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MODIFIED BIASED ANISOTROPIC DIFFUSION FOR COLOR IMAGE PROCESSING

Summary. In this paper a new approach to the problem of noise reduction in color images is presented. The new technique is based on a modification of the biased anisotropic diffusion. In the modified iterative scheme, the input noisy signal is replaced by an image processed by a nonlinear multichannel filter. The experiments revealed, that the proposed solution significantly excels over the standard anisotropic diffusion methods in case of the mixed Gaussian and impulse noise contaminating the color image. The main benefits of the proposed approach are its rapid convergence to the final stable state, low computational complexity and good performance in complex noise scenarios.

MODYFIKACJA OBCIĄŻONEJ DYFUZJI ANIZOTROPOWEJ DO PRZETWARZANIA BARWNYCH OBRAZÓW CYFROWYCH

Streszczenie. W artykule przedstawiono nowe podejście do problemu redukcji szumów w barwnych obrazach cyfrowych. Nowa metoda filtracji oparta jest na modyfikacji obciążonej dyfuzji anizotropowej. W zmodyfikowanym algorytmie iteracyjnym wejściowy obraz zakłócony zastępowany jest przez wyjście wielokanałowego filtru nieliniowego. Przeprowadzone eksperymenty wykazały, że zaproponowana technika znacząco przewyższa standardową metodę dyfuzji anizotropowej w przypadku obrazów zakłóconych przez mieszany szum gaussowski i impulsowy. Główną zaletą zaproponowanej metody jest jej szybka zbieżność do stabilnego stanu końcowego, niska złożoność obliczeniowa i duża efektywność w przypadku szumów o skomplikowanej strukturze.

1. Introduction

Recently, growing attention has been given to the nonlinear processing of vector valued noisy image signals through the anisotropic diffusion technique. Anisotropic diffusion is a powerful method which allows to reduce the image noise without blurring the edges between image regions of different chrominance or brightness.

The technique of anisotropic diffusion (AD), has been introduced in [1], in order to selectively enhance contrast and reduce image noise, using a modified heat diffusion equation and the concepts of scale space. The main idea of anisotropic diffusion is based on the modification of the isotropic diffusion equation, so that the smoothing across image edges can be inhibited. This modification is done by introducing a conductivity function that encourages intra-region over inter-region smoothing.

One of the problems encountered using the AD for practical implementations is the need of establishing a stopping criterion for the iterative AD process. Although various rather heuristic approaches based on the histogram of the image gradient values have been proposed in the literature [2], setting a stopping criterion which guarantees the optimal denoising results remains still a problem. Therefore, in this paper the extended AD scheme, called *biased* AD (BAD) is examined, as this scheme converges to a stable solution, which depends only on the parameters of the conductivity function used for the evolution of the AD process. In the paper we show that a slight modification of the classic biased AD, leads to very good denoising results, significantly outperforming those delivered by the standard schemes described in [3].

2. Biased anisotropic diffusion

Let $x(\xi,\eta,t)$ denotes a real-valued function representing the gray scale image, then the equation of linear and isotropic diffusion is

$$\frac{\partial x(\xi,\eta,t)}{\partial t} = c\left[\frac{\partial^2 x(\xi,\eta,t)}{\partial \xi^2} + \frac{\partial^2 x(\xi,\eta,t)}{\partial \eta^2}\right], \tag{1}$$

where ξ,η are the continuous coordinates, t denotes time and c is the constant conductivity (diffusivity) coefficient.

