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ABSTRACTS 1172-1348

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ABSTRACTS 1172–1348

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061.1.055.5 1172

Twenty-seventh Annual Report of the National Research Council, 1943-44. *A.R. Nat. Res. Coun. Can.*, 31 *pp.*, 1943–1944.

37 : 511 *see Abstr.* 1174

51 = 3 1173

Generalization in mathematics. BÄBLER, F. *Schweiz. Bauztg*, 124, *pp.* 215–218, *Oct.* 21, 1944.—An essay on the evolution and development of mathematical concepts, including number theory (prime number, algebraic number, hypercomplex number, transfinite number, number field, ideal, etc.). Other subjects discussed are geometry, set theory, topology and abstract algebra. L. S. G.

511 : 37 1174

On the teaching of mathematics. MURNAGHAN, F. D. *Science*, 100, *pp.* 479–486, *Dec.* 1, 1944.—An address in which the author suggests a change in the methods of teaching elementary arithmetic and algebra. Mechanical formalism should be replaced by more emphasis on the meaning of the elementary operations, and on a clear understanding of the different types of numbers, e.g. integers, fractions, ordered pairs of numbers. L. S. G.

511.21 1175

On m consecutive integers. IV. PILLAI, S. S. *Bull. Calcutta Math. Soc.*, 36, *pp.* 99–101, *Sept.*, 1944.—A proof, easier than those given earlier, is presented for the theorem that when $m \geqslant 17$ we can find m consecutive integers such that no number in the set is prime to all the other members of the set. L. S. G.

511.213 1176

Bertrand's postulate. PILLAI, S. S. *Bull. Calcutta Math. Soc.*, 36, *pp.* 97–99, *Sept.*, 1944.—The postulate states that if $n \geqslant 2$ there is at least one prime, p, such that $n < p \leqslant 2n$. A proof (simpler than previous proofs) is given and this does not involve the use of Stirling's approx. formula for $n!$ L. S. G.

511.32 1177

On some formulae in analytical theory of numbers. II. BANERJEE, D. P. *Bull. Calcutta Math. Soc.*, 36, *pp.* 107–108, *Sept.*, 1944.—Some further formulae are given in continuation of a previous paper [Abstr. 8 (1945)]. L. S. G.

511.5 1178

A method in rational diophantine analysis. BELL, E. T. *Proc. Nat. Acad. Sci., Wash.*, 30, *pp.* 355–359, *Nov.*, 1944.—By rational substitutions, the given diophantine system is reduced to a multiplicative system, which is completely solvable. If the substitutions are birational, the complete rational solution of the original system is obtained, but when the substitutions are not rationally reversible, a partial solution only is found. The method is general and elementary and is an alternative to the recent applications of algebraic geometry to the arithmetic of rational varieties [Abstr. 1925, 1926, 1928 (1944)]. Two examples are given. In the first, $(\xi+\eta+\zeta)^3 - d\xi\eta\zeta = m$ is solved rationally for ξ, η, ζ when d and m are given rational numbers. In the second, $(x+y)(y+z)(z+x) - dxyz = m$ is solved for x, y and z. L. S. G.

511.52 1179

On trinomial congruences and Fermat's last theorem. VANDIVER, H. S. *Proc. Nat. Acad. Sci., Wash.*, 30, *pp.* 368–370, *Nov.*, 1944.—Using the results of a previous paper [Abstr. 1180 (1945)] criteria are given for the solution of the equation $au^m + bv^m + 1 \equiv 0 \pmod{p}$ and the results obtained are applied to the equation $x^l + y^l + z^l = 0$. L. S. G.

511.52 : 519.48 1180

Some theorems in finite field theory with applications to Fermat's last theorem. VANDIVER, H. S. *Proc. Nat. Acad. Sci., Wash.*, 30, *pp.* 362–367, *Nov.*, 1944.—Two theorems are obtained concerning finite fields and from these explicit expressions follow for the number of sets of solutions of certain equations. An application is made, in continuation of a previous paper [Abstr. 2213 (1942)], to criteria for the solution of $x^l + y^l + z^l = 0$ (Fermat's last theorem). An expression is obtained for the number of distinct integral solutions (u^m, v^m) of $au^m + bv^m + 1 = 0 \pmod{p}$ where a and b are integers and p is prime. L. S. G.

511.92 1181

Distribution of the types of the product of two demlo-numbers. CHANDRATRAYA, M. L., AND KAPREKAR, D. R. *J. Univ. Bombay*, 13, *pp.* 4–6, *Nov.*, 1944.—A continuation of previous work [Abstr. 516 (1944)]. A table is given of the various types of products.

512.25 1182

The "escalator" process for the solution of Lagrangian frequency equations. MORRIS, J., AND HEAD, J. W. *Phil. Mag.*, 35, *pp.* 735–759, *Nov.*, 1944.—The process is described in detail, the analysis being kept on simple algebraic lines. The method is based on the successive introduction or elimination of each of the variables involved, by definite self-contained stages. It does not involve any iterative process nor the evaluation of determinants. A special feature consists of a check at each stage to ensure and, if necessary, adjust the accuracy. The method is particularly suitable when there is a large number of equations. The canonical form of the equations is discussed and there is a section dealing with latent-root equations arising from the solution of linear simultaneous equations by iterative processes. A numerical example of the escalator process is given. This relates to 6th-order symmetrical canonical equations. L. S. G.

512.831 1183

A note on the Hermitian matrix. GHOSH, N. N. *Bull. Calcutta Math. Soc.*, 36, *pp.* 87–90, *Sept.*, 1944.—A new method is given for reducing the general Hermitian matrix to the sum of simple Hermitian matrices. M is simple if it is of the form $H\overline{H}^*$ where H is a column vector (of complex elements) and $\overline{H}^*$ is the transpose of the column vector having as elements the complex conjugates of the elements of H. L. S. G.

512.831 : 530.145.63 *see Abstr.* 1241

512.831 : 621.3.012.8 : 531.25 *see Abstr.* 1253

512.86 1184

On substitutional equations. RUTHERFORD, D. E. *Proc. Roy. Soc. Edinb. A*, 62, 2, *pp.* 117–126, 1944.—The $n!$ permutations $\epsilon, \sigma_2, \ldots, \sigma_{n!}$ of n letters form a group and the substitutional expressions,

$$L = l_1\epsilon + l_2\sigma_2 + \ldots + l_{n!}\sigma_{n!}$$

are the elements of an algebra. It is shown that substitutional equations of the type $LX = 0$ where X is an unknown expression are intimately related to the theory of idempotents (L is idempotent if $L^2 = L$). An equation $LX = 0$ is shown to have the same solution as $LM = 0$ where $M = AL$ (A being suitably chosen) and where the minimum equation of M is $x\psi(x) = 0$, $\psi(x)$ being prime to x. The expression $\psi(M)$ is idempotent and the general solution of $LX = 0$ is $X = \psi(M)Y$, where Y is arbitrary. The number of linearly independent solutions of $LX = 0$ is $kn!$ where k is the coefficient of ϵ (the unit permutation) in $\psi(M)$. Corresponding results are obtained for the equation $LX = R$ and methods are given for solving sets of simultaneous equations of each type. L. S. G.

512.864 : 530.145.63 *see Abstr.* 1242

512.88 : 512.972 1185

On invariant theory under restricted groups. LITTLEWOOD, D. E. *Philos. Trans. A*, 239, *pp.* 387–417, *Nov.* 25, 1944.—In an earlier paper [Abstr. 773 (1944)] invariant theory was studied under the full linear group and the methods are here extended to restricted groups, especially the orthogonal group (O) and the symplectic group (S). It is shown that a knowledge of the characters of the group is an essential preliminary to the study of the group invariants, and the characters of O and S (previously obtained by transcendental methods) are now obtained by entirely algebraic methods. A fundamental theorem proved, concerning groups with a system of fundamental tensors, is that every concomitant is obtainable by multiplication and contraction of ground form tensors, tensor variables, fundamental tensors and the alternating tensor. An analysis is developed enabling the numbers and types of the concomitants of any given degree in any system of ground forms to be predicted. The determination of actual concomitants is also discussed. For O, the theory is applied to the quadratic, the ternary cubic and the quaternary quadratic complex, and for the ternary S, to the quadratic, the linear complex and the quadratic complex. Applications are also made to intransitive and imprimitive groups. L. S. G.

512.94 1186

Quaternion centenary celebration. *Proc. R. Irish Acad.*, 50 *A*, 6, *pp.* 69–121, *Feb.*, 1945.—Included are the following papers: The sequence of ideas in the discovery of quaternions, E. T. Whittaker; Quaternions and matrices, A. W. Conway [Abstr. 1241 (1945)]; A modern presentation of quaternions, F. D. Murnaghan [Abstr. 1187 (1945)]; The icosian calculus, J. Riversdale Colthurst. This calculus refers to the system generated by 3 elements i, j, k such that $i^2 = j^3 = k^5 = 1$, $k = ij$ and $ij \neq ji$. L. S. G.

512.94 1187

A modern presentation of quaternions. MURNAGHAN, F. D. *Proc. R. Irish Acad.*, 50 *A*, 6, *pp.* 104–112, *Feb.*, 1945.—When S is a skew-symmetric matrix, $\mathbf{M} = e^{\mathbf{S}}$ is orthogonal. When the order (n) of the matrices is 2 this result becomes de Moivre's theorem since

$$i = \sqrt{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Here i is "realized" by means of the second-order matrix. The case $n = 4$ is then studied and it is shown that S can only satisfy a quadratic equation $\mathbf{S}^2 + \theta^2\mathbf{E}_4 = 0$ when S is of the form

$$\begin{pmatrix} 0 & -c & b & a \\ c & 0 & -a & b \\ -b & a & 0 & c \\ -a & -b & -c & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & -c & b & a \\ c & 0 & a & -b \\ -b & -a & 0 & -c \\ -a & b & c & 0 \end{pmatrix}$$

Each of these matrices leads to a "realization" of the quaternion units, the first by means of the matrices

$$\mathbf{I} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{J} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

It is readily verified that these satisfy $\mathbf{I}^2 = \mathbf{J}^2 = \mathbf{K}^2 = -\mathbf{E}_4$ and $\mathbf{JK} = -\mathbf{KJ} = \mathbf{I}$, $\mathbf{KI} = -\mathbf{IK} = \mathbf{J}$, $\mathbf{IJ} = -\mathbf{JI} = \mathbf{K}$. L. S. G.

512.972 : 512.88 *see Abstr.* 1185

512.972 : 531.259 1188

Dyadic analysis of space rigid framework. CHEN, P. P. *J. Franklin Inst.*, 238, *pp.* 325–334, *Nov.*, 1944.—It is shown that any space framework consisting of prismatic beams and rigid joints may be analysed exactly by solving a set of simultaneous linear vector equations with dyadic coefficients. The fundamental vector equation for a beam is found, assuming that the external forces and couples are applied at the ends of the beam. A study is made of an externally hinged or slotted rigid joint. Examples are given and there is a discussion of the method of iteration. L. S. G.

513.468 1189

Open packing of spheres. MELMORE, S. *Nature, Lond.*, 154, *p.* 708, *Dec.* 2, 1944.

513.766.5 : 519.3 1190

Transversality in higher space. DUTKA, J. *J. Math. Phys.*, 23, *pp.* 126–133, *Aug.*, 1944.—A short historical review is given of transversality relations, which first arose in the calculus of variations, and later appeared in the theory of contact transformations. A transversality in n-space is defined as a (1,1) correspondence between lineal and hypersurface elements having a common base point. Such a correspondence is studied in detail, and it is shown that a necessary and sufficient condition for a given correspondence to be a transversality is that a certain induced correlation should be a polarity. L. S. G.

513.813 1191

On parallelism in Riemannian space. SEN, R. N. *Bull. Calcutta Math. Soc.*, 36, *pp.* 102–107, *Sept.*, 1944.—A new parallelism is set up and some of its properties are deduced. The curvature properties of the space are studied from the point of view of this parallelism which is also used to study some of the

fundamental invariants of differential quadratic forms. L. S. G.

513.88 : 517.948.35 = 4 *see Abstr.* 1208–1211

517.38 1192

On the integral theorems of Gauss and Stokes. WESTBERG, R. *K. fysiogr. Sällsk. Lund, Förh.*, 13, 15, 10 *pp.*, 1944.—Two general theorems are given by means of which many transformations of integral expressions met with in theoretical physics may be simplified. The 2- and 3-dimensional cases are considered and examples are taken from the theories of electricity and hydrodynamics. L. S. G.

517.432 : 530.145.6 1193

Eigen-values and eigen-functions for the operator $d^2/dx^2 - |x|$. BELL, R. P. *Phil. Mag.*, 35, *pp.* 582–588, *Sept.*, 1944.—The eigen-values and eigen-functions of the equation, $d^2\psi/dx^2 - |x|\psi = -\lambda\psi$, are found in a form involving Bessel functions of order $\frac{1}{3}$. A table of numerical values is also given for the eigen-values and these are compared with the values for the operator $d^2/dx^2 - |x^q|$ where q is 2, 4, or ∞ [for the case $q = 4$ see Bell [Abstr. 1534 (1945)]. Some possible physical applications are noted. L. S. G.

517.432.1 1194

The extension of the Heaviside expansion theorem to the equations of engineering and physics in curvilinear orthogonal co-ordinates. SMITH, J. J. *J. Franklin Inst.*, 238, *pp.* 245–272, *Oct.*, 1944.—Problems such as the propagation of electromagnetic or elastic waves, the conduction of heat through solids, the propagation of acoustic waves in air and the distribution of flux in the airgap of electrical machinery may be solved by the operational methods (similar to that of Heaviside) developed in this paper. Orthogonal curvilinear co-ordinates are used. Formulae are derived for the field due to a point source in a given space for the following cases: (i) Laplace equation with steady state conditions, (ii) equation of heat conduction, (iii) Maxwell's wave equation, (iv) the wave equation for a dissipative medium. It is shown that by restricting the type of problem to a certain form (Green's function) basic solutions of the equations may be expressed in separated variables. L. S. G.

517.531 1195

A note on the maximum modulus of the derivative of an integral function. SHAH, S. M. *J. Univ. Bombay*, 13, *pp.* 1–3, *Nov.*, 1944.—Let $f(z)$ be an integral function and write

$$M'(r) = \operatorname{Max}_{|z|=r}|f'(z)|,\ \Phi(r) = \frac{\log\{rM'(r)/M(r)\}}{\log r}$$

Some inequalities relating to $\Phi(r)$ are noted and 3 new theorems are given (with proofs). The first of these states that, if $f(z)$ is of order ρ, then

$$\overline{\lim_{r\to\infty}}\,\Phi(r) = \rho$$

L. S. G.

517.54 : 621.319.7 : 621.3.013 1196

The field between equal semi-infinite rectangular electrodes or magnetic pole-pieces. DAVY, N. *Phil. Mag.*, 35, *pp.* 819–840, *Dec.*, 1944.—[Abstr. 1063 B (1945)].

