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MODELLING OF WATER - SOLID MIXTURES BASED ON STATISTICAL AVERAGES

Summary: This paper is intended to be a reference on the basic formulation of various physical and mathematical models of suspension flow systems. A discussion on the formulation of the conservation laws of mass, momentum and energy is presented. Special emphasis has been on the local instant formulation and on the time and space averaged statistical models. These can be used to solve various practical problems by analytical means or to set up experimental facilities, to clarify the purposes of experiments and to establish the methods of measurements.

1. Introduction

As long as the particles of a particulate two - phase flow are small enough, they are following the fluid motion and the whole flow can be treated as a homogeneous flow with appropriate changed physical properties. This kind of particulate two - phase flow, which can be considered basically as a single phase flow, will not be analysed in this paper. It is evident that in this case the standard method of continuum mechanics are followed. In this paper it is presumed that the particles are large enough so that there exists a slip velocity between the two phases.

Most of the available experimental work on particulate flow is dealing, on the one hand, with the flow of and around single particles in gas or liquid and on the other hand, with the flow of suspensions, where only integral parameters are to characterize the flow properties. Through detailed experimental studies, it is also possible to see whether information on the flow around single particles can be transfered to particulate flow systems. In addition to this, numerical models of two - phase flow can only be developed if there exists reliable local experimental data, which can be compared with theoretical results.

It was the aim of some publications [2][9] the particulate two - phase flow theoretically and / or numerically dealt in two different ways and there are two different approaches to describe the transport equations for the particulate phase. The lagrangian approach treats the fluid phase as a continuum and predicts the trajectory of a single particle in the fluid flow as result of various forces acting on the particle. Assuming

different starting positions of the particles and following their trajectories, a solid - fluid flow can be simulated. As the locations of the particles are known, the mass-, momentum- and energy transfer of the solid to the fluid phase and vice versa can be calculated. The main problem here is that the equation for the conservation of the general variable contains the additional source term representing the net efflux of the general variable into the fluid phase due to the particle - fluid interaction. This source term is usually calculated [2] by solving the Lagrangian equation for the corresponding particle variable. This is the essence of the so-called PSI - cell (particle - source in cell) approach, which was used by Sharma [6] and others [1][7]. The calculation of the particle trajectories by solving the equations of motion of the particles, as well as the calculation of the particle source terms is performed in the same iteration loop after solution of the fluid flow equations, in order to handle the interaction between the fluid and the particles.

Treating also the particle phase as a continuum and solving the appropriate for the fluid and the particle phase makes up the basic feature of the eulerian approach. The volume concentration of each phase has to be introduced and the continuity equation for both phases solved. The back - influence of the solid particles to the fluid has then to be proportional to the void fraction of the solid phase. The influence of fluid turbulence on the particles or vice versa the influence of the particles on fluid flow turbulence are neglected. It is assumed that there are no particle - particle interactions and that the interaction of the particles on the wall can be neglected. This model has been completed by J. O. Hinze [3].

It was shown in [5] that for phenomenological modeling of two - phase flow by means of two - fluids model formulation, the same type of momentum equation for particulate phase can be obtained whether derived on the basis of a single particle motion or under the condition on continuous mechanics. Differences has been mentioned only by expressing of individual formulation the general force exerted by the fluid on a particulate phase and by various methods of variables averaging [6].

2. Purpose of averaging

It is well established in continuum mechanics that the conceptual models for single phase flow are formulated in terms of field equations which describe the conservation laws of mass, momentum, energy, charge, etc. These field equations are then completed by appropriate constitutive equations such as the constitutive equations of state, stress, chemical reactions etc., which specify the thermodynamic, transport and chemical properties of a given constituent material, i.e., of a specified solid or liquid.

In order to appreciate the difficulties in deriving balance equations for structured media, it should be recalled that in continuum mechanics the field theories are constructed on integral balances of mass, momentum

and energy. Thus, if the variables in the region of integration are continuously differentiable and the Jacobian transformation between material and spacial coordinates exists, then the Euler type differential balance can be obtained by using the Leibnitz's rule or more specifically the Reynolds transport theorem which allow to interchange differential and integral operations.

Depending on the basic physical concept used to formulate thermo-mechanical problems, averaging procedures can be classified into three main groups, namely the Eulerian averaging, the Lagrangian averaging and the Boltzmann statistical averaging. They can be further divided into subgroups based on a variable with which a mathematical operator of averaging is defined. The above classifications and the definitions of various averaging are given in Table 1, where $\xi = \xi(x,t)$, v_p means the phase velocity, i.e. the velocity of individual particles, and $\mathcal{F}_{k,n}$ is the probability or particle density function of k^{th} - phase containing n particles.