Perona and Malik, [1] suggested that the conductivity coefficient c should be dependent on the image structure and therefore they proposed the following partial derivative equation

$$\frac{\partial x(\xi,\eta,t)}{\partial t} = \nabla\left[c(\xi,\eta)\nabla x(\xi,\eta,t)\right]. \tag{2}$$

The conductivity coefficient $c(\xi,\eta)$ is a monotonically decreasing function of the image gradient magnitude and usually contains a free parameter β, which determines the amount of smoothing introduced by the nonlinear diffusion process. Various functions of $c(\xi,\eta)$ have been suggested in the rich literature [4, 5], however the most popular are those introduced in [1]

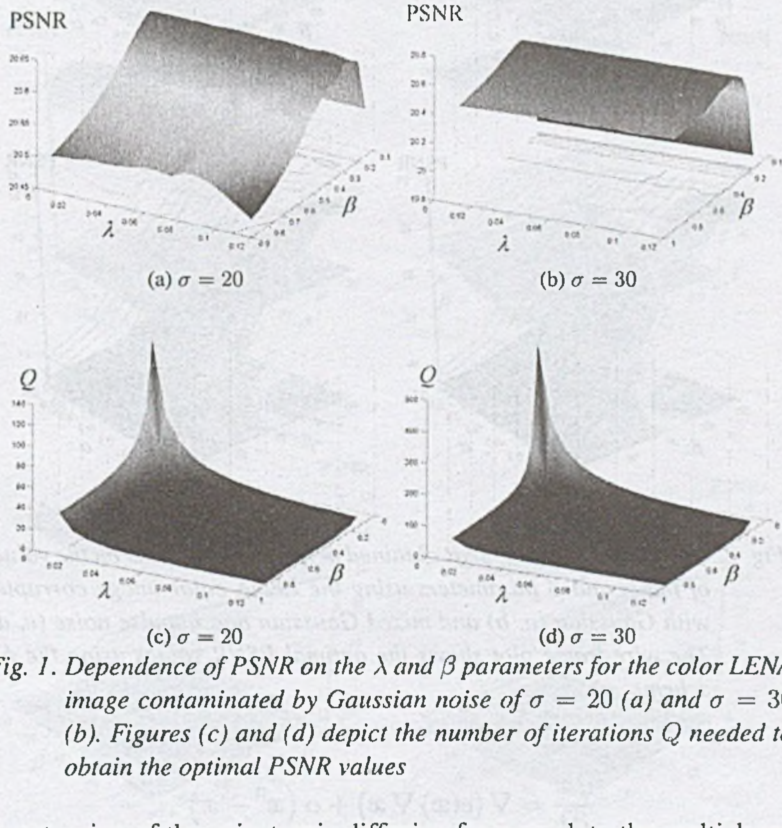
$$c_1 = \exp\left(-\frac{G^2}{2\beta^2}\right),\ c_2 = \left(1+\frac{G^2}{\beta^2}\right)^{-1}, \tag{3}$$

where $G = \nabla x(\xi,\eta)$. The conductivity function $c(\xi,\eta)$ is space-varying and it is chosen to be large in a relatively homogeneous regions to encourage smoothing, and small in regions with high gradients to preserve image edges. The discretized, iterative version of (2) is

$$x_1^{t+1} = x_1^t + \lambda\sum_{k=2}^{n} c_k^t\left[x_k^t - x_1^t\right],\quad \lambda \leqslant \frac{1}{n-1}, \tag{4}$$

where t denotes discrete time, (iteration number), n is the number of pixels in the filtering window, c_k^t for $k=2,\dots n$, are the diffusion coefficients in the directions of the neighbors x_k^t of the central pixel x_1^t in the filtering window, λ is the time increment.

One of the drawbacks of the anisotropic diffusion approach is that the optimal values of parameters β and λ are unknown. Although β can be estimated using some heuristic rules, [2] the algorithm is relatively slow and requires many iterations to achieve the desired solution and also some stopping criterion is needed to finish the iteration process, before the image converges to the trivial solution, (average value of the image pixels). Another disadvantage of the Perona-Malik approach is that this algorithm is not able to cope with impulsive noise and as a result the noisy image goes through the diffusion process without perceptible improvement.