517.562 : 518.2 *see Abstr.* 1214

517.564.3 1197

The tabulation of some Bessel functions $K_\nu(z)$ and $K'_\nu(z)$ of fractional order. CARSTEN, H. R. F., AND MCKERROW, N. W. *Phil. Mag.*, 35, *pp.* 812–818, *Dec.*, 1944.—The functions $K_\nu(x)$, where $\nu = \frac{1}{3}, \frac{2}{3}$ and $\frac{5}{3}$, are tabulated over the range of values 0·1 (0·1) 5·0 for x and the values for $x = 6·0$, 8·0 and 10·0 are also given. Tables are listed of $K_{1/3}(x)$, $K_{3/3}(x)$, $K'_{1/3}(x)$ and $K'_{3/3}(x)$ for $x = 0·1$ (0·1) 5·0 and for $x = 6·0$, 8·0, and 10·0. The method of calculation is stated, and a theory in which the functions appear is given. This relates to the temperature field within a cylindrical rod subjected to a sudden change in temperature. L. S. G.

517.912.2 1198

On the method of collocation. SAIBEL, E. *J. Franklin Inst.*, 238, *pp.* 107–110, *Aug.*, 1944.—The problem treated is the approx. determination of the characteristic numbers λ_i associated with a differential equation E containing a parameter λ. The method of collocation and Galerkin's method are explained and Grammel's converse to Galerkin's method is mentioned. In this we start with the homogeneous integral equation I associated with the problem, instead of with E. A new method, which in many cases is an improvement, is proposed. It consists of a collocation of I instead of E. An example, illustrating the method, is that of finding an approx. value for the lowest natural frequency of vibration of a uniform cantilever beam. This is treated in detail and the result is compared with that obtained through collocation of the differential equation. L. S. G.

517.942.4 = 4 1199

Fuchs's theorem and linear equations with periodic coefficients. PATRY, J. *Arch. Sci. Phys. Nat.*, 24, *July–Aug.* (*Suppl. No.* 2, *C.R. Soc. Phys. Hist. Nat. Genève*, 59, *pp.* 118–122), 1942.—The equation with periodic coefficients is taken to be

$$\sum_{m=0}^{n}(e_m + f_m e^{-ix} + g_m e^{ix})\frac{d^m u}{dx^m} = 0$$

and, by writing $z = e^{ix}$, this may be written in the form

$$\sum_{m=0}^{n}(F_m + E_m z + G_m z^2)z^m\frac{d^m u}{dz^m} = 0$$

Fuchs's theorem may be applied to this equation under certain conditions which are noted, and the solution is written in the form

$$u = \sum_{k=0}^{\infty} D_k e^{i(k+\mu)x}$$

where the D_k are given by a recurrent system of difference equations. It is noted that the condition that the series for u converge is that the singular points of the differential equation (roots of the equation $f_n + e_n z + g_n z^2 = 0$) should both lie outside or inside the unit circle (in the z-plane). L. S. G.

517.942.4 = 4 1200

A numerical method for solving linear equations with periodic coefficients. PATRY, J. *Arch. Sci. Phys. Nat.*, 24, *July–Aug.* (*Suppl. No.* 2, *C.R. Soc. Phys. Hist. Nat. Genève*, 59, *pp.* 122–126), 1942.—A method is presented for use when the equation cannot be solved by an application of Fuchs's theorem [Abstr. 1199 (1945)]. The solution is developed into a Fourier series the coefficients of which are found from an infinite set of linear equations. These cannot be solved directly, but a method is given for deriving the

solution approximately (using the "echo" principle). Conditions are found in order that the series solution converge. L. S. G.

517.942.4 = 4 1201

On the solution of linear differential equations with periodic coefficients. PATRY, J. *Arch. Sci. Phys. Nat.*, 24, *Nov.-Dec.* (*Suppl. No.* 3, *C.R. Soc. Phys. Hist. Nat. Genève*, 59, *pp.* 225–229), 1942.—A continuation of a previous paper [Abstr. 1200 (1945)]. A further examination is made of the infinite set of equations giving the coefficients in the Fourier development, and the conditions for convergence are made more precise. L. S. G.

517.942.9 1202

The numerical solution of Laplace's and Poisson's equations. MOSKOVITZ, D. *Quart. Appl. Math.*, 2, *pp.* 148–163, *July*, 1944.—The equations are written in finite-difference form according to the Liebmann procedure [Abstr. 2703 (1938)] and the system of difference equations thus obtained is solved symbolically by using the inverse of a certain operator. Proceeding from its definition, the properties of this operator are established, and the solution of the original equation is given as a series involving the boundary values of the function in question. Numerical tables of the coefficients in the series are given to facilitate application of the method to particular cases. There is a discussion of the procedure to be followed when the boundary is non-rectangular. L. S. G.

517.946.9 : 539.214 1203

A boundary value problem in plane plasticity for the Coulomb yield condition. COBURN, N. *J. Math. Phys.*, 23, *pp.* 117–125, *Aug.*, 1944.—The method of a preceding paper [Abstr. 63 (1945)] is modified for use in the present paper, where the normal stresses σ_x, σ_y and the shear stress σ_{xy} are supposed known. The problem solved is that of determining the stresses in the interior of the half-space $x > 0$. The stresses $\sigma_x, \sigma_y, \sigma_{xy}$ and the functions $\sin 2\gamma$, $\cos 2\gamma$ are expanded into power series of the friction coefficient and these, upon substitution into Levy's equations, yield an infinite set of equations for the various approximations to the stress components. It is shown that the latter may be determined by a step-by-step method of computation. L. S. G.

517.947.42 : 534.121.2 1204

Relaxation methods applied to engineering problems. XI. Problems governed by the 'quasi-plane-potential equation.' ALLEN, D. N. DE G., SOUTHWELL, R. V., AND VAISEY, G. *Proc. Roy. Soc. A*, 183, *pp.* 258–283, *Feb.*, 1945.—The quasi-plane-potential equation $\frac{\partial}{\partial x}\left(X\frac{\partial \Psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(X\frac{\partial \Psi}{\partial y}\right) + Z = 0$ is discussed in relation to problems where Z and X are known functions of x and y. It governs, *inter alia*, the small transverse displacement of a membrane in which the tension $T\alpha_\gamma$, and its finite-difference approximation governs the small transverse displacements of nodal points of a net in which, similarly, the string tension T varies from node to node. (Equilibrium in the directions of x and y can be maintained, both in the membrane and in the net, by forces acting in those directions and accordingly having no effect on the transverse equilibrium.) The relaxational treatment, based on this mechanical (net) analogue, reduces, when X is const., to the treatment developed for problems governed by the plane-harmonic (Poisson) equation [Abstr. 323 (1944)].

517.947.44 : 518.5 1205

Numerical solution of initial-value problems by means of punched-card machines. KORMES, J. P., AND KORMES, M. *Rev. Sci. Instrum.*, 16, *pp.* 7–9, *Jan.*, 1945.—The method is illustrated on the equation: $\partial^2 u/\partial t^2 - c^2 \partial^2 u/\partial x^2 = 0$ with given initial conditions $u(0, x)$ and $u_t(0, x)$ as well as boundary conditions $u(t, x_0) = f_1(t)$ and $u(t, x_n) = f_2(t)$. The plane continuum is replaced by a rectangular net and the differential equation by a finite-difference equation. A value of the function $u(t, x)$ in a given point of the net is expressed by its values on two lower rows of the net [see Abstr. 2662 (1943)].

517.948 1206

On the integral equations of continuous dynamical systems. INGRAM, W. H. *Proc. Nat. Acad. Sci., Wash.*, 30, *pp.* 370–376, *Nov.*, 1944.—The systems are those considered previously [Abstr. 2330 (1940)]. The kernels are represented in a certain form which is known to be valid in the case of the vibrating string, the electrical transmission line, etc. The paper is devoted to a determination of the modal vectors and numbers of such kernels. L. S. G.

517.948.3 : 532.532 *see Abstr.* 1293

517.948.3 = 5 1207

On a functional equation. ASCOLI, G. *Portugaliae Mathematica*, 4, 4, *pp.* 145–157, 1944.—It is observed that the logarithmic derivative, $\Psi(z)$, of the gamma function, $\Gamma(z)$, satisfies the functional equation

$$\Psi\left(\frac{z}{n}\right) + \Psi\left(\frac{z+1}{n}\right) + \ldots + \Psi\left(\frac{z+n-1}{n}\right) = n\{\Psi(z) - \log n\}$$

This result is generalized. The function $f(x, y) = \Psi(x + iy) - \log y$ satisfies

$$\text{(A)}\quad f\left(\frac{x}{n}, y\right) + f\left(\frac{x+1}{n}, y\right) + \ldots + f\left(\frac{x+n-1}{n}, y\right) = nf(x, ny)$$

and a study is made of some properties of the general solution of (A). It is shown that a necessary and sufficient condition for a function $f(x, y)$ to satisfy (A) is that it permit a Fourier series development of the form

$$f(x, y) \sim c_0 + \sum_{n=-\infty}^{\infty}{}' \alpha(ny) e^{2\pi i n x}$$

where the dash (′) denotes omission of the term $n = 0$. Here $\alpha(t) = \int_0^1 f(\xi, t) e^{-2\pi i \xi} d\xi$ for t greater than a certain value. An asymptotic expansion of $f(x, y)$ is given in terms of the functions $\beta_s(x, y) = B_s(x)/y^s$ where the $B_s(x)$ are the Bernoullian polynomials. Some examples are given. L. S. G.

517.948.35 : 513.88 = 4 1208

The spectral decomposition of Hermitian operators. WAVRE, R. *Arch. Sci. Phys. Nat.*, 24, *July–Aug.* (*Suppl. No.* 2, *C.R. Soc. Phys. Hist. Nat. Genève*, 59, *pp.* 112–115), 1942.—A process is given for decomposing an element f of a space which is either a

functional space or is isomorphic to a Hilbert space. This proceeds according to the proper elements f^α obtained with the aid of the operator A^2 where A is a Hermitian operator [Abstr. 783 (1943)]. A sequence of elements $x_0, x_1, x_2, \ldots$ is obtained from a normalized element $x_0(||x_0|| = 1)$ by iteration according to $l_i x_i = A(x_{i-1})(i = 1, 2, \ldots)$ and we have $l_1 \leqslant l_2 \leqslant \ldots$, $l = \lim_{i\to\infty} l_i$. Write $\tilde{\omega} = \frac{l_1}{l} \cdot \frac{l_2}{l} \ldots$ If $\tilde{\omega} \neq 0$, the operator A is said to be regular and in this case the elements x_{2r} converge strongly towards a limiting element x^* satisfying the equation $l^2 x^* = A^2 x$. If $\tilde{\omega} = 0$, the x_{2r} converge weakly to zero. Several theorems are stated and these are used to deduce the main result that, when A is regular, the element f may be decomposed in the form $f = \sum_\alpha \tilde{\omega}(f_0^\alpha) f^\alpha + h$ where $A(h) = 0$ and f^α is the limit of the elements obtained by an iteration involving the operator A^2 acting upon f_0^α. Here we have $f_0^\alpha = \tilde{\omega}(f_0^\alpha) f^\alpha + f_0^{\alpha+1}$. L. S. G.

517.948.35 : 513.88 = 4 1209

On linear equations involving a Hermitian operator. WAVRE, R. *Arch. Sci. Phys. Nat.*, 24, *July–Aug.* (*Suppl. No.* 2, *C.R. Soc. Phys. Hist. Nat. Genève*, 59, *pp.* 157–159), 1942.—Using the results of a previous paper [Abstr. 1208 (1945)] an equation of the type $\phi = f + \frac{1}{\nu} A(\phi)$ is considered where f is a given element of E, ϕ an unknown element, and ν is a constant. Any solution of this equation is also a solution of $\phi = f^* + \frac{1}{\nu^2} A^2(\phi)$, where $f^* = f + \frac{1}{\nu} A(f)$ and conversely. By decomposing f^* into $f^* = \sum_\alpha f_\alpha^* f^\alpha + h$ where $A(h) = 0$ the solution of the original equation is obtained in the form $\phi = f^* + \sum_\alpha \frac{l_\alpha^2 f_\alpha^*}{\nu^2 - l_\alpha^2} f^\alpha$. Some remarks are made concerning possible relationships between ν and the numbers l_α and the consequences of these. It is shown how the theory of integral equations of the Fredholm type with a symmetric kernel may be rapidly obtained (e.g. the Hilbert–Schmidt theorems) by the methods of iteration involving Hermitian operators. L. S. G.

517.948.35 : 513.88 = 4 1210

A note on the iteration of Hermitian operators. WAVRE, R. *Arch. Sci. Phys. Nat.*, 24, *Nov.–Dec.* (*Suppl. No.* 3, *C.R. Soc. Phys. Hist. Nat. Genève*, 59, *pp.* 229–233), 1942.—A supplement to previous papers [Abstr. 1208–1209 (1945)]. Some further remarks are made concerning the numbers $l_1, \ldots, l_r, \ldots$ obtained from the formula

$$A^r x_0 = l_1 \ldots l_r x_r \; (r = 0, 1, 2, \ldots)$$

and a study is made of certain infinite products that arise in the spectral decomposition of an element of a functional or Hilbert space. L. S. G.

517.948.35 : 513.88 = 4 1211

Some results complementary to the theory of iteration of operators by M. Wavre. VIGIER, J. P. *Arch. Sci. Phys. Nat.*, 24, *July–Aug.* (*Suppl. No.* 2, *C.R. Soc. Phys. Hist. Nat. Genève*, 59, *pp.* 159–162), 1942.—An extension of the work of Wavre [Abstr. 1208 (1945)] to skew Hermitian operators, i.e. operators defined by $(Ax, x) = -(x, Ax)$. The theory of Lalesco relating to skew symmetric kernels is shown to follow rapidly from the point of view taken. In particular, a series solution of the equation $\phi = \frac{1}{r} A\phi + f$ is given and this extends the theorem of Hilbert–Schmidt to skew Hermitian operators. L. S. G.

517.949 : 531.234 *see Abstr.* 1250

517.949 : 624.18 = 3 1212

Application of difference equations in calculating ferro-concrete protection pillars. KOLLBRUNNER, C. F., AND DUBAS, C. *Schweiz. Bautzg*, 124, *pp.* 191–194, *Oct.* 7, 1944.—In calculating protective pillars of goalpost form, ⊓, to be placed near machinery in a power station, the whole pillar is treated as a lamina and the problem is solved mathematically, using Airy's function. G. E. A.

518.2 1213

Preparation of punched-card tables of logarithms. THOMAS, G. B., AND KING, G. W. *Rev. Sci. Instrum.*, 15, *p.* 350, *Dec.*, 1944.

518.2 : 517.562 1214

Punched-card tables of the exponential function. KING, G. W. *Rev. Sci. Instrum.*, 15, *pp.* 349–350, *Dec.*, 1944.

