. 4 contains 3 lines and 4 columns.

$$A = \{a_{ij}\} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{array} \begin{array}{ccc} t_1 & t_2 & t_3 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \quad (5)$$

The task of finding optimum control of the disassembly process leading to the maximisation of profit, or the minimisation of losses, may be recorded as mathematical programming task.

Each transition t_j is assigned the binary decision variable y_j which will express the execution or failure of the disassembly operation j. The variable y_j assumes value 1 if t_j is executed, or $t_j = 0$ in the opposite case.

The criterion function should represent profit resulting from conducting subsequent disassembly operations. The execution of a certain transition t_j, or completion of a process stage, which is expressed by the network marking status $M(p_i)$, brings profit z_j, being the sum of subassembly values or single part values obtained in j operations, decreased by the value of disassembled subassembly and the costs of the operation concerned. For example, the execution of the transition t_2 of the Petri Net from Fig. 4 is connected with the following profit z_2:

$$z_2 = c(p_4) + c(p_5) - c(p_1) - kd(t_2) \quad (6)$$

The profit from conducting a sequence of disassembly operations can be expressed by equation (7):

$$z = \sum_{j=0}^{|T|}\sum_{i=1}^{|P|} a_{ij} c(p_i) y_j - \sum_{j=0}^{|T|} kd(t_j) y_j \quad (7)$$

Limitations of the selection of transitions result from the Petri Net structure. As we mentioned above, it is not possible to launch two transitions originating from the same vertexes at the same time. That condition applied to the example from Fig. 4 can be recorded as follows:

$$t_1 \geq t_3 + t_4, \text{ or } t_1 - t_3 - t_4 \geq 0$$

The condition for the whole network is:

$$\sum_{j=0}^{|T|} a_{ij} y_j \geq 0, \tag{8}$$

while:

$y_j \in \{0,1\}$, $j = 1,\ldots,|T|$,
$y_1 = 1$

The last condition assures that we will have assemblies designated for disassembly, although the disassembly itself does not have to be executed.

4. Part and Subassembly Grouping

The problem of disassembly sequence selection can be associated with the issue of material segregation, or part grouping with regard to types of materials. Also that problem can be modelled using a Petri Net, and solved using linear programming methods.

The necessity to carry out material grouping results mainly from the generally known law which correlates the value of the material allocated to the weight unit with the general number of units made of that material [11]. Insignificant quantities of materials do not present any market value.

A simple Petri Net for grouping the parts which result from the disassembly of the assembly from Fig. 1 is presented on Fig. 5.

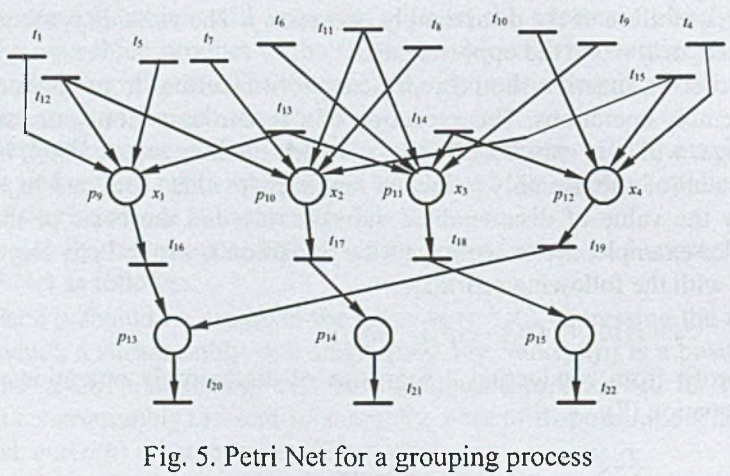

Fig. 5. Petri Net for a grouping process

Single parts x_1, x_2, x_3, x_4 are placed in three containers. Each of them contains various parts made of the same material. Parts x_1 and x_4 are made of steel. Of course there can be subassemblies made of the same materials in the containers when further disassembly is not economically justified. Also the parts and subassemblies which are not sources of materials and must be disposed of have one container.

The place capacities located in that Petri Net fragment which models the grouping process should be expressed in the units weight or volume. Also, the values of parts and subassemblies and the costs of conducting particular operations should be converted to part weight or volume. The enforcement of the grouping operation may be obtained by assigning a negative figure to the costs of the given operation, so that the value of the criterion function (7) should be increased by the increase of the value of segregated materials in comparison to the value of mixed materials.

Transitions t_{20}, t_{21}, t_{22} represent the transportation operations of a full container with the same material from the assembly position. The multiplication factor is equivalent to the number of containers. In this paper, that fragment of the Petri Net has been omitted.