(a) $\sigma = 20$ (b) $\sigma = 30$

(c) $\sigma = 20$ (d) $\sigma = 30$

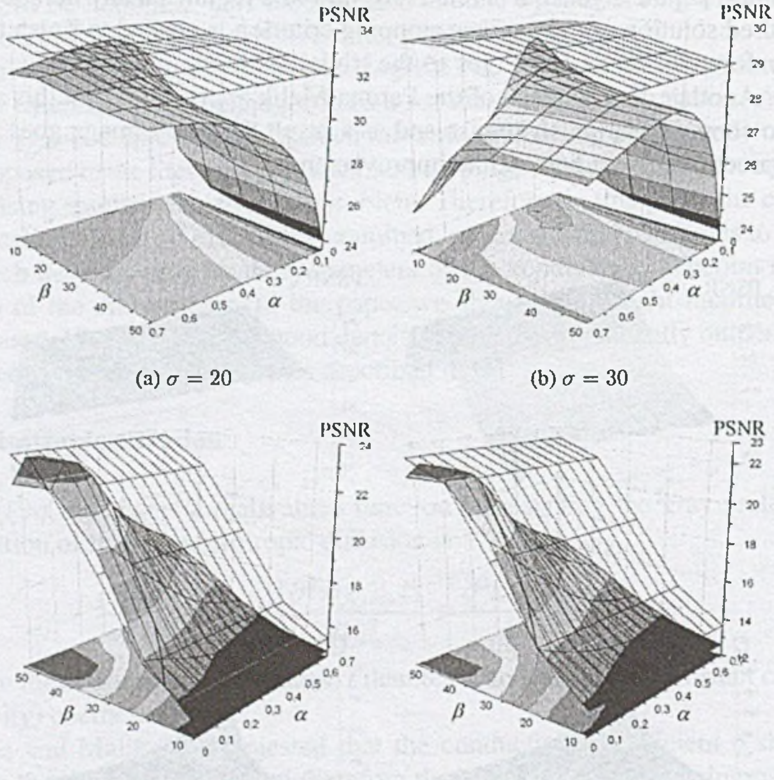
Fig. 1. Dependence of PSNR on the λ and β parameters for the color LENA image contaminated by Gaussian noise of $\sigma = 20$ (a) and $\sigma = 30$ (b). Figures (c) and (d) depict the number of iterations Q needed to obtain the optimal PSNR values

The extension of the anisotropic diffusion framework to the multichannel case is not a very difficult task. Let $x(\xi,\eta,t) = [x_R(\xi,\eta,t), x_G(\xi,\eta,t), x_B(\xi,\eta,t)]$ denotes a color image pixel at position (ξ,η), where $x_R(\xi,\eta,t)$, $x_G(\xi,\eta,t)$, $x_B(\xi,\eta,t)$ are the *red*, *green* and *blue* channels respectively. Equation (2) can be written for the multichannel case as

$$\frac{\partial x(\xi,\eta,t)}{\partial t} = \nabla[\hat{c}(\xi,\eta,t)G], \quad (5)$$

where $\hat{c}(\xi,\eta,t) = f(\|G\|)$ is the conductivity function, the same for each image channel, dependent on the magnitude of the local gradient $G = \nabla x(\xi,\eta,t)$, which couples the three color image channels, [6].

In [3] the so called *biased anisotropic diffusion* has been proposed. This scheme differs from the standard AD approach (4) in an additional term expressing the deviation between the initial color image x^0 and the processed image x



(a) $\sigma = 20$ (b) $\sigma = 30$

(c) $\sigma = 20, p = 0.2$ (d) $\sigma = 30, p = 0.3$

Fig. 2. Dependence of the PSNR obtained performing the BAD on the values of the α and β parameters using the LENA color image corrupted with Gaussian (a, b) and mixed Gaussian and impulse noise (c, d). The wire-frame plot shows the optimal PSNR values using the AD scheme

$$\frac{\partial x}{\partial t} = \nabla\left(c(x)\nabla x\right) + \alpha\left(x^0 - x\right), \tag{6}$$