Table 1

Function	Euler	Lagrange	Boltzmann
Time mean value	$\frac{1}{\Delta t}\int_{\Delta t} F(x,t)\,dt$	$\frac{1}{\Delta t}\int_{\Delta t} F(\xi,t)\,dt$	
Spatial mean value	$\frac{1}{\Delta V}\int_{\Delta V} F(x,t)\,dV(x)$		
Statistical mean value	$\frac{1}{n}\sum_{n=1}^{n} F_n(x,t)$	$\frac{1}{n}\sum_{n=1}^{n} F_i(\xi,t)$	
Molecular density function			$f(x, v_p, t)$
Transport properties			$\frac{\int \psi(v_p)\,\mathcal{F}_{k,n}\,dv_p}{\int \mathcal{F}_{k,n}\,dv_p}$

The most important and widely used group of averaging continuum mechanics is the Eulerian averaging, because it is closely related to human observations and classical instrumentations. The basic concept of Lagrangian averaging is directly related to the Lagrangian description of mechanics. As the particle coordinate ξ displaces the spatial variable x of the Eulerian description, this averaging is naturally fitted to a study of the dynamics of a particle. Thus if our interest is on the behaviour of an individual particle rather than the collective mechanics of a group of particles, the Lagrangian average is important and useful for analyses. In opposite to them, the Boltzmann statistical averaging is important when the collective mechanics of large number of particles are in question. As the number of particles and their interactions between them increase, the behaviour of any single particle becomes so complicated and diversified, it is not practical to solve for each particle. In such a case the behaviour of a group of many particles increasingly exhibits some peculiar characteristics which are different from a single particle as the collective particle mechanics becomes a governing factor.

3. Balance equation and model formulation

The obvious method is to average the balance laws in time and space and the results applied to the conservation laws of mass, momentum and energy of each phase. We consider the general balance equation of an arbitrary quantity ψ in the following form [6]

$$\frac{\partial \rho \psi}{\partial t} + \nabla.(\rho \psi v) + \nabla.\tau - \rho\phi = 0 \tag{1}$$

Here τ and ϕ represent the generalized tensor efflux and the source of ψ, respectively. Since it is multiplied by the density ρ, the quantity ψ is expressed as the amount per unit mass. Thus the above equation itself is a mathematical statement of the balance of the quantity in an unit volume of the medium. This is an important point to be remembered when we compare it to the surface balance equation in the course of the time averaging.

Using for example the Eulerian time and phase average the balance equation (1) for k^{th} -phase becomes

$$\frac{\partial \alpha_k \langle \bar{\rho}_k \rangle \psi_k}{\partial t} + \nabla.\left[\alpha_k \langle \bar{\rho}_k \rangle \bar{\psi}_k \bar{v}_k\right] + \nabla.\left[\alpha_k\left(\langle \bar{\tau}_k \rangle + \langle \overline{\rho_k \psi_k' v_k'} \rangle\right)\right] - \alpha_k \langle \bar{\rho}_k \rangle \phi_k - I_k = 0 \tag{2}$$

The time averaged phase density function α_k - the local time fraction - has been defined as

$$\alpha_k(x,t) = \frac{1}{\Delta t}\int_{\Delta t} H_k(x,t)\,dt = \bar{H}_k \tag{3}$$

where H_k is the state density function defined for two phase flow as

$$H_k(x,t) = \begin{cases} 1 \\ 0 \end{cases} \qquad k = 1 \text{ or } 2 \tag{4}$$

The function α_k which appears only after the integral operation is a fundamental parameter in studying the time averaged field equations. Physically α_k represents a probability of finding the k^{th} -phase, thus it expresses the geometrical importance of that phase. Apart from the time fraction α_k the concentration based on mass can be defined. In analogy with the theory of diffusion, the mass concentration c_k is given by

$$c_k = \frac{\alpha_k \langle \bar{\rho}_k \rangle}{\rho_m} \tag{5}$$

where the notation $\langle\ \rangle$ means the phase average, so that

$$\langle \bar{\rho}_k \rangle = \frac{\bar{\rho}_k}{\alpha_k} = \frac{\overline{H_k \rho_k}}{\bar{H}_k} \tag{6}$$

In developing a general model based on the mixture properties, it is necessary to express an average convective flux by various mean values. Assuming the turbulence fluctuating components of every physical quantities, it shows that the average convective flux can be split into two parts according to the different transport mechanisms. The turbulent flux

is included into the third term of (2) and the last term I_k represents the interfacial for k^{th} -phase.

So the fundamental purpose of averaging has been accomplished. Thus the original two phases which are alternatively occupying a point have been transformed into two coexisting continua.

Now we shall present balance equations for the two - fluid model and for the diffusion model separately.