518.3 1215

Nomograph for equations of the form $yx^n = z$. BARNES, J. C. *Engng J., Montreal*, 27, *pp.* 543–545, *Oct.*, 1944.

518.4 : 536.2 *see Abstr.* 1466

518.5 : 517.947.44 *see Abstr.* 1205

519.1 : 536.77 *see Abstr.* 1502

519.217 : 532.72 1216

The diffusion problem and the theory of connected events. DARLING, D. A. *Proc. Amer. Phys. Soc., Pasadena, Cal., Dec.* 16, 1944. *Abstr. in Phys. Rev.*, 67, *p.* 65, *Jan.* 1 *and* 15, 1945.—An extension of the 1-dimensional random walk to the case where the particles possess inertia leads to a statistical problem in the theory of non-independent, or Markoff, processes. The modification of the homogeneous random process in this case is deduced, and the meaning to be attached to random events extended to the continuum is studied. The analogue to the ordinary diffusion equation becomes an integro-differential equation $\frac{\partial p(t, x)}{\partial t} = k \frac{\partial^2 p(t, x)}{\partial x^2} \int_0^\infty \Phi(t, y) y^2 dy$, where $p(t, x)$ is the probability that the particle (possessing inertia) has a displacement x at a time t, and $\Phi(t, y)$ is a solution to the ordinary diffusion equation. Solutions of the equation are deduced by considering the moments of the probability distribution. Some of the equations used in turbulence and Brownian-movement problems are special cases of the integro-differential equation, with a suitable interpretation of the dependence function Φ.

519.281.2 1217

A method for the solution of certain non-linear problems in least squares. LEVENBERG, K. *Quart. Appl. Math.*, 2, *pp.* 164–168, *July*, 1944.—A method is developed which is of use in cases where the standard

method of least squares fails. Let $h(x)$ be approximated by $H(x; \alpha)$ where $x = (x, y, z, \ldots)$ are the variables and $(\alpha) = (\alpha, \beta, \gamma, \ldots)$ are the unknown parameters. The residuals at the points $(x_i)(i = 1, 2, \ldots, n)$ are $f_i(\alpha) = H(x_i; \alpha) - h(x_i)$ and the least-squares criterion requires that $s(\alpha) = \sum_{i=1}^{n} f_i^2$ be a minimum. If the initial solution is $(\alpha_0) = (\alpha_0, \beta_0, \gamma_0, \ldots)$, the 1st-order Taylor expansions of the residuals taken about (α_0) yield

$$f_i(\alpha) \simeq F_i(\alpha) = f_i(\alpha_0) + \frac{\partial f_i}{\partial \alpha}\Delta\alpha + \frac{\partial f_i}{\partial \beta}\Delta\beta + \ldots \quad (1)$$

where $\Delta\alpha = \alpha - \alpha_0$, $\Delta\beta = \beta - \beta_0, \ldots$. The standard method consists of minimizing

$$S(\alpha) = \sum_{i=1}^{n} F_i^2$$

but this may lead to values of $\Delta\alpha$, $\Delta\beta$, $\Delta\gamma, \ldots$ which are so large that the approx. (1) becomes invalid. In this case the values of $\Delta\alpha$, $\Delta\beta$, $\Delta\gamma, \ldots$ are limited or "damped" by making a minimum the expression

$$\bar{S}(\alpha) = wS(\alpha) + a(\Delta\alpha)^2 + b(\Delta\beta)^2 + \ldots$$

where $a, b, \ldots$ and w are weighting factors. This leads to "damped normal equations" which only differ from the ordinary type in that the coefficients of the principal diagonal are altered. The 2 systems approach coincidence as $w \to \infty$. These normal equations are discussed and several methods are given for determining the weight factors $w, a, b, c, \ldots$. A simple geometric interpretation is given of the damping imposed on the parameter variables in the above process. L. S. G.

519.283 1218

A second approximation to Soper's epidemic curve. WILSON, E. B., AND WORCESTER, J. *Proc. Nat. Acad. Sci., Wash.*, 30, *pp.* 37–44, *Feb.*, 1944.—A differential equation is considered which expresses Soper's formulation of the course of an epidemic. Various approx. methods are considered for solving the equation and an example is given relating to an epidemic of measles. L. S. G.

519.283 1219

The epidemic curve with no accession of susceptibles. WILSON, E. B., AND WORCESTER, J. *Proc. Nat. Acad. Sci., Wash.*, 30, *pp.* 264–269, *Sept.*, 1944.—A continuation of a previous paper [Abstr. 1218 (1945)], more accurate calculations being now given together with some numerical results. L. S. G.

519.3 : 513.766.5 *see Abstr.* 1190

519.444 1220

Groups involving a small number of sets of conjugate operators. MILLER, G. A. *Proc. Nat. Acad. Sci., Wash.*, 30, *pp.* 359–362, *Nov.*, 1944.—The groups are classified in the cases where the number of sets of conjugate operators is 2, 3, 4 and 5 respectively, e.g. if a group involves exactly 4 sets of conjugate operators, it is one of 2 groups of order 4 when it is abelian, and when it is non-abelian it is either the dihedral group of order 10 or the tetrahedral group. L. S. G.

519.48 : 511.52 *see Abstr.* 1180

519.5 = 4 1221

Characterization of the operation of closure by means of a single axiom. MONTEIRO, A. *Portugaliae Mathematica*, 4, 4, *pp.* 158–160, 1944.—The closure of a subset, $X \subset 1$, is a subset $X^* \subset 1$, such that (i) $X \subset X^*$, (ii) $X \subset Y$ implies $X^* \subset Y^*$, (iii) $X^{**} = X^*$. It is proved that these 3 conditions are equivalent to the single condition $Y + Y^* + X^{**} \subset (X + Y)^*$. L. S. G.

521.031 1222

Energy liberation in red giant stars. VAN ALBADA, G. B. *Physica, 's Grav.*, 10, *pp.* 604–612, *July*, 1943.—The theory of energy liberation (Gamow and Teller) [*Phys. Rev.*, 55, p. 791, 1939)] based on suggestions by Bethe [Abstr. 1550 (1939)] is criticized and it is shown that the rate of evolution predicted by the theory is not in agreement with secular variations in the periods of the Cepheids. The conclusions of Greenfield [Abstr. 2267 (1941)] are also shown to be erroneous. It is suggested that, for the explanation of the great energy output of the red giant stars, it is probably necessary to adopt the hypothesis of superdense cores and highly inflated envelopes (homology transformations being invalid in this case). A method for further research is suggested. L. S. G.

521.14 1223

A note on the minimum radius for degenerate stellar masses. KOTHARI, D. S., AND AULUCK, F. C. *Phil. Mag.*, 35, *pp.* 783–786, *Nov.*, 1944.—Previous work is discussed [Abstr. 3088 (1930)] and it is shown that the theory of degenerate stellar masses composed of relativistic degenerate gas predicts a non-zero radius for any mass. The radius attains a min. value for a mass which is about 10 times larger than the "critical" mass introduced previously by Chandrasekhar. L. S. G.

523.11 : 536.422.15 *see Abstr.* 1475

523.746 1224

Provisional sunspot-numbers for June to July, 1944. BRUNNER, W. *Terr. Magn. Atmos. Elect.*, 49, *p.* 238, *Dec.*, 1944.

523.746 1225

Provisional sunspot-numbers for August to November, 1944. BRUNNER, W. *Terr. Magn. Atmos. Elect.*, 50, *p.* 56, *March*, 1945.

523.746 1226

A table of secular variations of the solar cycle. GLEISSBERG, W. *Terr. Magn. Atmos. Elect.*, 49, *pp.* 243–244, *Dec.*, 1944.

523.746 : 621.396.11 1227

Sunspot minimum. BENNINGTON, T. W. *Wireless World*, 51, *pp.* 81–82, *March*, 1945.—[Abstr. 1279 B (1945)].

523.841.37 : 523.872 1228

The spectra of the Cepheid variables. STRUVE, O. *Observatory*, 65, *pp.* 257–273, *Dec.*, 1944.—Recent advances in stellar spectroscopy are applied to the interpretation of Cepheid spectra. Conflicting evidence as to the extent of the atmospheres of supergiants throws doubt on the validity of conventional theories of absorption-line formation in these stars, and raises the question whether the variation in velocity and in light are at all closely related to any phenomena in the deeper interiors of Cepheids. Observations at McDonald Observatory of 7 Cepheids

are used to determine whether Cepheid spectra at any stage exactly resemble supergiant spectra. Close resemblance is found at min. light, but anomalies occur towards max. which cannot be matched in normal stellar spectra and suggest the formation of an extended atmosphere. This model requires some modification of the accepted view of the observed radial velocities, on which is based the pulsation theory. The differential motions observed in certain spectral lines are probably related to the mechanism of shell formation at max. light. It is suggested that a close study of the spectra of cluster variables should now be undertaken. A. HU.

523.872 : 523.841.37 *see Abstr.* 1228

525.24 : 537.591.5 : 538.691 *see Abstr.* 1531

526.918.523 : 771.35 : 535.317.1 *see Abstr.* 1370

530.12 1229

The two-body problem in Birkhoff's and Einstein's theories. BERENDA, C. W. *Phys. Rev.*, 67, *p.* 56, *Jan.*, 1945.—Einstein and others have given a solution of the two-body problem, which gives results agreeing closely with those of Birkhoff's theory [Abstr. 488 (1945)]. The maximum difference between the two theories is within the experimental error, and may arise from differences in the approximation methods employed. A. J. M.

530.12 : 530.145 = 4 1230

A new method for the quantization of fields. I. STUECKELBERG, E. C. G. *Arch. Sci. Phys. Nat.*, 24, *pp.* 193–222, *Sept.–Oct.*, 1942.—Previous work on the theory of fields is discussed [Abstr. 3489 (1936), 983 (1939)] and, in particular, the recent work of Pauli [Abstr. 35 (1941), 22 (1942)] who showed that particles of integral or half-integral spin must obey the Fermi–Dirac and Bose–Einstein statistics respectively. Pauli's study however is incomplete and, in the present paper, the foundations are laid for a new presentation of the classical and quantum theory of both charged and uncharged fields. Considerable use is made of the tensor and spinor calculus. The properties of the two fundamental solutions of the wave equation [Abstr. 1352 (1941), 1475 (1942)] are studied and the classical theory of non-charged fields is presented. The interaction between two fields is considered and a particular example is discussed in detail. L. S. G.

530.12 : 530.145 = 4 1231

A new method for the quantization of fields. II. Quantum theory of non-charged fields. STUECKELBERG, E. C. G. *Arch. Sci. Phys. Nat.*, 24, *pp.* 261–271, *Nov.–Dec.*, 1942.—A continuation of a previous paper [Abstr. 1230 (1945)]. The symmetrical quantization (Bose–Einstein statistics) and the skew-symmetric quantization (Fermi–Dirac statistics) are discussed and a study is made of the different possible theories of non-charged fields. A condition is found in order that the quantization be uniquely determined. This relates to the proper values of the wave energy. L. S. G.

530.12 : 530.145 = 4 1232

A new method for the quantization of fields. III. Classical and quantum theory of charged fields. STUECKELBERG, E. C. G. *Arch. Sci. Phys. Nat.*, 24, *pp.* 5–34, *Jan.–Feb.*, 1943.—A continuation of previous papers [Abstr. 1231 (1945)]. The world-lines followed by wave packets of charged and non-charged fields are studied and these lead to the classical theory. In the quantum theory there are 16 *a priori* theories, but reasons are given for admitting only 4 of these. The creation of quantum pairs is then discussed, and it is finally concluded that only 2 theories are permissible. L. S. G.

530.12 : 531.51 1233

On Milne's theory of gravitation. SCHILD, A. *Phys. Rev.*, 66, *pp.* 340–342, *Dec.*, 1944.—It is argued that the equations which Milne gives in his kinematical relativity for the motion of two gravitating particles are meaningless so long as no rule is given for associating each event on the world-line of one particle with a corresponding event on the world-line of the other. Milne himself uses his definition of simultaneity to relate the pairs of events but, since this definition is not invariant under Lorentz transformations, neither is the resulting gravitational theory. Nevertheless, invariant rules can be adopted, e.g. the retarded potential, but this introduces an *ad hoc* non-epistemological element into the theory. An exception occurs when one of the two gravitating particles is a fundamental particle, in which case the equation of motion of the other is Lorentz-invariant. G. C. McV.

530.12 : 539.153 1234

Mass-energy relation. DUSHMAN, S. *Gen. Elect. Rev.*, 47, *pp.* 6–13, *Oct.*, 1944.—Experimental evidence for the two relations which follow from the principle of relativity is given. The first states that the mass of a particle increases with velocity and becomes infinity for speeds that approach that of light. The second relation states that energy and mass are equivalent.

530.145 : 530.12 = 4 *see Abstr.* 1230–1232

530.145.6 1235

Unification of the theories of photon and meson. BORN, M. *Nature, Lond.*, 154, *pp.* 764–765, *Dec.* 16, 1944.—A simple example is given of the interaction of 2 fields which may be studied rigorously by means of simple mathematics. The Lagrangian densities for fields having a spin 0 and 1, respectively, are written down and the simplest possible interaction is taken, assuming that in each field the mass term is zero. The field equations are deduced, and a discussion based on these leads to the conclusion that particles of spin 0 and 1 without rest mass are transformed by the simplest interaction into 2 new types of particles, one having the rest mass zero (photon), the other a finite rest mass (meson). Thus photon and meson seem to be different phenomena of the same quantized field. L. S. G.

530.145.6 1236

Expansion of positive-energy Coulomb wave functions in powers of the energy. BECKERLEY, J. G. *Phys. Rev.*, 67, *pp.* 11–14, *Jan.*, 1945—The non-relativistic wave function (in spherical polar co-ordinates) for a charged particle in a Coulomb field is expressed in a form suitable for problems in which the particle has a small positive energy. This formulation amounts to expanding the radial part of the wave function in powers of the energy E and is achieved by simple algebraic manipulation of power series and recurrence formulae. The coefficients of the expansion are functions of the radial co-ordinate and are identified with Bessel functions of integral order.

530.145.6 : 517.432 *see Abstr.* 1193

530.145.6 : 538.3 1237

On the commutativity of the Dirac electron wave function with the electromagnetic field. BELINFANTE, F. J. *Physica, 's Grav.*, 10, *pp.* 720–724, *Oct.*, 1943.—It is shown that the non-vanishing of $(\text{div } \mathbf{E} - 4\pi\rho)$, a quantity of importance in quantum electrodynamics, is not a particular feature of a Maxwellian or Proca field, but is just a consequence of the anticommutation relations between the components of the Dirac electron wave function (an undor) and of the invariance of these relations. L. S. G.