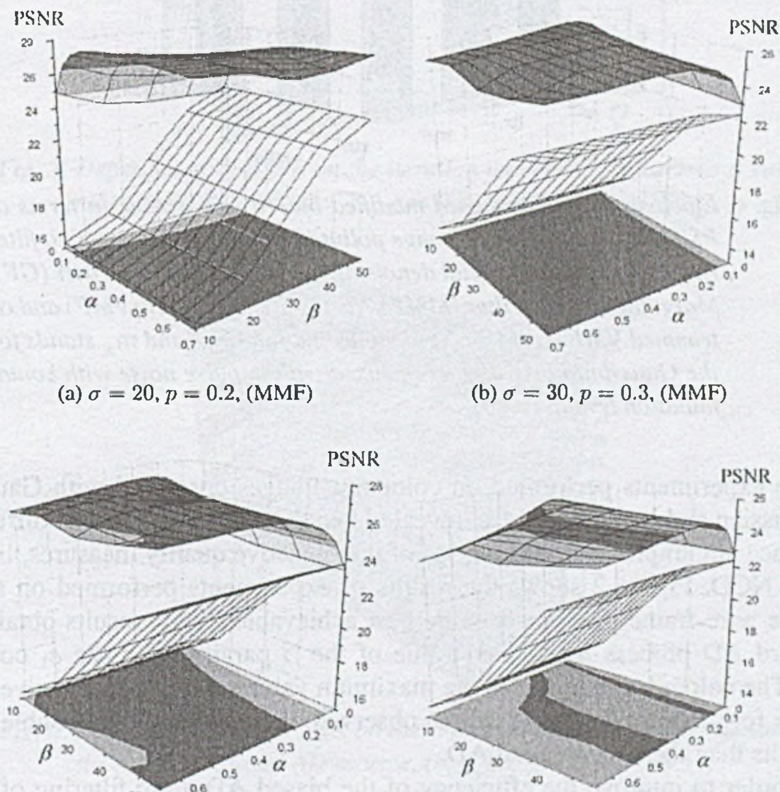
where α is a parameter. The discrete, iterative scheme is then given by

$$x_1^{t+1} = x_1^t + \lambda\alpha(x_1^0 - x_1^t) + \lambda\sum_{k=2}^{n} c_k^t\left(x_k^t - x_1^t\right). \tag{7}$$

The main advantage of the biased AD is its convergence to non-trivial stable solution, which eliminates the need of setting the number of iterations or some stopping criterion to obtain the desired optimal denoising result.

3. Experiments

As can be derived from Eq. (7) the iterative process depends on the parameters λ, β in the definition of the conductivity functions (3) and α. The performed experiments revealed that the dependence on λ is not significant, as it does not influence the final quality of the restoration. Figure 1 shows the dependence of the PSNR restoration quality measure on the parameters λ and β for the AD process performed on the color test LENA image contaminated by Gaussian noise of $\sigma = 20$ and $\sigma = 30$. As can be observed the best possible values of PSNR do not depend on λ, however the value of this parameter influences the number of iterations Q needed to obtain the optimal denoising results, (see Figs. 1c, d). For the experiments reported in this paper the value $\lambda = 0.1$ was chosen, as it yields the desirable denoising results after approximately 10 iterations.



(a) $\sigma = 20, p = 0.2$, (MMF) (b) $\sigma = 30, p = 0.3$, (MMF)

(c) $\sigma = 20, p = 0.2$, (VMF) (d) $\sigma = 30, p = 0.3$, (VMF)

Fig. 3. Dependence of PSNR obtained performing the modified BAD using the LENA color image distorted by mixed Gaussian and impulsive noise on the α and β parameters using: (a, b) marginal median filter (MMF), (c, d) vector median filter (VMF)

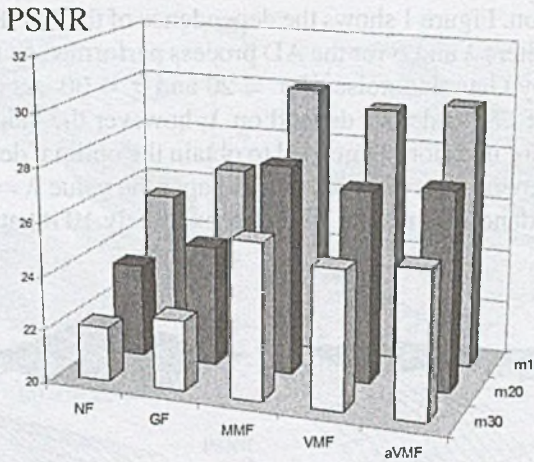


Fig. 4. Efficiency of the proposed modified biased AD scheme in terms of PSNR for the test LENA image polluted by mixed noise with no filtering (NF), and with initial denoising using the Gaussian Filter (GF), Marginal Median Filter (MMF), Vector Median Filter (VMF) and α-trimmed VMF (αVMF). NF denotes 'no filtering' and m_χ stands for the Gaussian noise of $\sigma = \chi$ mixed with impulse noise with contamination level $p = \chi\%$