In order to obtain mass balance equations for two - fluid model, we set

$$\psi_k = 1, \quad \tau_k = 0, \quad \phi_k = 0, \quad I_k = \Gamma_k \tag{7}$$

and by substituting (7) into (2) we get

$$\frac{\partial \alpha_k \langle \bar{\rho}_k \rangle}{\partial t} + \nabla \cdot \left(\alpha_k \langle \bar{\rho}_k \rangle \bar{v}_k \right) = \Gamma_k \tag{8}$$

where $\sum_{k=1}^{2} \Gamma_k = 0$. Equation (8) is the continuity equation for each phase with the interfacial mass source Γ_k appearing on the right hand side due to phase changes.

Analogously the macroscopic momentum balance can be obtained from (2) by setting

$$\psi_k = v_k, \quad \tau_k = p_k \delta - \pi_k, \quad \phi_k = g_k \tag{9}$$

where π_k is the stress tensor of the phase k, p means the pressure and δ the unit tensor.

The macroscopic balance equation which have been derived from the time and phase averaging and applied to the conservation laws of mass, momentum and energy for both models, have been shown in Tab. 2 with following definitions:

Table 2

Balance	Two - Fluid Model	Diffusion Model
Mass	$\frac{\partial \alpha_k \langle \bar{\rho}_k \rangle}{\partial t} + \nabla \cdot \left(\alpha_k \langle \bar{\rho}_k \rangle \bar{v}_k \right) = \Gamma_k$	$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \left(\rho_m v_m \right) = 0$
Momentum	$\frac{\partial \alpha_k \langle \bar{\rho}_k \rangle \bar{v}_k}{\partial t} + \nabla \cdot \left(\alpha_k \langle \bar{\rho}_k \rangle v_k v_k \right) + \nabla \left(\alpha_k \langle \bar{p}_k \rangle \right) - \nabla \cdot \left[\alpha_k \left(\langle \bar{\tau}_k \rangle + \langle \overline{\rho_k v_k' v_k'} \rangle \right) \right] - \alpha_k \langle \bar{\rho}_k \rangle g_k - I_k = 0$	$\frac{\partial \rho_m v_m}{\partial t} + \nabla \cdot \left(\rho_m v_m v_m \right) + \nabla p_m - \nabla \cdot \left(\pi_m + \pi_m^T + \pi_m^D \right) - \rho_m g_m - I_m = 0$
Energy	$\frac{\partial}{\partial t} \left[\alpha_k \langle \bar{\rho}_k \rangle \left\langle \frac{\bar{v}_k^2}{2} + \frac{\overline{v_k'^2}}{2} \right\rangle \right] + \nabla \cdot \left[\alpha_k \langle \bar{\rho}_k \rangle \left\langle \frac{\bar{v}_k^2}{2} + \frac{\overline{v_k'^2}}{2} \right\rangle \bar{v}_k \right] + \nabla \cdot \left(\alpha_k \langle \bar{p}_k \rangle \bar{v}_k \right) - \nabla \cdot \left[\alpha_k \langle \bar{\pi}_k \rangle \bar{v}_k \right] - \alpha_k \langle \bar{\rho}_k \rangle \bar{g}_k \bar{v}_k - E_k = 0$	$\frac{\partial}{\partial t} \left[\rho_m \left(\varepsilon_m + \frac{v_m^2}{2} \right) \right] + \nabla \cdot \left[\rho_m \left(\varepsilon_m + \frac{v_m^2}{2} \right) v_m \right] + \nabla \cdot \left(p_m v_m \right) - \nabla \cdot \left(\pi_m \cdot v_m \right) - \rho_m g_m v_m - E_m = 0$

$$p_m = \sum_{k=1}^{2} \alpha_k \langle \bar{p}_k \rangle \qquad \rho_m = \sum_{k=1}^{2} \alpha_k \langle \bar{\rho}_k \rangle$$

$$g_m = \frac{\sum_{k=1}^{2} \alpha_k \langle \bar{\rho}_k \rangle g_k}{\rho_m} \qquad v_m = \frac{\sum_{k=1}^{2} \alpha_k \langle \bar{\rho}_k \rangle \bar{v}_k}{\rho_m} = \sum_{k=1}^{2} c_k \bar{v}_k$$

$$\pi_m = \sum_{k=1}^{2} \alpha_k \langle \bar{\pi}_k \rangle \qquad \varepsilon_m = \frac{\sum_{k=1}^{2} \alpha_k \langle \bar{\rho}_k \rangle \left(\bar{u}_k + \frac{\bar{v}_k^2}{2} \right)}{\rho_m} \qquad (10)$$

$$\pi_m^T = -\sum_{k=1}^{2} \alpha_k \langle \overline{\rho_k v_k' v_k'} \rangle$$

$$\pi_m^D = -\sum_{k=1}^{2} \alpha_k \langle \bar{\rho}_k \rangle \overline{(v_m - v_k')(v_m - v_k')}$$

and where term's expressing the heat transfer has been neglected.