530.145.61 1238

A calculus of finite precision. LIEBOWITZ, B. *Phys. Rev.*, 66, *pp.* 343–350, *Dec.*, 1944.—This paper inverts certain quantum-mechanical ideas by constructing a probability of presence from first principles. If $x_i = f_i(s, a_1, a_2)$ represents a 2-parameter family of curves in 3-dimensional space, the density of the functions f_i with respect to the parameters a_1, a_2 is defined by an expression for the normal cross-section of a sheaf of curves. This density plays the rôle of a probability. Its derivative along a curve of the family is independent of the parameters a_1, a_2. This permits a special density to be constructed, which is independent of the parameters and which can be found by solving a linear second-order partial differential equation. The extension to n-dimensional space and $n-1$ parameters is given. A new approach to planetary dynamics is provided and yields an equation that is very similar to Schrödinger's equation for the stationary states of n particles; it has the same basic interpretation. For a non-conservative system of n particles an equation is derived which differs from Schrödinger's equation for stationary states only in certain respects; conflicts in interpretation and formalism arise which are briefly discussed. To account for the stationary states, an electromagnetic wave system is proposed which has the property that a particle moving in it may be in phase-mechanical resonance with it. Macroscopic examples of this kind of resonance are given.

530.145.61 : 537.228.5 *see Abstr.* 1508

530.145.61 : 621.3.012.8 1239

Electric circuit models of the Schrödinger equation. KRON, G. *Phys. Rev.*, 67, *pp.* 39–43, *Jan.*, 1945.—Equivalent circuits are developed to represent the Schrödinger amplitude equation for 1, 2, and 3 independent variables in orthogonal curvilinear coordinate systems. The networks allow the assumption of any arbitrary potential energy and may be solved, within any desired degree of accuracy, either by an a.c. network analyser, or by numerical and analytical circuit methods. The electrical model may be replaced by a mechanical model containing moving masses and springs.

530.145.61 : 621.3.012.8 : 621.316.313 1240

A.c. network-analyser study of the Schrödinger equation. CARTER, G. K., AND KRON, G. *Phys. Rev.*, 67, *pp.* 44–49, *Jan.*, 1945.—Utilizing the type of circuit [Abstr. 1239 (1945)], tests were carried out on the a.c. network analyser on one-dimensional circuits. Measurements were made of eigenvalues and eigenfunctions for the particular cases of the linear oscillator, the rectangular potential well, the double rectangular barrier, the single barrier, and the rigid rotator. These tests show the circuit representation to be valid and to be adaptable to solution on the network analyser.

530.145.63 : 512.831 1241

Quaternions and matrices. CONWAY, A. W. *Proc. R. Irish Acad.*, 50 *A*, 6, *pp.* 98–103, *Feb.*, 1945.—A demonstration is given of quaternion combinations equivalent to matrices which have been employed in recent years with reference to the particle aspect of certain equations arising in atomic and nuclear theory, e.g. the Dirac–Eddington matrices. The Duffin–Kemmer meson matrices [Abstr. 80 (1944)] are also related to quaternions and simplified meson matrices are introduced. L. S. G.

530.145.63 : 512.864 1242

Unitary representations of the Lorentz group. DIRAC, P. A. M. *Proc. Roy. Soc. A*, 183, *pp.* 284–295, *Feb.*, 1945.—The Lorentz group, consisting of all the transformations with matrix A such that $AGA^* = G$ where

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

possesses some infinite unitary representations in addition to the well-known finite representations. These are studied by a method suggested by Fock's quantum theory of the harmonic oscillator. A new kind of tensor quantity (called an expansor) in space-time is introduced. This has an infinite enumerable number of components and a positive definite (invariant) quadratic form for its squared length. Some important properties of expansors are studied and a quantum mechanical application is made to (i) the states of a four dimensional harmonic oscillator, (ii) the spins of particles. The possibility arises of a particle having no spin when at rest but acquiring a spin when moving. L. S. G.

530.145.63 : 538.3 1243

A note on the Hamiltonian theory of quantization. CHANG, T. S. *Proc. Roy. Soc. A*, 183, *pp.* 316–328, *Feb.*, 1945.—A study is made of the field equations obtained from varying a Lagrangian subject to auxiliary conditions. With proper choice of conjugate variables and the Hamiltonian, they can be brought to canonical forms and thus permit quantization in the usual way. For fields with some conjugate variables missing, it is sometimes possible to introduce such auxiliary conditions without affecting the field equations, but with the result that all the new canonical variables are present, thus allowing the application of the standard methods of quantization. To illustrate this, the Maxwell field is quantized subject to $\Delta A + c^{-1}\,\partial\phi/\partial t = 0$. The usual supplementary condition on ψ, $(\Delta A + c^{-1}\,\partial\phi/\partial t)\psi = 0$, is shifted to other conditions on ψ.

531.19 : 532.7 : 533.7 1244

On the statistical mechanics of liquids, and the gas of hard elastic spheres. RICE, O. K. *J. Chem. Phys.*, 12, *p.* 521, *Dec.*, 1944.—Errata [see Abstr. 822 (1944)].

531.224 1245

Stresses in a reinforced monocoque cylinder under concentrated symmetric transverse loads. HOFF, N. J. *J. Appl. Mech.*, 11, *A*235–*A*239, *Dec.*, 1944.—The

conventional assumptions of a linear normal stress distribution and a corresponding shear-stress distribution in the bent cylinder are replaced by a least-work analysis. Application of the theory to the numerical example of a cantilever monocoque cylinder yields a max. shear stress in the sheet covering and a max. b.m. in the ring amounting to 900% and 33%, respectively, of the values obtained by the conventional analysis.

531.224/.225 1246

On combined flexure and torsion, and the flexural buckling of a twisted bar. GOODIER, J. N. *Quart. Appl. Math.*, 2, *pp.* 93–101, *July*, 1944.—When a straight uniform slender bar is twisted, the straight form becomes unstable at a certain value of the twisting couple and the centre line of the bar becomes a space curve, the buckled form then possessing both flexural and torsional energy. The nature of the combined torsion and flexure are investigated to a higher order of small quantities than formerly and the new terms in the strain energy are found. The energy method is made available for more difficult problems of buckling from a twisted state such as those of non-uniform bars. L. S. G.

531.224.4 = 3 1247

The deformation problem for transversely loaded beams. STÜSSI, F. *Schweiz. Bautzg*, 123, *pp.* 149–150, *March* 25, 1944.—The continuous problem is treated by dividing the beam into a large number of small sections and expressing the equations of equilibrium in finite-difference form. The solution is then obtained approximately. Some examples are given. L. S. G.

531.225 : 535.55 : 620.171.5 1248

Studies in three-dimensional photo-elasticity. Torsional stresses by oblique incidence. FROCHT, M. M. *J. Appl. Mech.*, 11, *A*129–*A*234, *Dec.*, 1944.—A photo-elastic method is described for the study of stresses in cylindrical shafts due to pure torsion. The basic equation is derived. The max. stresses can be determined from a single stress pattern, and the 2 stress components which define the complete stress system in pure torsion at an interior point can be determined from 2 stress patterns obtained from one section of a shaft into which the pure-torsion system has been frozen. The method is applied to a circular shaft, and the experimental results agree with the theory.

531.23 1249

Moment-distribution analysis for three-dimensional pipe structures. DEHART, R. C. *J. Appl. Mech.*, 11, *A*240–*A*244, *Dec.*, 1944.—The Cross method of moment distribution is mainly used for determining the moments and shears produced in a 3-dimensional pipe structure in which expansion results from change of temperature. The method has the advantage of reducing the calculations and the time required for these. G. E. A.

531.234 : 517.949 1250

An application of the method of finite-difference equations to a problem of bending moments. (Continuous beam of *N* equal spans under uniform loading.) DURANT, N. J. *Phil. Mag.*, 35, *pp.* 848–850, *Dec.*, 1944.—The bending moments at the supports of the beam are obtained explicitly from a formula which is the solution of a finite-difference equation. The bending moments at any 3 successive supports are connected by the equation

$$M_{n-1} + 4M_n + M_{n+1} = -\tfrac{1}{4}\omega L^2$$

and this may be reduced to the form

$$u_{n-1} + 4u_n + u_{n+1} = 0$$

This equation has the solution $u_n = AE_1^n + BE_2^n$ where E_1 and E_2 are solutions of $1 + 4E + E^2 = 0$. This leads to the desired solution. A simple example is given to show the saving in time and it exhibits the power of the finite-difference approach in solving such problems. L. S. G.

531.25 1251

Moving loads on beams fixed at both ends and on propped cantilevers. O'DONOVAN, J. J. *Engineering*, 158, *pp.* 481–483, *Dec.* 22, 1944.—With the more general use of welding, the number of beams fixed at both ends is increasing, and as the method usually adopted is to work from the deflection curve, the author details an alternative method which employs the bending-moment equation only. Bending moments are determined and graphed for a single concentrated load and for two such loads; these two cases are also worked out for propped cantilevers. G. E. A.

531.25 1252

Loads borne on elastic supports. LAMB, E. H. *Proc. Instn Mech. Engrs, Lond.*, 151, 3, *pp.* 265–273, 1944.—Describes an exact method for application to problems in connection with the wheel loading of railway rolling stock and locomotives, motor vehicles, etc., and to the adjustment of the springs necessary to produce a desired distribution of the loads upon them. If the initial loading on the springs is known by direct measurement, the equations which give the amounts of the adjustments to be made in terms of the desired increments of load take a simple form, and the adjustments can be calculated and made once for all. A few examples are worked out and a short account of an extension of Castigliano's second theorem, from which the equations in the paper may be derived, is given.

531.25 : 621.3.012.8 : 512.831 1253

Tensorial analysis and equivalent circuits of elastic structures. KRON, G. *J. Franklin Inst.*, 238, *pp.* 399–442, *Dec.*, 1944.—The electrical analogy is extended to mechanical structures in which each member has 6 degrees of freedom, and equivalent circuits are developed to enable their solution, by means of a network analyser, under steady small deformation and under forced or natural vibration. The elastic structure is assumed to consist of long thin beams and rigid bodies with elastic coefficients, interconnected into an arbitrary network. Southwell's relaxation method gives an approx. solution, and an exact one is obtained by a tensorial method. Both methods employ matrices. G. E. A.

531.25 = 3 1254

A contribution to the calculation of arches which are rigidly fixed on each side and which are curved in ground plan. RUDMANN, K. *Schweiz. Bauztg*, 122, *pp.* 268–269, *Nov.* 20, 1943.—The arches may occur in such structures as a balcony or a bay. The bending moments and torsion moments are calculated when the arch is symmetrical and is symmetrically loaded. L. S. G.

531.252 1255

The intrinsic theory of thin shells and plates. I. General Theory. II. Application to thin plates. III. Application to thin shells. CHIEN, W. Z. *Quart. Appl. Math.*, 1, *pp.* 297–327, *Jan.*; 2, *pp.* 43–59, *April, and pp.* 120–135, *July*, 1944.—In part I the 3 equations of equilibrium and the 3 equations of compatibility are obtained for the 6 unknowns $p_{\alpha\beta}$, $q_{\alpha\beta}$ which represent the extension and change of curvature of the middle surface of the plate or shell. When these quantities are found, the strain stress throughout the plate or shell may be calculated. The displacement does not appear explicitly. A tensor notation is used, since stress, strain and curvature are all tensors. In II and III the various approximate forms are considered of the equations which arise from consideration of the thinness of the shell or plate and the smallness (or vanishing) of its curvature. A complete classification is made of all shell and plate problems, there being 12 types of plate problems according to the relative orders of magnitude of $p_{\alpha\beta}$, $q_{\alpha\beta}$ and the thickness of the plate in the unstrained state. The first 4 types represent problems of finite deflection and the results obtained in these cases are new. In the case of thin shells there are 35 types of problems, the basis of the classification being the type of differential equation arising in the problem.

L. S. G.

531.252 1256

Membrane stresses in shells of constant slope. MORKOVIN, V. *Quart. Appl. Math.*, 2, *pp.* 102–112, *July*, 1944.—The shell has for its middle surface one generated by a straight line sliding along a plane curve C (in the x-y plane), maintaining a right angle with the tangent to C and a constant angle with its binormal (the z-axis). Some formulae relating to the differential geometry of this surface are written down and then such effects as fall within the membrane theory of shells are studied for the shells generated by the process. These include a determination of the stress components and the resultant moment about the origin, due to certain forces acting on the shell. The effects of taper, as exhibited in a conical shell of circular cross-section, are also studied, and the stresses in a shell of constant slope under torsion are determined when the load is applied in such a manner that only shearing stresses are generated at the end-sections.

L. S. G.

531.252 1257

Relaxation methods applied to engineering problems. X. The graphical representation of stress. ALLEN, D. N. DE G., AND SOUTHWELL, R. V. *Proc. Roy. Soc. A.*, 183, *pp.* 125–134, *Nov.* 30, 1944.—A vector quantity (e.g. the shear stress on a specified plane) is represented by (i) contours which exhibit its resultant magnitude and (ii) "trajectories" which have its direction at each point. Suppose the components, $X = S\cos\theta$ and $Y = S\sin\theta$, of the shear-stress on a given plane are known. The contours are easily constructed because $S^2 = X^2 + Y^2$. To construct the trajectories, the latter are regarded as contours of a function β which has zero gradient in the direction θ. Then β is determined (not uniquely) from $X\partial\beta/\partial x + Y\partial\beta/\partial y = 0$. This equation is solved by the relaxation method in 3 typical examples and diagrams are given of the contours and the trajectories. The graphical representation of tensors is discussed. L. S. G.

531.255 1258

On the deflection of a cantilever beam. BARTEN, H. J. *Quart. Appl. Math.*, 2, *pp.* 168–171, *July*, 1944.—The deflection is computed in the case where the squares of the first derivatives cannot be neglected as is done in classical beam theory. A solution is given which may be applied to a cantilever of any stiffness, but the difference between the deflection thus found and that found by the classical beam theory is only noticeable when the stiffness is small. The method is of use in spring theory. L. S. G.

531.259 : 512.972 *see Abstr.* 1188

531.259 : 624.624 = 3 1259

The binding effect between pre-stressed and unstressed concrete and its application to a beam with pre-stressed support. SOUTTER, P. *Schweiz. Bauztg*, 124, *pp.* 103–108, *Aug.* 26, *and Erratum, p.* 126, *Sept.* 2, 1944.—The author gives the results of experiments with a concrete beam resting on a pre-stressed support, with detailed theory which takes account of bending, shrinkage and the necessary amount of pre-stressing. The support must, with all loads, be free from tension; compression stresses must not exceed 5 kg/cm^2, stresses higher than this must be taken up by iron; and the breaking moment should be 3 times the moment of the safe load. An example is worked out for a case in which the loading is carried to the breaking point. G. E. A.

531.259.22 : 621.643 1260

The overstrain of tubes by internal pressure. MANNING, W. R. D. *Engineering*, 159, *pp.* 101–102, *Feb.* 9, *and pp.* 183–184, *March* 9, 1945.—An approx. method is given for determining the ultimate strength of thick-walled ductile cylinders, and the mechanism of their rupture is described. It is shown that the internal pressure reaches a max. for a bore strain of about 20%, depending on the diameter ratio. If the pressure increases, the overstrain spreads outwards from the bore surface, the bore radius increases and wall thickness decreases and eventual fissure follows a 45° helical course along the line of max. shear. An arithmetical method is described by which the stress curves in overstrained tubes can be derived. G. E. A.