The experiments performed on color test images corrupted with Gaussian and mixed Gaussian and impulsive noise, revealed that the biased anisotropic diffusion does not introduce any improvements in terms of the objective quality measures, like PSNR, MAE and NCD. Figure 2 shows the results of experiments performed on the LENA image. The wire-frame plot presents the best achievable PSNR results obtained using the standard AD process for a fixed value of the β parameter in the c_1 conductivity function. The colored plot provides the maximum values of PSNR for a given β and α parameters for the biased AD. As can be observed, the biased AD is not able to deliver better results than the conventional AD.

In order to improve the efficiency of the biased AD a pre-filtering of the noisy images was performed. In this way Eq. (6) is changed into

$$\frac{\partial x}{\partial t} = \nabla \left(c(x) \nabla x \right) + \alpha \left(\tilde{x} - x \right) , \qquad (8)$$

where $\tilde{x}$ is the output of a noise reduction filter applied on the initial noisy image x^0. In this way the biased anisotropic diffusion scheme utilizing the pre-filtered image as a reference, leads to a significant improvement of the modified scheme.

Figure 3 shows the results of experiments performed on the color LENA image contaminated by Gaussian and mixed Gaussian and impulse noise. Again the wire-frame plots show the best possible PSNR results obtained for a given β value and the color plots show the dependence on the α and β parameters in (8).

A significant increase of the noise reduction capabilities of the proposed modified scheme was achieved using the non-linear filters intended for the denoising of color images. For the experiments the Gaussian Filter, (GF) Vector Median Filter (VMF), the

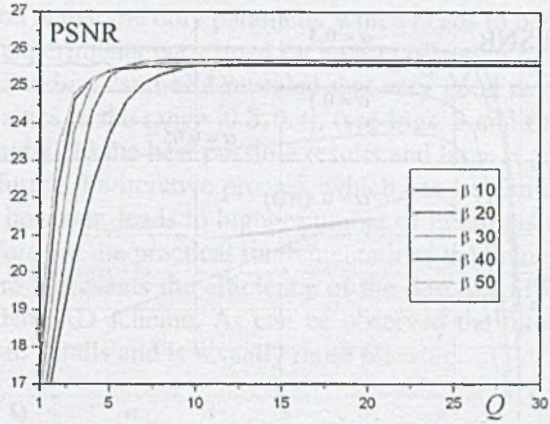


Fig. 5. Dependence of PSNR on the iteration number Q for various β values, (LENA, $\alpha = 0.3$, $\sigma = 30$, $p = 0.3$, MMF as initial filter)

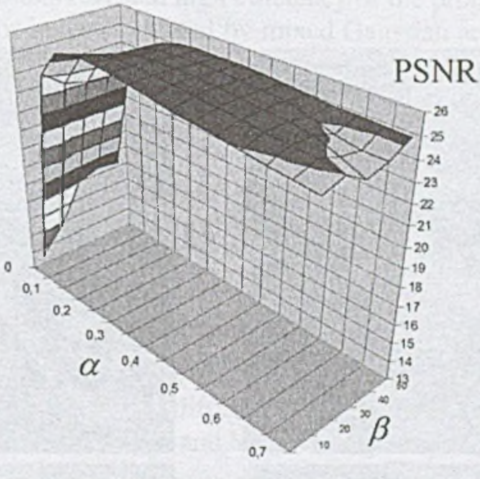


Fig. 6. Dependence of the PSNR on the α and β parameters when iterating the modified biased AD scheme, ($\sigma = 30$, $p = 0.3$, MMF)

α-trimmed VMF (αVMF) and the Marginal Median Filter (MMF) which performs the independent filtering of the image channels with the median filter were utilized, [7].