The limitations imposed on the part and subassembly grouping process have the same form as the limitations existing in the disassembly problem. For example, the grouping condition of vertexes p_9, p_{12} may be formulated as follows:

$$t_1 + t_{12} + t_5 + t_{13} \geq t_{16} \tag{9}$$

$$t_{14} + t_{15} + t_{10} + t_4 \geq t_{18} \tag{10}$$

In a general case, such limitations are expressed by the previously quoted equation (8).

To illustrate the proposed method, we considered the assembly shown in Fig. 1. In our calculations, we used the approximate values of particular parts and costs of disassembly operation execution. We assumed that the values characterising the disassembly process are known. Therefore, we know the market value vectors of the parts and subassemblies used in the product KD = [2, 1.5, 2.5, 0.5, 1.5, 1, 0.5, 0.5, 2, 1.5, 0.5, 0.5, 0.5, 0.5, 0.5], together with the costs of execution of each operation C = [0, 5, 10, 10, 15, 15, 15, 2, 15, 9, 20, 3, 2, -1, -0.5, -1, -0.5]. The solution of thus formulated linear programming problem, using the LINGO or AMPL packages, leads to finding the following decision variable values: Y = [0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1]. The set of transitions which creates an optimum operation sequence has the form of: $\{t_4, t_7, t_{13}, t_{16}, t_{17}, t_{18}\}$. If all the disassembly operations are completed correctly, the profit will amount to: z = 35.5.

Despite the use of approximate values, we hope that the example presented above has demonstrated a general approach to the problem of economic disassembly planning.

5. Conclusion

The determination of the type of disassembly and the method of its execution is an essential issue in the planning of the process of gaining useful parts or materials. The procedure presented here allows for selection of the operation sequence which is the best as regards the disassembly costs, course and degree. That procedure determines the moment of interrupting the process when you may not expect obtaining any further benefits from the disassembly process continuation.

The tools applied to solving disassembly planning problems were the elements of the graph theory which concern the determination of the set if all cuts of the graph which models a complex product, the Petri Nets theory and linear programming which finds the optimum dendrite describing the best disassembly method. The useful side of the Petri Net is the fact that we can present all possible sequences in it. That network, in comparison to e.g. a directed graph [4], contains fewer vertexes and resultant edges, and it clearly shows all assembly operations which can be executed concurrently.

The optimisation of decisions in each possible process status should be treated as a parametric problem. That is the reason of the exponential complexity of the calculation process and of the limitation of the sizes of tasks to the sets composed of several dozens of parts at most.

Finding of an optimum operation sequence is not the only benefit of the process design system presented here. The Petri Net proposed for that process here allows for the designation of the graph of available markings, and, consequently, we can examine the process properties, for example with respect to the existence of cycles, blockades, concrete action sequences and relationships between sequences, and thus we can test the correctness of the modelled assembly system operation.

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Omówienie

Ustalenie rodzaju demontażu i sposobu jego przeprowadzenia jest zagadnieniem ważnym w planowaniu procesu odzyskaniu z niego wartościowych części czy materiałów. Przedstawiona procedura umożliwia wybór sekwencji operacji optymalnej ze względu na koszt, przebieg i stopień demontażu. Procedura określa moment zatrzymania procesu, moment, w którym nie można się już spodziewać uzyskania dodatkowych korzyści z kontynuowania procesu demontażu.

Narzędziami w rozwiązywaniu problemów planowania demontażu były elementy teorii grafów dotyczące sposobu wyznaczenia zbioru wszystkich cięć grafu, który modeluje złożony wyrób, teoria sieci Petri oraz programowanie liniowe znajdujące optymalny dendryt, opisujący najlepszy sposób demontażu. Znalezienie optymalnej sekwencji operacji nie jest jedynym atutem przedstawianego systemu projektowania procesu. Zaproponowana w tym artykule sieć Petri procesu umożliwia wyznaczenie grafu osiągalnych znakowań, a tym samym badanie własności procesu m.in. pod kątem istnienia cykli, blokad, konkretnych sekwencji czynności, relacji między sekwencjami i tym samym sprawdzenia poprawności działania modelowanego systemu montażowego.

Optymalizację decyzji w każdym możliwym stanie procesu traktować należy jako problem parametryczny. To właśnie jest przyczyną wykładniczej złożoności procesu obliczeniowego i tym samym ograniczenia wielkości rozwiązywanych zagadnień do zespołów zbudowanych z co najwyżej kilkudziesięciu części.