531.3 : 656.1 1261

Principles of physics applied to traffic movements and road conditions. HERREY, E. M. J., AND HERREY, H. *Amer. J. Sci.*, 13, *pp.* 1–14, *Feb.*, 1945.—[Abstr. 1389 B (1945)].

531.382 1262

Boundary-value problems of circular discs under body forces. II. SEN, B. *Bull. Calcutta Math. Soc.*, 36, *pp.* 83–86, *Sept.*, 1944.—A continuation of a previous paper [Abstr. 847 (1945)]. The problem now solved is that of a heavy circular disc rotating in a vertical plane about an eccentric horizontal axis. It is not necessary to use Airy's stress function as was done by a previous author [Abstr. 4777 (1938)].

L. S. G.

531.383 1263

Gyroscopic principles and applications. INGLIS, C. E. *Proc. Instn Mech. Engrs, Lond.*, 151, 3, *pp.* 223–235, 1944. *Engineer, Lond.*, 176, *pp.* 428–430,

Nov. 26; 452–453, *Dec.* 3, *and pp.* 464–465, *Dec.* 10, 1944.—Description of experiments shown at the Hawksley Lecture, designed to illustrate the more important principles determining gyroscopic behaviour, and explained in general terms. The presence of a component of acceleration which is induced by a change in the direction of the axis of spin is explained, and demonstrated by its effect when a belt is run over 2 vertical pulleys mounted on a rotating turntable. The acceleration of a rotating disc when its axis is precessing in a horizontal plane causes the axis of spin to move at right angles to the plane of the forces acting on the axis. Other effects are shown. A gyro free to precess can act as an automatic quick-acting counterweight; hastening the precession causes an added load to rise; the turn indicator, useful in the case of an aeroplane flying blind; precession effects in motor-cars; stabilization of a monorail; and stability of a spinning top. Mathematical appendices are given. G. E. A.

531.383 : 621.398 1264

The distant-reading gyro-magnetic compass. *Engineer, Lond.*, 179, *pp.* 177–179, *March* 2, *and pp.* 188–190, *March* 9, 1945.—A distant-reading compass installation comprises a master compass unit, generally located in the tail of the aircraft remote from armour plating or electrical circuits liable to cause local magnetic disturbance, various repeater compasses for pilot, navigator, etc., and a variation-setting corrector by means of which local variation up to 30° E. or W. of true N. can be allowed for. The various units and their special features are described in detail, with particulars of the electrical transmission system and of the way in which effective operation is obtained with angles of bank or pitch up to 75°. Many thousands of these installations, made by the Automatic Telephone and Electric Co., Ltd., and by Ferranti, Ltd., have proved their worth in aircraft of Coastal, Transport and Bomber Commands of the R.A.F., of the R.N. and the Dominions Air Forces and also the U.S.A. Air Force. Credit for the initial conception of the apparatus is due to the Royal Aircraft Establishment, Farnborough. A. W.

531.383 : 629.135 : 621.34 1265

The Gyrosyn compass. Esval, O. E. *Trans. Amer. Inst. Elect. Engrs*, 63, *pp.* 857–860, *Nov.*, 1944.—[Abstr. 1230 B (1945)].

531.51 : 530.12 *see Abstr.* 1233

531.552 1266

Minimum conditions necessary to achieve the velocity of escape. Seifert, H. S. *Proc. Amer. Phys. Soc., Pasadena, Cal., Dec.* 16, 1944. *Abstr. in Phys. Rev.*, 67, *p.* 65, *Jan.* 1 *and* 15, 1945.—The max. velocity achieved by a rocket directed vertically upward depends upon: the velocity (relative to the rocket) with which mass is ejected; the fraction of the total mass which is finally ejected, or "loading factor"; the initial acceleration; the dependence of acceleration upon time (flight law); the dependence of frictional resistance upon time and velocity. The flight laws of constant acceleration and constant thrust are considered. Although constant thrust produces higher terminal velocities than constant acceleration, the terminal accelerations produced in an efficient rocket by constant thrust are greater than a human passenger could tolerate. Assuming an initial acceleration of g, an energy loss due to friction equal to the kinetic energy of escape, constant thrust, and a step-rocket consisting of 3 parts of mass ratios 16 : 4 : 1, each with a loading factor of 0·9, the attainment of escape velocity would require an ejection velocity over 3 000 m./sec.

531.66 1267

Methods for measuring the coefficient of restitution and the spin of a ball. Briggs, L. J. *J. Res. Nat. Bur. Stand., Wash.*, 34, *pp.* 1–24, *Jan.*, 1945.—Four methods are discussed and employed experimentally: The 2-pendulum ballistic method, in which the ball is struck by a flat-nosed projectile driven from an air-gun; a method based on spark photography, by means of which the ratio of the speed of the ball to that of the projectile is determined; the measurement of the vertical rebound of a ball from a massive horizontal plate, when dropped from a known height, correction being made for air resistance; the measurement of the angle of reflection of a ball rebounding from a smooth inclined plate, the angle of incidence being known. A correction for spin is necessary in method 4 if the plate is not ideally smooth. Methods are described for measuring the spin velocity, and an approx. method for computing the spin is given, provided the coefficient of restitution is known. The variation of the coefficient of restitution of golf balls with impact speed and with temperature is experimentally determined, and a method for determining the time interval during which the ball remains in contact with the club is described. The coefficient of restitution of a golf ball when hit hard is roughly 0·7; the corresponding value for a baseball of pre-war construction is about 0·45.

531.715.1 1268

Apparatus for the production and measurement of small motions. Hull, G. F., Jr. *Rev. Sci. Instrum.*, 15, *pp.* 340–342, *Dec.*, 1944. *Proc. Amer. Phys. Soc., Providence, R.I., Oct.* 28, 1944. *Abstr. in Phys. Rev.*, 66, *p.* 353, *Dec.* 1 *and* 15, 1944.—A cantilever bar for producing small motions of the order of 10^{-5} cm. and greater is described, together with an interferometer for measuring these motions, using an uninterrupted monochromatic light source in which the interference fringes are visible and stationary for particular amplitudes, or a stroboscopic light source synchronized with the oscillating bar.

531.717.1 : 539.23 : 535.514.2 *see Abstr.* 1433

531.724 1269

A comparison of two methods for determining the surface area of a powder. Langille, R. C., Braid, P. E., and Kenrick, F. B. *Canad. J. Res. B*, 23, *pp.* 31–39, *Jan.*, 1945.—The surface area of the same Ba crown-glass powder was determined by: (*a*) comparison of adsorption of methylene blue on the powder and on large measurable broken surfaces; (*b*) measurement of the projection areas of randomly oriented particles. The averaged results were 682 cm.^{2}/g. by (*a*), and 568 cm.^{2}/g. by (7). Since only smooth surfaces were selected for comparison in the dye method, all small surface irregularities and striae on the surface of the powder would be taken into account. Such is not the case in the projection method.

531.752 1270

Recent improvements in a precision balance and the efficacy of rhodium plating for standard weights. MANLEY, J. J. *Proc. Phys. Soc., Lond.*, 57, *pp.* 136–144, *March*, 1945.—Various improvements are described which contribute to increased sensitivity: these are (1) the fitting of an accurately graduated scale in place of that engraved on the beam; (2) the use of pladuram wires for carrying the riders; (3) the use of Pt wire grids for ascertaining when the loaded pans have the same temperature; (4) a screened bridge wire, and (5) a new Al cell for enclosing the 2 pans. Tests show how the balance can be used to determine the cooling curve for a 100 g. weight and also how the sensitivity varies with the load. A series of measurements carried out on rhodium-plated weights, after storage for 18 months over wood charcoal heated at intervals, showed only in one case the formation of any film. Another rhodium-plated weight, stored over charcoal which was not reheated, was found to have acquired a film about 3 times greater than normal. A. W.

531.771 1271

A speed indicator for high-speed shafts. WILKIE, M. J. *J. Sci. Instrum.*, 22, *pp.* 36–37, *Feb.*, 1945.—A mirror, with one half blacked, is mounted on the end of the shaft. Near to it is a box containing a lamp and lens, and a photo cell which receives reflected light at a frequency equal to the r.p.m. Output is read on a valve-amplifier type of frequency meter, the indicator of which may be in any location. No load whatever is imposed on the drive. Speeds up to 30 000 r.p.m. have been measured with an accuracy of $\pm 0{\cdot}1\%$. Calibration procedure is described. E. H. W. B.

531.787 : 532.62 *see Abstr.* 1302

531.791 : 621.317.39 1272

A low-impedance counter circuit. LANGBERG, E. L. *Rev. Sci. Instrum.*, 16, *p.* 14, *Jan.*, 1945.—[Abstr. 1199 B (1945)].

532.1 = 3 1273

The liquid state. I. Two-phase diagrams of internal friction. TRAUTZ, M. *Kolloid Z.*, 100, *pp.* 405–424, *Sept.*, 1942.—A critical survey is presented of the results obtained by various methods of measuring the coefficient of friction of the 2-phase system consisting of a liquid and its saturated vapour in the neighbourhood of, and at points removed from, the saturation line. In the methods considered, either a capillary, an oscillating disc, a falling body or a nozzle was used. The relationships between viscosity or fluidity and pressure and temperature, and their relation to the saturation line, are discussed. Equi-fluid lines, i.e. those for which the ratio between ideal and actual viscosity is constant, are defined, and their location in the density/temperature and density/pressure diagrams is described. R. N.

532.13 1274

Viscosity of liquid styrene and butadiene. EDWARDS, D. A., AND BONILLA, C. F. *Industr. Engng Chem.*, 36, *pp.* 1038–1040, *Nov.*, 1944.—The viscosity was determined in Ostwald type viscometers. The primary purpose of the experiments was to obtain values of viscosity which are sufficiently accurate for the design of heat transfer equipment involving these compounds.

532.13 : 536.2 1275

The transport phenomena of gaseous helium at very low temperatures. BOER, J. DE. *Physica, 's Grav.*, 10, *pp.* 348–356, *May*, 1943.—Theoretical expressions for the viscosity and heat conductivity [Abstr. 2496, 3940 (1933), 5140 (1934)] are used to calculate these quantities at temperatures below 4°K. and the results are in reasonable agreement with recently obtained experimental data. The effect of certain correction terms at very low temperatures is investigated. L. S. G.

532.13 : 537.29 1276

On the experimental study of the effect of transverse h.f. electric field on the viscosity of liquids. SINGH, B. N., AND GHOSE, S. *Indian J. Phys.*, 17, *pp.* 252–256, *Oct.*, 1943.—A special apparatus is described to study the effect on the rate of flow of liquids of fairly low conductivity through narrow channels. For xylene, with fields of the order 10^6 c/s no change in the rate of flow was found for fields up to ~5 kV/cm. For amyl alcohol and ethyl acetate, fields up to 1 kV/cm. produced no change in the rate of flow. N. M. B.

532.13 = 3 1277

On the viscosity of mixtures of linear high-polymerized substances. BROSER, W., AND KRÜGER, H. E. *Kolloid Z.*, 100, *pp.* 327–331, *Sept.*, 1942.—A general equation is derived for calculating the viscosity of a mixture from the viscosities of its components and their gravimetric ratio. This equation is valid for both solutions and melts, and explains some of the phenomena observed with both. Philipoff's rule of mixture forms a simple special case. Calculated results are in good agreement with experimental values. A comparison is made with Flory's graphical method of determination [Abstr. 2353 (1940)]. R. N.

532.133 1278

The influence of velocity gradient on the relation between viscosity and concentration in cuprammonium solutions of cellulose. LYONS, W. J. *J. Chem. Phys.*, 13, *pp.* 43–52, *Jan.*, 1945.—A modification of the Baker–Philipoff equation is developed, giving a uniform value for the intrinsic viscosity of a given solution regardless of changes in velocity gradient. This new equation, $\eta_r = (1 + c/\lambda)^8 + (k_i - 8/\lambda)c$, agrees with data on cuprammonium solutions of cellulose in concentrations $< 0{\cdot}5$ g./100 ml. The parameter λ, interpreted as a function of velocity gradient, increases with gradient, while k_i, the intrinsic viscosity, has a constant value characteristic of the solute. The second constant in other equations may be interpreted as a velocity-gradient adjustment term.

532.133 : 532.72 = 3 1279

The coefficient of self-diffusion and the viscosity of monatomic liquids. SCHÄFER, K. *Kolloid Z.*, 100, *pp.* 313–320, *Sept.*, 1942.—On the basis of the "hole theory," postulating the formation of a number of holes in the structure of the liquid phase on fusion of a crystalline structure, the self-diffusion of monatomic liquids is calculated from the relationship between diffusion and the mean square of the displacement of the particles. Stokes's friction formula is modified by a factor 2 for calculating the viscosity. Satisfactory agreement between calculated and experimental values is found for monatomic metals. R. N.

532.133 : 539.133 : 661.713 1280

Viscometric chain length of wood cellulose in Triton F solution. LOVELL, E. L. *Industr. Engng Chem. (Analyt. Edit.)*, 16, *pp.* 683–685, *Nov.* 21, 1944.—Use of aq. dimethyldibenzylammonium hydroxide (Triton F) as a cellulose solvent for the determination of average chain length by the viscometric method is suggested. The experimental technique is described. Intrinsic viscosity data for a number of cotton and wood celluloses in Triton F are compared with the corresponding degree of polymerization values obtained viscometrically after nitration, and a linear relationship is established. The results are compared with cuprammonium viscosity values, as used in the wood cellulose industry.

532.133 : 541.18 = 3 *see Abstr.* 1570

532.137 1281

Viscosity measurement. CANNON, M. R. *Industr. Engng Chem. (Analyt. Edit.)*, 16, *pp.* 708–710, *Nov.* 21, 1944.—Describes the construction and operation of 2 types of master viscometers designed for accurately calibrating routine viscometers. The first instrument is calibrated directly with water at 20°C. The remaining instruments of larger bore are then calibrated against the first by means of hydrocarbons of suitable viscosity. These calibrated master viscometers are used to establish accurately the viscosity of a series of calibrating fluids to be used in calibrating routine viscometers. Since some fluids change in viscosity with age, the master instruments are employed to check the standard fluids 2 or 3 times per year. The opaque-type master is useful in handling very dark or opaque liquids and in studying the drainage effect in viscosity measurements.

532.137 1282

Miniature and micro suspended-level viscometers. LAPITZKY, M. M., AND THOMPSON, S. *J. Inst. Petrol.*, 30, *pp.* 349–350, *Nov.*, 1944.