As can be observed evaluating the results shown in Fig. 3 and summarized in Fig. 4, the application of non-linear filters significantly improved the efficiency of the biased AD. It is worth noticing that the convergence of the modified biased AD scheme is quite quick. Generally after about 10 iterations the optimal restoration effect is achieved, as illustrated in Fig. 5, which depicts the PSNR convergence for a LENA image contaminated by mixed noise. This Figure shows that the output of the BAD scheme does not significantly depend on the β parameter, as for various values of β the scheme converges to the final image with the same quality described in terms of PSNR. This very beneficial behavior is confirmed by Fig. 6, which depicts a plot of the dependence of the PSNR of the final restored image as function of the α and β values in (3) used in (7).

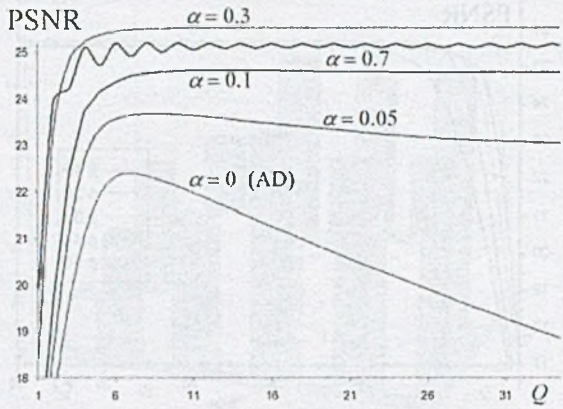


Fig. 7. Dependence of the convergence of the modified biased AD scheme, ($\sigma = 30$, $p = 0.3$, MMF) on the α parameter value

(a) (b) (c) (d)

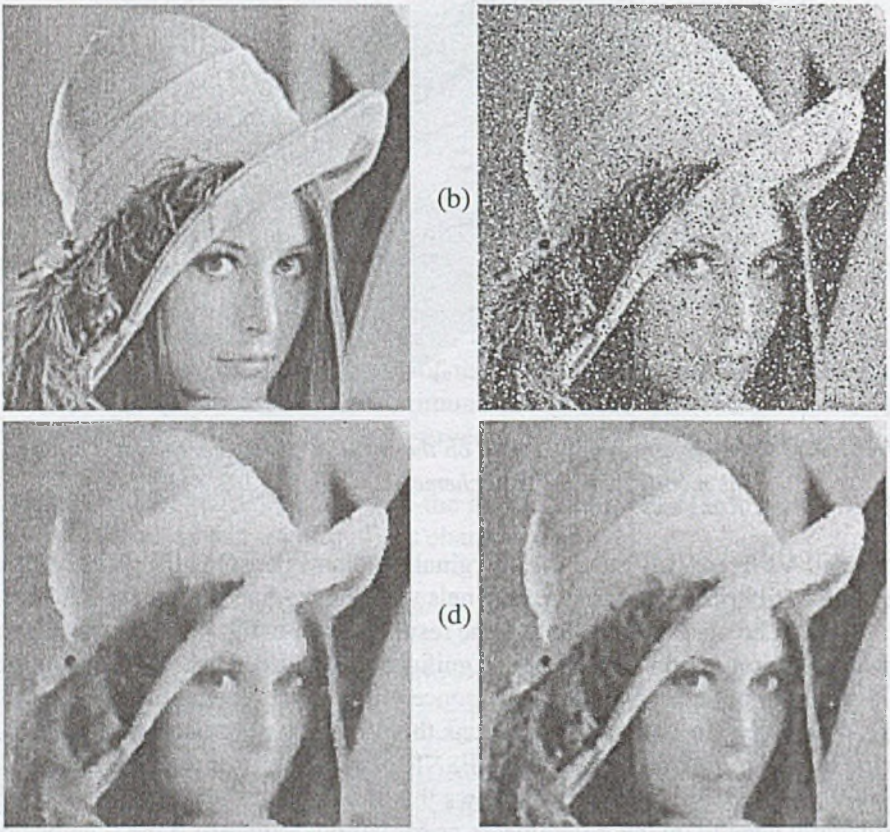


Fig. 8. Comparison of the efficiency of the standard AD with the new method: (a) LENA image, (b) noisy image: Gaussian ($\sigma = 30$) and impulse noise ($p = 0.3$), (c) optimal output of AD, (d) best output of the modified BAD with VMF prefiltering