It should be mentioned, that the basic concept of the diffusion model is to consider the mixture as a whole, rather than two phases separately. It is evident that the diffusion model formulation will be simpler than the two - fluid model, however it requires some drastic constitutive assumption and with them some of the important characteristics of two - phase flow will be lost.

It is evident, that equations of both models have the same form but the physical quantities have another meaning. It would be possible to apply through the same way the Lagrangian or Boltzmann averaging and become another form of balance equations with more complicated terms and another definitions of physical quantities.

In summarizing this paper it can be said that the usually used models, describing the motion of suspended solid particles in a liquid stream can be formulated on the basic of general balance equation by means of various method of averaging.

Notation

- c - mass concentration
- E - energy source from interfaces
- g - acceleration due to gravity
- I - interfacial source term
- M - state density function
- p - pressure
- t - time
- u - internal energy
- v - velocity
- x - spatial coordinate
- α - local time fraction
- Γ - mass generation
- δ - unit tensor

∇ - Laplace operator
ξ - particle coordinate
$\mathcal{T}$ - stress tensor
ϱ - density
τ - tensor efflux
ψ - arbitrary quantity

Subscript
k - k^{th} -phase
m - mixture
p - particle

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MODELOWANIE MIESZANIN WODA - CIAŁO STAŁE
NA PODSTAWIE STATYSTYCZNYCH WARTOŚCI ŚREDNICH

S t r e s z c z e n i e

W artykule opisano możliwości podstawowego fizycznego i matematycznego formułowania modelowania ruchu przepływu dwóch wielofazowych ośrodków w stosunku do przepływu mieszaniny dwufazowej składającej się z unoszącej cieczy z dyspergowanymi w niej stałymi cząstkami.

Podstawę wszystkich proponowanych modeli jest idea mechaniki kontynium.

W proponowanych modelach opisujących potoki z zawiesinami uwzględnia się uśredniony względny ruch komponentów potoku i hydrodynamiczne oddziaływanie fragmentów zawiesin z unoszoną cieczę.

Na podstawie znanych metod uśrednienia wielkości fizycznych według Eulera, Lagrange'a i Boltzmanna pokazana jest możliwość ułożenia podstawowego równania bilansu dowolnej wielkości.

Przypuszcza się, że w przypadku dwufazowego potoku ruch osobno wziętych cząstek z każdego składnika jego jego komponentu jest chaotyczny, dlatego w dowolnie obranym punkcie w obszarze potoku w dowolnym momencie czasu może znajdować się przypadkowo jakakolwiek stała cząstka lub unosząca się ciecz.

Z punktu widzenia tego prawdopodobieństwa pokazano uśrednienie według jednakowych faz i czasu, jako przykład określone prawa zachowania masy, pędu i energii dla dwóch różnych modeli.

Model stały rozpatruje przepływ dyskretnej struktury zawiesiny jako przepływ dwucieczowy, model drugi opisuje zawiesinę jako jednocieczowy kontynium. Porównując otrzymane równania można wskazać na poszczególne zasady rozwiązania z uwzględnieniem określenia podstawowych charakterystyk.

МОДЕЛИРОВАНИЕ СМЕСИ ВОДА - ТВЁРДОЕ ТЕЛО НА ОСНОВЕ
СТАТИСТИЧЕСКИХ СРЕДНИХ ЗНАЧЕНИЙ

Р е з ю м е

В статьи описаны возможности основной формулировки физического и математического моделирования течения двух многофазных сред с отношением к течению двухфазных смесей, состоящих из несущей жидкости с диспергированными в ней твердыми частицами, в основе всех предложенных моделей лежит идея механики континуума. В предложенных моделях, описывающих взвесенесущие потоки учитывается осредненое относительное движение компонентов потока и гидродинамическое взаимодействие элементов взвеси с несущей жидкостью.

На основе известных метод осреднений физических величин по Эйлеру, Лагранжу и Больтзманну показана возможность оформления основного уравнения баланса любой величины. Предполагается, что в случае двухфазного потока, движение отдельно взятых частиц каждого из составляющих его компонентов является хаотическим, поэтому в любой фиксированной точке в области потока в любой момент времени может находиться случайным образом или какая-нибудь твердая частица или несущая жидкость. С точки зрения этой вероятности показано осреднение по одинаковым фазам и времени и как пример определенны законы сохранения массы, количества движения и энергии для двух разных моделей. Твердая модель рассматривает течение дискретной структури взвеси как течение двухжидкостное, вторая модель описивает взвес только одножидкостным континуумом. Сравнением изисканных уравнений можно показать отдельные принципы решения с учетом определения основных характеристик.