532.137 : 541.182.025 = 3 1283

Vibration-viscosimetric measurements on concentrated suspensions. ERBRING, H., AND BROESE, S. *Kolloid Z.*, 100, *pp.* 332–335, *Sept.*, 1942.—A vibration viscosimeter with a steel diaphragm electromagnetically excited by a.c. of 50 c/s and adjustable amplitude is described. Suspensions of bentonite and of mixtures of bentonite and blanc fixe or satin white were investigated, and the viscosity was measured at different amplitudes and different concentrations during thixotropic solidification and liquefaction. An attempt to obtain thixotropic liquefaction by supersonics was unsuccessful. R. N.

532.14 1284

Liquid densities of volatile hydrocarbon mixtures. HANSON, G. H., KUIST, B. B., AND BROWN, G. G. *Industr. Engng Chem.*, 36, *pp.* 1161–1164, *Dec.*, 1944.—The use of apparent densities for methane and ethane in the liquid phase for estimating the density of crude oil saturated with natural gas at high pressures has been extended to liquid mixtures of volatile hydrocarbons as found in natural gasoline; the average error is −0·5%. The modified expansion-factor method also appears satisfactory for estimating the density of a complex liquid mixture, such as natural gasoline, from the composition of the mixture; the average error is −2·0%. Curves and data are given. N. M. B.

532.292 : 627.8 = 4 1285

The stability of regulating reservoirs and of systems of these. JAEGER, C. *Schweiz. Bauztg*, 122, *pp.* 255–257, *Nov.* 20; 297–300, *Dec.* 11; 314–317, *Dec.* 18, *and pp.* 323–325, *Dec.* 25, 1943.—The differential equations of motion of the water in a regulating reservoir are studied, particularly with reference to the oscillations of the level of the water. By eliminating the velocity, the equations lead to an equation which can always be written in the form

$$d^2z/dt^2 + \phi(t)dz/dt + \psi(t)z = 0$$

This is solved approximately in the case of a cylindrical reservoir and various numerical and graphical results are given. The theory of a system of 2 reservoirs is also given, and some methods of coupling which lead to stable systems are discussed. L. S. G.

532.3 1286

The forces on a body in an incompressible viscous fluid. WESTBERG, R. *K. fysiogr. Sällsk. Lund, Förh.*, 14, 1, 15 *pp.*, 1944.—Using the results of a previous paper [Abstr. 1192 (1945)], formulae are deduced for the force and couple resultants acting upon a body of arbitrary shape in a viscous incompressible fluid having an arbitrary non-steady motion. The results are a generalization of Blasius's formula for the force resultant in the sense that the surface of integration F in the integrals depending on the motion of the fluid may be chosen arbitrarily, except that the principle of linear momentum must be valid for the fluid enclosed between F and the body, and the hydrodynamic equations must be valid in the neighbourhood of F. The case in which the fluid has an irrotational motion is treated in detail. After transformation to a moving system of co-ordinates, formulae for the force and the couple are obtained in the three-dimensional case. The equivalent problem, when the body is an infinitely long cylinder and the fluid motion is two-dimensional, is also studied. L. S. G.

532.516 : 621.891 1287

Investigation of friction and wear under quasi-hydrodynamic conditions. LARSEN, R. G., AND PERRY, G. L. *Trans. Amer. Soc. Mech. Engrs*, 67, *pp.* 45–50, *Jan.*, 1945.—[Abstr. 1360 B (1945)].

532.516 : 621.891 1288

The friction properties of various lubricants at high pressures. BOYD, J., AND ROBERTSON, B. P. *Trans. Amer. Soc. Mech. Engrs*, 67, *pp.* 51–59, *Jan.*, 1945.—[Abstr. 1361 B (1945)].

532.517 1289

On theoretical determination of fluid motions at small viscosities. GUSTAFSON, T. *K. fysiogr. Sällsk. Lund, Förh.*, 10, 2, 10 *pp.*, 1944.—An attempt is made to unite the Prandtl theory of boundary layers with Oseen's asymptotical theory. Each of these involves one empirical assumption, the first the distribution of pressure round the body, the other the position of the limits of the wake. The possibility is investigated of choosing the motion of the asymptotical theory as a starting point and, with the aid of the boundary-layer theory, determining the position of the wake, in order to avoid the empirical assumption. An experimental and theoretical investigation is made of the subcritical

flow around a rotating cylinder. It appears possible to determine the flow, including the position of the wake, theoretically with good approximation. L. S. G.

532.517 1290

Some recent investigations on hydrodynamic stability. LIN, C. C. *Proc. Amer. Phys. Soc., Pasadena, Cal., Dec.* 16, 1944. *Abstr. in Phys. Rev.*, 67, *p.* 64, *Jan.* 1 *and* 15, 1945.—Recent investigations of the stability of 2-dimensional parallel flows bring out that the physical mechanism underlying hydrodynamic stability in an incompressible fluid can be understood in terms of vorticity transfer. The condition of vanishing curvature of the velocity profile in Rayleigh's criterion is identified with an extremum of vorticity distribution, and the physical picture describing the associated instability is worked out in detail. Conditions similar to Rayleigh's criterion have been obtained for curved and axially symmetrical flows, and these can also be interpreted in a similar manner.

532.517.4 1291

Pressure flow of a turbulent fluid between two parallel infinite planes. CHOU, P. Y. *Proc. Amer. Phys. Soc., Pasadena, Cal., Dec.* 16, 1944. *Abstr. in Phys. Rev.*, 67, *pp.* 64–65, *Jan.* 1 *and* 15, 1945.—Two solutions of the problem are given. The first is the consequence of the equations of mean motion and of double correlation, while, in the second solution, all the equations of mean motion and of double and triple correlations are utilized. The first solution holds good only in the central portion of the channel, while the second covers the entire channel except the immediate vicinity of the channel wall. The behaviour of the mean squares of turbulent fluctuations is discussed qualitatively. The present calculation indicates that the velocity defect, which is independent of the Reynolds number of the mean flow, according to von Karman, may also be independent of the magnitudes of the turbulent velocity fluctuation, after the flow has reached its steady turbulent state.

532.525 1292

The turbulent jet. CHOU, P. Y. *Proc. Amer. Phys. Soc., Pasadena, Cal., Dec.* 16, 1944. *Abstr. in Phys. Rev.*, 67, *p.* 66, *Jan.* 1 *and* 15, 1945.—A brief review of previous investigations.

532.532 : 517.948.3 1293

The mathematics of weir forms. COWGILL, A. P. *Quart. Appl. Math.*, 2, *pp.* 142–147, *July*, 1944.—Abel's integral equation is used to study the problem of the weir form when the flow is proportional to the nth power of the depth of flow h. If $y = f(x)$ expresses the distribution of the width of the weir with the depth x, the function $f(x)$ is given by

$$\int_0^h \frac{f(x)dx}{(h-x)^{\frac{1}{2}}} = 2Kmh^{m-1}$$

which leads to

$$f(x) = \frac{2K\Gamma(m+1)}{\pi^{\frac{1}{2}}\Gamma(m-\frac{1}{2})}x^{m-\frac{3}{2}} (m \geqslant 2)$$

It is evident from the form of $f(x)$ that when $m < \frac{3}{2}$ the weir forms have infinite width at the bottom. This is impossible in practice, and since the case $m = 1$ (flow proportional to the depth) is of engineering value, it is important to have a correction. This is obtained by starting with a rectangular cross-section at the bottom of depth a and width w. It is then required that in the upper section, given by $y = \phi(x)$ the flow be $\propto$ the depth x, when $x \geqslant a$. The use of Abel's equation, again, leads to the result

$$\phi(x) = w - 2w\pi^{-1} \tan^{-1} (x/a)^{\frac{1}{2}}$$

The forms of weirs are also considered when the rate of flow is expressible as a power series (convergent) in h. L. S. G.

532.542 1294

Friction factors for pipe flow. MOODY, L. F. *Trans. Amer. Soc. Mech. Engrs*, 66, *pp.* 671–684, *Nov.*, 1944.—Accepted conclusions are represented by graphs enabling engineers to estimate, simply, values of the friction factor (f in the Darcy formula $h_f = fLV^2/2gD$) in calculations relating to the loss of head in clean new pipes and in closed conduits running full under steady flow conditions. The graphs are applicable to both laminar and turbulent flow of fluids in general, while the cases of the flow of water and air receive somewhat more particular attention. Manning's formulae are recommended for the calculations of fluid flow in open channels, in preference to the use of values of the Chezy coefficient, C (derived from pipe friction factors). In such calculations, Reynolds' number should be replaced by a new criterion, the Froude number, representing the ratio of mean velocity in the open channel to the gravitational critical velocity, i.e. the velocity of propagation of surface waves. A bibliography is given. J. S. G. T.

532.57 : 591.112.3 1295

An electronic recording flowmeter. CRITTENDEN, E. C., JR., AND SHIPLEY, R. E. *Rev. Sci. Instrum.*, 15, *pp.* 343–346, *Dec.*, 1944.—An instrument for measuring and automatically recording small steady liquid flows or the mean value of small pulsating liquid flows is described. The detecting unit employs a rotameter, or variable-area meter, which consists of a vertical tube having a tapered bore in which a metal float is supported by the stream. Damping of motion of the float is obtained by means of an attached plunger whose cross-sectional area is slightly less than that of a chamber which communicates with the measuring tube. The position of the float, which varies almost linearly with flow, is determined electromagnetically with the aid of an electronic circuit and is optically recorded on moving sensitized paper. Interchangeable detecting units allow a wide range of flows to be measured. A unit covering the range of 1 ml./min. to 200 ml./min. is described in detail.

532.582.7 1296

The movement of bed particles in a turbulent stream. EINSTEIN, H. A. *Proc. Amer. Phys. Soc., Pasadena, Cal., Dec.* 16, 1944. *Abstr. in Phys. Rev.*, 67, *p.* 66, *Jan.* 1 *and* 15, 1945.—The current theory of suspensions in turbulent streams uses an equilibrium condition between the continuous settling of the particles in the surrounding water and the vertical turbulent exchange of water masses to establish the law for the vertical distribution of sediment concentration. At the bed itself, this argument breaks down

because here the exchange of liquid approaches zero. An exchange of particles between the bed and the moving layers exists and is governed by the general rules of probability. The probability of a particle moving out of its position in the bed can be related directly to the probability of the adjoining liquid having a certain velocity. If one introduces the measured probabilities for instantaneous velocities near the bed, the law for the movement of particles out of the bed can be predicted up to some general constants.

532.612 : 666.1 = 3 1297

Relationships between surface tension and structure of glass melts. DIETZEL, A. *Kolloid Z.*, 100, *pp.* 368–380, *Sept.*, 1942.—Examination of the ion arrangement at the surface of salt crystals shows that the average degree of incompleteness of a co-ordination in the surface of a melt is about 0·20, independent of the co-ordination number. The value decreases slightly with increasing binding energy between anion and cation. Polarizability of the anion has a marked influence on surface tension. For glasses, the curve connecting the activity of the oxides with the quotient r/z (r = radius, z = valency of the cation) has an ascending branch containing the cations forming anionic complexes in the glass, and a descending branch containing the pure cations. Easily deformable ions are forced to the surface of the glass. The absence of the so-called boric-acid anomaly as regards surface tension, and the relationships between surface tension and the linear coefficient of expansion, are discussed. R. N.

532.613.2 : 537.221 1298

The measurement of potentials at the interface between vitreous silica and solutions of potassium chloride by the streaming potential method. JONES, G., AND WOOD, L. A. *J. Chem. Phys.*, 13, *pp.* 106–121, *March*, 1945.—Improvements in the technique for the determination of the electric potential at the interface between a non-conducting solid and an aqueous solution of an electrolyte (commonly called the ζ-potential) have been developed and are described in detail. Zeta-potentials of vitreous silica capillaries in contact with 3 dilute solutions of KCl (10^{-3}, 10^{-4}, $10^{-5}N$) were determined. These data have been used for the interpretation of surface-tension data [Abstr. 1250 (1942)] by using Langmuir's theory for the thickness of wetting films as a function of the ζ-potential.

532.62 1299

Monolayers of egg albumin on concentrated salt solutions. BULL, H. B. *J. Amer. Chem. Soc.*, 67, *pp.* 4–8, *Jan.*, 1945.—The force area curve of egg albumin for films spread on 35% $(NH_4)_2SO_4$ was measured and the film pressures were determined by a Wilhelmy balance. The area of the spread film for any given film pressure is independent of the time which elapses between the spreading and the start of the compression up to an elapsed time of 20 min. The film is gaseous at low film pressures and the estimated mol. wt. is approx. 44 000. The egg albumin molecule probably does not dissociate on the surface. The area of the gaseous uncompressed film is 0·97 m.²/mg. of protein, suggesting that there is extensive orientation of the side chain residues in the uncompressed state. The film pressure at the point of min. compressibility was determined and the measured area at this point is 0·82 m.²/mg. The area per egg albumin molecule and the area per residue are calculated. These data support the theory that native protein molecules of the egg-albumin type have laminated structures. W. R. A.

532.62 1300

Monolayers of β-lactoglobulin on concentrated salt solutions. BULL, H. B. *J. Amer. Chem. Soc.*, 67, *pp.* 8–10, *Jan.*, 1945.—A Wilhelmy balance was used to investigate the force area curve for films of β-lactoglobulin on 35% $(NH_4)_2SO_4$ solution. The film is gaseous at l.p., and the mol. wt of the protein in this state is ~44 000, indicating no association or dissociation in the film. The uncompressed film area of 1·2 m.²/mg. points to extensive orientation of the side-chain residues in the uncompressed state. The film pressure and the film area (0·83 m.²/mg.) were determined at the point of min. compressibility. The data for the films are compatible with a molecule of duplex structure, of thickness 18·1 Å and top face area 2 900 Å². W. R. A.

532.62 1301

Mixed monolayers of egg albumin and lauryl sulphate. BULL, H. B. *J. Amer. Chem. Soc.*, 67, *pp.* 10–12, *Jan.*, 1945.—Force area curves for films of Na lauryl sulphate (I) on 35% $(NH_4)_2SO_4$ and film areas of mixtures of egg albumin and (I) on 35% $(NH_4)_2SO_4$ as a function of the weight fractions of the two components were measured. The partial area in m² mg is calculated for both components. The number of molecules of (I) bound per molecule of egg albumin is calculated and there is evidence of complex formation between them. W. R. A.

532.62 : 531.787 1302

A simple single-wire surface pressure balance. PANKHURST, K. G. A. *Trans. Faraday Soc.*, 41, *pp.* 156–157, *March*, 1945.—[See Abstr. 1246 (1942)].

532.62 : 536.658 : 541.183.56 1303

The relationship between the Brunauer–Emmett–Teller adsorption isotherm and the new isotherm of Jura and Harkins. LIVINGSTON, H. K. *J. Chem. Phys.*, 12, *p.* 466, *Nov.*, 1944.

532.62 : 536.7 1304

Equations of state and phase changes in films on liquids and solids. HARKINS, W. D., AND JURA, G. *Proc. Amer. Phys. Soc., Chicago, Dec.* 1 *and* 2, 1944. *Abstr. in Phys. Rev.*, 67, *p.* 61, *Jan.* 1 *and* 15, 1945.—Monolayers on water exhibit 3 condensed and 3 non-condensed phases. The relation between film pressure (π) and molecular area (σ) is linear ($\pi = b - a\sigma$, equivalent to $\log p/p_0 = B - A/v^2$) for any condensed phase, but for liquid intermediate or expanded films or for gaseous films the equations are more complicated. At sufficiently low temperatures films adsorbed on solids are often condensed at the higher v.p., but gaseous at low pressures. At higher corresponding temperatures either liquid expanded or liquid intermediate phases may occur. Phase changes which are first order in 3 dimensions usually become second order in 2 dimensions, though in the latter, the condensation of a vapour may be either first or second order.

532.692 : 535.668.1 *see Abstr.* 1443

532.692 : 621.793.72 : 539.231 *see Abstr.* 1548

532.7 : 533.7 : 531.19 *see Abstr.* 1244

532.72 1305

A new method for measuring diffusion constants of biologically active substances. POLSON, A. *Nature, Lond.*, 154, *p.* 823, *Dec.* 30, 1944.

532.72 : 519.217 *see Abstr.* 1216

532.72 : 532.133 = 3 *see Abstr.* 1279

532.72 : 536.2 : 533.15 *see Abstr.* 1311

532.739 : 536.445 *see Abstr.* 1480

532.74 : 534.22 1306

Velocity of sound and molecular association in liquids. LAGEMANN, R. T. *J. Chem. Phys.*, 12, *pp.* 464–465, *Nov.*, 1944.—Two empirical criteria for determining molecular association in liquids are indicated. One is the quantity P_cR/T_c in which P_c and T_c are the critical pressure and temperature respectively and R is a function of the velocity of sound; the other is the ratio of the velocity of sound in a liquid to the velocity of sound in the gas of the same compound. In both cases low numerical values of the ratios are qualitative indicators of association. The two methods do not possess high correlation; it is difficult to compare the results from these methods with those from previous methods.

532.74 : 539.26 = 3 1307

Organic compounds of silicic acid. III. Saponification and polymerization of silicic acid esters in organic media. HOLZAPFEL, L. *Kolloid Z.*, 100, *pp.* 380–388, *Sept.*, 1942.—The association mechanism and structural formulae of esters of primary, secondary, tertiary and polyesters of silicic acid are discussed. Viscosity, conductivity, refractive index and speed of solidification of different aliphatic and aromatic compounds were determined. Electron-microscopic investigations showed a honeycomb structure. The results of X-ray investigations are discussed. R. N.

532.77 1308

Cryoscopy of solutions. BANKS, W. H., BRACKNER, A. V., AND LEACH, S. J. *Nature, Lond.*, 154, *pp.* 739–740, *Dec.* 9, 1944.—[See Abstr. 1775 (1944)].

532.771 : 541.64 : 536.77 : 536.753 *see Abstr.* 1501

532.785 1309

Sucrose-salt solutions. VAN HOOK, A., AND SHIELDS, D. *Industr. Engng Chem.*, 36, *pp.* 1048–1051, *Nov.*, 1944.—The activity theory [Abstr. 1310 (1945)] is applied to the crystallization in the presence of electrolytes and non-electrolyte impurities. Interionic theory indicates a max. in the salt effect and a common denominator in terms of the ionic strength to explain the effects of different valence type electrolytes. Molasses-forming constituents are accounted for in a semi-empirical way.

532.785 : 541.127.1 1310

Kinetics of sucrose crystallization. VAN HOOK, A. *Industr. Engng Chem.*, 36, *pp.* 1042–1047, *Nov.*, 1944.—The enhanced velocity of crystallization of sucrose from pure aq. solutions at high supersaturations may be accounted for on the basis of the increased activity of sucrose in solution. The fundamental rate equation is: velocity $= k(a - a_{satd.})$. The activities are estimated by extrapolations of v.p. data. Published and new data on velocity of growth, even in the presence of pronounced false grain, can be accounted for by this means.

532.872 : 523.841.37 *see Abstr.* 1228

533.15 : 532.72 : 536.2 1311

Diffusion in spherical shells, and a new method of measuring the thermal diffusivity constant. BARRER, R. M. *Phil. Mag.*, 35, *pp.* 802–811, *Dec.*, 1944.—A group of exact solutions of the diffusion equation is obtained. These relate to the flow of heat or matter in spheres and spherical shells. A number of cases of practical importance are considered, and numerous examples are given of processes in which diffusion may be the limiting factor, e.g. permeability of bubble films; solvent extraction by one liquid of a solute dissolved in a second liquid which is immiscible in the first and is dispersed in it as an emulsion; nitriding or carburizing spherical iron shells or spheres such as ball bearings. The method for measuring the diffusivity constant, K (involving the use of Bessel functions) is a continuation of previous work [Abstr. 1704 (1940)]. It is well suited for measuring K for poor conductors and insulators. The values of K are given for various substances, e.g. ebonite, brick, glass, granite. L. S. G.

533.15 = 3 1312

On the diffusion of inert gases through different organic foils and its relation to aqueous permeability. Diffusion of gases through high polymers. II. MÜLLER, F. H. *Kolloid Z.*, 100, *pp.* 355–361, *Sept.*, 1942.—[See Abstr. 1956 (1941)]. The diffusion of gases and vapours through foils of polystyrol, polyvinylchloride, cellulose-triacetate and cellophane was investigated. The diffusion coefficient decreases with increasing diameter of the molecules. The percentage decrease is nearly the same for the rare gases, independent of the material of the foil. While the diffusion coefficients for H_2O differ between wide limits for the different foils, those for the rare gases, O and N differ but slightly, at most by a factor of 100. The mechanism of diffusion as conditioned by a micro-Brownian movement, the temperature dependence of the diffusion coefficient, and the relationship between the concentration conditions within a foil and the shape of the absorption isotherm, are discussed. R. N.

533.16 1313

The viscosity of gases at high pressures. COMINGS, E. W., MAYLAND, B. J., AND EGLY, R. S. *Bull. Univ. Ill. Engng Exp. Sta., No.* 354, 68 *pp.*, *Nov.* 28, 1944.—The present bulletin contains a study of the effect of pressure on the viscosity of gases, for which purpose CO_2, ethylene, propane, and methane were examined in a short capillary viscometer of the Rankine type suitably modified for work at high pressures. A critical survey of the viscosity data in the literature is also included. Theoretical considerations come first and then follow description and discussion of experiments and apparatus. 14 tables of data are given, and the uncertainty of the results is claimed to be not more than 1 or 2% at the most. A correlation of viscosity data is made, based on the principle of corresponding states; this can be used to predict the viscosity of a gas at any temperature at an elevated pressure, if the critical temperature and pressure and the viscosity at the same temperature and at a pressure

sufficiently low so that the viscosity is independent of the pressure, are available. The theory of the rolling-ball viscometer is included. H. H. HO.

533.21 1314

Compressibilities of nitrogen-carbon dioxide mixtures. HANEY, R. E. D., AND BLISS, H. *Industr. Engng Chem.*, 36, *pp.* 985–989, *Nov.*, 1944.—Compressibility factors and residuals are reported for mixtures of N_2 and 25·13 mole % or 50·48 mole % CO_2 at 25°, 50°, 75°, 100°, and 125°C. and at up to 500 atm. Determinations were made in a const. volume apparatus. Below 200 atm. the Dalton law seems best. Between 250 and 400 atm. Bartlett's proposal is best, and above 400 atm. the Amagat law is superior.

533.6.011 : 536.24 1315

Flow of gas through a tube of constant cross-section with heat exchange through the tube walls. SZCZENIOWSKI, B. *Canad. J. Res. A.*, 23, *pp.* 1–11, *Jan.*, 1945.—The influence of the exchange of heat between a gas flowing through a tube and the outside atmosphere on the pressure in the gas stream is marked in the case of large stream velocities, approximating the velocity of sound. The heat exchange is possible only when the stream velocity is maintained beyond certain limits. The velocity of flow in the tube, if the tube is heated or cooled, shows a tendency to reach the velocity of sound, after which the heat exchange is no longer possible. The heat conductance coefficient is zero when the stream velocity reaches the velocity of sound. This means that it will reach a certain maximum value corresponding to a value of stream velocity which is not exactly known but which will be less than that of sound.

533.6.013 1316

The general motion of the aeroplane. BRODETSKY, S. *Philos. Trans. A*, 238, *pp.* 305–355, *Jan.* 8, 1940.—The aim of the paper is to initiate a systematic mathematical study of the equations of motion of the aeroplane, the equations being solved by a process of successive approximation. The time is not used as the independent variable. Instead one of the Eulerian angles, the pitch, the roll or the yaw, is used, the choice depending upon the nature of the particular motion considered, e.g. the yawning angle when dealing with the spinning motion. In order to obtain the process of successive approximations it is assumed that one component of velocity of the centre of the gravity and also the angular velocity is much larger than the other 2 components. All velocity components are expressed as ratios of the gliding velocity of the machine. It is found that, under certain conditions, Lanchester's phugoids represent a first approximation to general longitudinal normal flight. About one half of the paper is devoted to the longitudinal motion without screw thrust. Then longitudinal motion with engines in action is considered briefly. The three-dimensional motion is finally considered in detail.

L. S. G.

533.6.013 = 6 1317

Statistical data concerning parasitic resistances of aircraft. RE, J. J. *Publ. Fac. Cienc. Fis.-Mat. La Plata, No.* 179, *pp.* 285–292, *Dec.*, 1944.—Based on curves provided by the U.S.A. Dept. of Trade, and combining the results of a number of calculations for different planes, the author extends the scope of the work to include weights up to 30 tons. Theoretical curves are produced from data for British, American, Italian and German designs, showing the relationship of weight to parasitic resistance "equivalent area," for each country. R. M.

533.6.013.22 1318

On an extension of Reynolds' method of finding apparent stresses and the nature of turbulence. CHOU, P. Y. *Chinese J. Phys.*, 4, 1, *pp.* 1–33, 1940.—A new theory of turbulence is developed. It is based upon a set of differential equations of turbulent fluctuation and the solution is obtained by constructing the double and triple correlation functions and the differential equations satisfied by these along the lines followed by Taylor and von Kármán in their statistical theory of isotropic turbulence. L. S. G.

533.6.013.22 1319

Velocity and temperature distributions in turbulent wakes behind an infinite cylinder and a body of revolution. HU, N. *Chinese J. Phys.*, 5, 1, *pp.* 1–29, *July*, 1944.—The distributions are investigated by means of Chou's theory of turbulence [Abstr. 1318 (1945)]. The results for the mean squares of velocity fluctuations across the wake and for the distributions of mean velocity and temperature behind the wakes are in good agreement with experiment [Abstr. 3087 (1932), 3474 (1936), 3511 (1938)]. L. S. G.

533.6.013.22 1320

Velocity and temperature distributions in turbulent wakes behind a row of equally spaced parallel rods and behind a square grid. HU, N. *Chinese J. Phys.*, 5, 1, *pp.* 30–48, *July*, 1944.—Another application of Chou's theory of turbulence [Abstr. 1318 (1945)]. It is found that the velocity and temperature distributions in the first kind of wake are the same cosine functions of the co-ordinates across the wake and those for the second kind of wake are just the superpositions of the wakes behind 2 perpendicular systems of parallel rods. In each case Taylor's law of linear decay of turbulent fluctuation and quadratic increase of scale of micro-turbulence are obtained. L. S. G.

533.6.013.42 : 534.13 = 6 1321

Structural vibrations of the aeroplane. DELFINO, O. P. *Publ. Fac. Cienc. Fis.-Mat. La Plata, No.* 179, *pp.* 293–309, *Dec.*, 1944.—Reviews available information to date on the causes of vibration and countermeasures, dealing mainly with forced vibrations in the wing span resulting from aerodynamic stresses and engine vibration. Theoretical and experimental methods are applied for determining the oscillating frequencies of the plane structures. As a result of work by Pugley, simple formulae can be applied for determining the factors contributing to flutter speed. Dynamic balancing of the masses of adjustable components obviates flutter. R. M.

533.691 1322

The aerodynamics of a ring airfoil. STEWART, H. J. *Quart. Appl. Math.*, 2, *pp.* 136–141, *July*, 1944.—The problem studied is that of the axially symmetric flow past a ring airfoil, the flow, in this case, being especially simple, since the vortex lines in the lifting surface are circular rings and there are no trailing vortices. The differences between this problem and the corresponding two-dimensional problem are dis-

cussed. If the curvature of the chord plane is small, the effect of this curvature on the chordwise lift distribution of a wing is extremely small. If the radius of curvature is small compared with the chord, as it is near the vertex of a cranked wing, this curvature may cause comparatively large changes in the lift distribution. The mathematical analysis used in the paper involves the vector potential method. The vector potential for a vortex ring and for a ring airfoil are found and this leads to an integral equation for the determination of the vortex strength. L. S. G.

533.7 1323

The forces between a hydrogen molecule and a hydrogen atom. MARGENAU, H. *Phys. Rev.*, 66, *pp.* 303–306, *Dec.*, 1944.—The repulsive exchange forces and the attractive van der Waals forces between H^2 and H atoms are calculated [see Abstr. 2162 (1943)]. Results for small distances of separation differ considerably from published data on interaction energies deduced from collision cross-sections. Reasons for the discrepancy are discussed. The potential-energy curves for the region around the van der Waals minimum exhibit the asymmetry of the interaction.

533.7 : 532.7 : 531.19 *see Abstr.* 1244

533.7 : 539.132 *see Abstr.* 1533

533.7 : 539.133 1324

The forces between water molecules and the second virial coefficient for water. MARGENAU, H., AND MYERS, V. W. *Phys. Rev.*, 66, *pp.* 307–315, *Dec.*, 1944.—All long-range forces are computed in a semi-empirical way from dipole moment, quadrupole moment in accordance with the Bernal–Fowler analysis, and optical dispersion data. Interactions caused by quadrupole moments cannot be neglected. Short-range repulsive forces are thus left undetermined; these are chosen such that the second virial coefficient, in its dependence on temperature, approximates its measured values as closely as possible. A reasonable fit cannot be obtained with the hard-sphere model, nor with any model which permits the polar component of the force to be effective at arbitrarily small distances of separation. More satisfactory agreement results if the polar forces are eliminated at all distances smaller than the potential min.

534.013 : 621–272 : 621.316.313 : 621.3.012.8 1325

Network- and differential-analyser solution of torsional oscillation problems involving non-linear springs. CONCORDIA, C. *J. Appl. Mech.*, 12, *A*43–*A*47, *March*, 1945.—[Abstr. 1062 B (1945)].

534.112 = 4 1326

Transverse vibrations of vertically hanging heavy strings. FAVRE, H. *Schweiz. Bauztg*, 122, *pp.* 253–254, *Nov.* 20, *and pp.* 285–287, *Dec.* 4, 1943.—The case considered is that where the mass per unit length of the string is constant but the force along the length is variable. At first the general differential equation of transverse vibration of a heterogeneous string is derived and the special case of a homogeneous vertical string is obtained by a limiting process. The equation, in the latter case, is solved and the physical interpretation of this is discussed. Some simple types of vibrations are discussed as examples. L. S. G.

534.12 : 621.165 1327

The axial vibration of turbine discs. MOODY, A. M. G. *J. Appl. Mech.*, 12, *A*48–*A*54, *March*, 1945.—[Abstr. 1043 B (1945)].

534.121.2 : 517.947.42 *see Abstr.* 1204

534.13 : 533.6.013.42 = 6 *see Abstr.* 1321

534.133 1328

Simultaneous oscillations of a piezo-electric quartz at two frequencies. PANDE, A., PANCHOLY, M., AND PARTHASARATHY, S. *J. Sci. Industr. Res.*, 3, 2 *pp.*, *Aug.*, 1944.—Diffraction patterns are given (*a*) for a quartz plate excited simultaneously at frequencies of 2·83 and 7·90 Mc/s in CCl_4, (*b*) for 2 different crystals excited one at each of the above frequencies in 2 vessels placed close together in the path of the beam of light between the collimator and telescope. The essential identity of the patterns in the two cases, and also when the 2 crystals were excited in the same liquid, indicates that the Bergmann pattern is attributable to a combination of 2 transmission gratings. Experiments are described illustrating the "ziehen" effect obtained by varying the tuning of the oscillator used for exciting a crystal at its 2nd and 3rd harmonics. Both harmonics were obtained for a considerable tuning range, the intensities varying according to the proximity of the oscillator frequency to one or the other harmonic. A. W.

534.142.4 1329

A theory of singing flames. JONES, A. T. *Proc. Amer. Phys. Soc., Providence, R.I., Oct.* 28, 1944. *Abstr. in Phys. Rev.*, 66, *p.* 354, *Dec.*1 *and* 15, 1944.—It is found that flames may sing without standing waves in the gas tube, any vibration in the air tube causing progressive waves to run backward through the gas tube, so that the phase of max. efflux of gas is approx. that of min. pressure in the air tube. When the gas tube ends below in a vessel of some size, standing waves are superposed on the running waves.

534.154 : 621.317.39 1330

The nature of vibration in electric machinery. GRAYBEAL, T. D. *Trans. Amer. Inst. Elect. Engrs*, 63, *pp.* 712–718, *Oct.*, 1944.—[Abstr. 1200 B (1945)].

534.154 : 629.135 : 621.317.39 1331

Aircraft vibration analyser. MARBLE, F. G. *Electronics*, 17, *pp.* 98–99, 180, 184 *and* 188, *Oct.*, 1944.—[Abstr. 1201 B (1945)].

534.2 1332

On the theory of the directional patterns of continuous source distributions on a plane surface. JONES, R. C. *J. Acoust. Soc. Amer.*, 16, *pp.* 147–171, *Jan.*, 1945.—The directional pattern at any distance from an arbitrary continuous distribution of sources is obtained. The approximate relation which holds at large distances is derived, and the simpler formulae which holds when the surface distribution has several different kinds of symmetry are developed. The case is studied in which the sources are distributed along a line. Directional patterns corresponding to specific source distributions are given. Although it is easy to specify a source distribution which will yield any directional pattern satisfying certain mathematical conditions of convergence, such distributions are usually of infinite extent, and therefore do not correspond to distributions of practical interest. A method of attack is

suggested for the problem of determining the distribution of specified extent which yields the directional pattern of optimum characteristics for any particular application. Tables of the directional patterns are given.

534.213 : 534.321.9 *see Abstr.* 1339

534.22 1333

Doppler effect—a lecture demonstration. JENSEN, A. S. *Amer. J. Phys.*, 13, *pp.* 39–40, *Feb.*, 1945.—Two loudspeakers emitting notes of different frequency are used, one is stationary, the other may be swung as a pendulum, thereby altering the beat frequency.

534.22 : 532.74 *see Abstr.* 1306

534.22 : 534.321.9 1334

Ultrasonic velocities in some vegetable oils. PANCHOLY, M., PANDE, A., AND PARTHASARATHY, S. *J. Sci. Industr. Res.*, 3, 2 *pp.*, *Sept.*, 1944.—Vegetable oils were tested to elicit relationships between chemical constitution and sound velocity. Ultrasonics of about 7 Mc/s were employed in the diffraction of a parallel monochromatic beam, and the velocities found varied between 1 381 m/s (olive oil) and 1 825 m/s (mustard oil), for temperatures just over 30°C. The adiabatic compressibilities of the oils were also measured. The low velocities obtained for these glycerides of fatty acids are explained as a known effect in esters; and glycerine ($V > 1\,900$ m/s) takes 3 acid radicals during esterification. G. E. A.

534.221 = 4 1335

Measurement of the speed of propagation of sound in CO and D^2 as a function of the pressure at the temperatures of liquid oxygen and liquid hydrogen. The second virial coefficient of D^2. VANDONINCK, W. *Physica, 's Grav.*, 10, *pp.* 481–492, *July*, 1943.—The experimental method (using an acoustic interferometer) is described, and the method is given for preparing the CO and D^2. It is found that the speed of propagation (W) may be written in the form $W = W_0(1 + sp)$ for CO and $W^2 = W_0^2(1 + Pp + Qp^2)$ for D^2. Here s, P and Q are functions of the temperature and p is the pressure, and their numerical values are given over the ranges 90°–70° abs. and 21°–19° abs. For the second virial coefficient B an empirical formula is obtained, giving B as a function of temperature thus: $B = a_0 + \sum_{l=1}^{5} a_l/T^l$ where $a_0, a_1, \ldots, a_5$ are certain numerical constants. This formula holds over the range 42°–19° abs. L. S. G.

534.222.2 1336

Note on Becker's theory of the shock front. THOMAS, L. H. *J. Chem. Phys.*, 12, *pp.* 449–457, *Nov.*, 1944.—Becker [Abstr. 1481 (1922)] concludes, inter alia, that violent shocks have a thickness small compared with a free path and that the relevant Boltzmann equation is not applicable. It is shown that this last conclusion rests on an oversight; the thickness of a shock front is always at least of the order of magnitude of a free path, and it is to be expected that the Boltzmann equation can be applied even for the most violent shocks.

534.3 : 788.6 1337

Intonation of the Boehm clarinet. MCGINNIS, C. S., AND PEPPER, R. *J. Acoust. Soc. Amer.*, 16, *pp.* 188–193, *Jan.*, 1945.—When a note is played on the clarinet, the pitch goes down 0·1 semitone in 10 sec. to a steady state. The authors' results refer to the steady state. The testing was done by use of standard forks and Lissajous figures. The range of the clarinet scale is from E_3 to G_6 [C_4 is middle C] and two series of tones were tested, (i) from E_3 to F_4, and (ii) the higher range, from B_4 to C_6, this being such that when a tone in series (i) is produced, a tone in series (ii), a twelfth above, is sounded by pressing a key with the left thumb. The ideal condition of correct twelfths is not attained in practice and the common trend of the errors is indicated by a composite graph in which the pitch is sharp at both ends of the scale and flat in the middle. Suggestions drawn from the experimental results corroborate the ideas of Potter who had improved the intonation by the addition of 3 auxiliary holes. G. E. A.

534.321.9 1338

Some applications of ultrasonics in high-polymer research. MARK, H. *J. Acoust. Soc. Amer.*, 16, *pp.* 183–187, *Jan.*, 1945.—Conc. solutions of high polymers, e.g. 5–10% gelatine in water or rubber in toluene, when subjected to an ultrasonic field decrease in viscosity. The system may or may not recover its viscous property. This has been interpreted as follows: High polymer solutions do not possess viscosity as defined by the fundamental laws. Their long and irregular entangled chain-like molecules form a flexible framework inside which the solvent is entrapped and immobilized, and standing ultrasonic waves destroy this network of molecules. Irreversible decrease of viscosity is explained by a decrease in mol. wt., i.e. a chemical degradation. An estimation of the average shape of the macromolecules gives their length as 1 500 Å and dia. 15 Å for a polystyrene molecule of mol. wt. 100 000. It is concluded that chemical degradation of high-polymer molecules by ultrasonic waves can be explained by present knowledge of the strength of covalent bonds and the structure of high-polymer molecules. G. E. A.

534.321.9 : 534.213 1339

Wave-front determination in a supersonic beam. LABAW, L. W. *Proc. Amer. Phys. Soc., Providence, R.I., Oct.* 28, 1944. *Abstr. in Phys. Rev.*, 66, *p.* 354, *Dec.* 1 *and* 15, 1944.—The electric signal generated by the piezo-electric action of a Rochelle-salt microphone which has the phase of the compressional wave in the liquid, is modulated by a small electric signal which has the phase of the voltage impressed across the quartz ultrasonic generator. The sum of these signals gives a sinusoidal space variation of the r.m.s. value of the h.f. signal generated by the microphone as it is moved in the direction of propagation of the beam. The locus of points of equal phase is found by following a max. in this sinusoidal space variation as the microphone is moved at right angles to the direction of propagation.

534.321.9 : 534.22 *see Abstr.* 1334

534.321.9 : 534.511.1.08 1340

Theory of ultrasonic interferometer. KRASNOOSHKIN, P. E. *J. Phys., U.S.S.R.*, 7, 2, *pp.* 80–91, 1943.—A theory is developed which takes into account the

lack of uniformity in the distribution of amplitudes of vibrations over the radiator of the interferometer. Corrections in existing methods of measuring the absorption and velocity of ultrasonic waves in gases are proposed. E. R. A.

534.321.9 : 536.633 *see Abstr.* 1491

534.372 : 621.3.012.8 : 629.123.037 : 621.313.2–9 1341

Damping effect of d.c. marine propulsion motors on vibrations produced in drive shafts by large propellers. LITMAN, B. *Trans. Amer. Inst. Elect. Engrs*, 64, *pp.* 31–37, *Jan.*, 1945.—[Abstr. 1110 B (1945)].

534.511.1.08 : 534.321.9 *see Abstr.* 1340

534.522.5 1342

Recording Lissajous figures. HALES, W. B. *J. Acoust. Soc. Amer.*, 16, *pp.* 137–146, *Jan.*, 1945.—The apparatus consisted of a Blackburn pendulum of length 325 cm. The bob was a 12 lb. brass ball with a rack to carry 2 dry cells and a housing for the flashlight globe. A piece of 11 × 14 in. double weight Kodabromide F5 paper was held in place by a printing board. Several patterns are illustrated.

534.61 1343

An investigation of the performance of the Rayleigh disc. SCOTT, R. A. *Proc. Roy. Soc. A*, 183, *pp.* 296–316, *Feb.*, 1945.—Describes an investigation of the law which governs the torque on a Rayleigh disc in a sound field. The torque follows an expression of the form obtained theoretically by König, provided the finite thickness and inertia of the disc are taken into account. The measurements were made over the range 250–4 000 c/s. The correction for thickness of the disc is large and has a sign opposite to that determined theoretically by König for ellipsoidal discs. The smoke-particle method of Andrade and of Carrière was adapted to the measurement of the oscillatory velocity of the field and determinations of oscillatory velocity within 1% for up to 4 000 c/s were made. The experimental work confirms that the stability of behaviour of the Rayleigh disc justifies its continued use as a reference standard of acoustical intensity.

534.611 1344

Acoustic intensity distribution from a piston source. WILLIAMS, A. O., JR., AND LABAW, L. W. *Proc. Amer. Phys. Soc., Providence, R.I., Oct.* 28, 1944. *Abstr. in Phys. Rev.*, 66, *pp.* 353–354, *Dec.*1 *and* 15, 1944.—The expression deduced permits determination of the cone on the surface of which the intensity falls to one-tenth the axial value. General agreement with quantitative data is satisfactory.

534.64 1345

A wide-range adjustable acoustic impedance. MEEKER, W. F., AND SLAYMAKER, F. H. *J. Acoust. Soc. Amer.*, 16, *pp.* 178–182, *Jan.*, 1945.—A device whose impedance may be calculated from the dimensions. It consists of 3 adjustable tubes with dia. 1 in., 0·38 in. and 0·13 in. These slide telescopically and attached collars maintain an acoustically tight connection. The relation between the input impedance and the length of a tube is represented by a spiral obtained by analogy with the input impedance of a length of electrical transmission line. The range of the device is from $R = 0{\cdot}02$ acoustic ohm to $R = 340$ acoustic ohms. In certain regions the input impedance is very sensitive to changes of frequency and temperature. G. E. A.

534.773 : 612.85 1346

Air- and bone-conduction audio testing assembly. WATSON, N. A. *J. Acoust. Soc. Amer.*, 16, *pp.* 194–196, *Jan.*, 1945.—The difficulties and errors encountered in testing the hearing of a person by means of the usual equipment are detailed, and an assembly is designed to overcome these. It consists of an adjustable chair and chin rest, a pair of large adjustable acoustically treated air-conduction source boxes to be placed one over each ear, and an adjustable bone-conduction vibrator for the centre of the forehead. The system provides at small cost, and without a sound-proof booth, the means for making, in a quiet but not silent room, most of the tests required for diagnosis and specification of hearing amplifiers. G. E. A.

534.773.2 1347

Fundamentals of hearing-aid design. PENN, W. D. *Trans. Amer. Inst. Elect. Engrs*, 63, *pp.* 744–749, *Oct.*, 1944.—The fundamental characteristics of speech and hearing which are of importance in the development and design of hearing aids are discussed. Selective amplification in which the l.f. are attenuated is necessary in some cases and beneficial in most. A valve circuit is described and its theory given for obtaining this type of amplification. The pressure of the loudest vowel sounds at the microphone is about 0·6 dynes/cm.^{2}, and to overcome the severest case of deafness this must be increased to 200. The acoustical amplification is 333·3, corresponding approx. to 50 db. The max. usable gain depends on the surrounding noise level and on the acoustic feedback, and may amount to 60 to 70 db. G. E. A.

534.845 1348

Absorption of sound by porous material. II–V. ZWIKKER, C., VAN DEN EIJK, J., AND KOSTEN, C. W. *Physica, 's Grav.*, 8, *pp.* 469–476, *May;* 1094–1106, *Dec.*, 1941, *and* 10, *pp.* 239–247, *April*, 1943.—A new theory of absorption is proposed, according to which the acoustic behaviour of porous materials with a rigid skeleton may be described by means of 3 constants, (i) the specific resistance of the air, (ii) the cavity-factor, and (iii) the structure-factor. Tests of the theory were made on a sound-absorbing material made from lemonade-straws and from small glass tubes placed at angles of 90° and 60° to the wavefront. More detailed experiments are described relating to samples composed of glass tubes of 1·6, 4·0 and 9·4 mm. diameter. The results are compared with those of other authors. L. S. G.

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