The conclusion is that the only parameter which needs to be set by the user is the value of the α. The experiments performed on a vast collection of test images polluted by mixed Gaussian and impulse noise revealed that very good restoration results were obtained for the α values in the range $[0.3, 0.4]$, (see Figs. 3 and 6). As shown if Fig. 7 low values of α do not yield the best possible results and large α values lead to oscillations of the PSNR during the iterative process, which can be eliminated by decreasing the value of λ. This however, leads to higher number of iterations needed to obtain the convergence. Therefore for the practical implementations the value of $\alpha = 0.35$ can be recommended. Figure 8 presents the efficiency of the described filtering method compared with the standard AD scheme. As can be observed the filtering output is much sharper, contains more details and is visually more pleasing.

4. Conclusions

In the paper a modification of the biased anisotropic diffusion has been introduced. The experimental results revealed high efficiency of the proposed filtering scheme for the denoising of color images polluted by mixed Gaussian and impulse noise. The novel filtering technique converges quickly to its root so that the described method can be applied for applications in which the prefiltering of images corrupted by mixed noise is crucial for the further image processing steps.

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Omówienie

Redukcja szumów występujących w obrazach cyfrowych jest najważniejszym etapem przetwarzania wstępnego. Od wielu już lat prowadzone są intensywne badania nad nowymi, efektywnymi metodami filtracji różnego rodzaju szumów zakłócających informację wizyjną.

Jedną z najskuteczniejszych metod filtracji jest tak zwana dyfuzja anizotropowa, która daje zadowalające wyniki w przypadku obrazów zakłóconych szumem gaussowskim. Metoda ta zawodzi jednak w przypadku obrazów zdegradowanych przez szumy impulsowe, ponieważ impulsy wprowadzone przez proces szumu są traktowane przez algorytm filtracyjny jak silne krawędzie i nie są usuwane z obrazu.

W niniejszej pracy przedstawiono nową metodę redukcji szumów w barwnych obrazach cyfrowych, która umożliwia filtrację obrazów zakłóconych przez szum o charakterze impulsowym. Wprowadzona technika filtracji szumów stanowi modyfikację tak zwanej obciążonej dyfuzji anizotropowej. Technika obciążonej dyfuzji anizotropowej polega na uwzględnieniu w nieliniowym równaniu dyfuzyjnym informacji o obrazie oryginalnym, co prowadzi do zbieżności procesu iteracyjnego.

Przedstawione w pracy badania symulacyjne wskazują, że metoda dyfuzji obciążonej nie daje niestety lepszych wyników w sensie standardowych miar jakości obrazu niż standardowa metoda dyfuzji anizotropowej. Jednakże, znacznie lepsze rezultaty filtracji można uzyskać, zastępując w obciążonym równaniu dyfuzyjnym obraz zaszumiony przez obraz poddany wstępnej filtracji za pomocą standardowych filtrów wektorowych. Ta prosta modyfikacja, która jest najważniejszym elementem niniejszej pracy, prowadzi do znacząco lepszych wyników filtracji obrazów zakłóconych przez mieszany szum gaussowski i impulsowy. Ponadto, jak wykazały liczne symulacje, zapewnione są zbieżność procesu iteracyjnego oraz duża niezależność wyników filtracji od parametrów występujących w równaniu dyfuzyjnym.

Zaproponowana metoda ze względu na swoją niską złożoność obliczeniową, szybką zbieżność do stanu ustalonego oraz dużą efektywność może być stosowana do redukcji szumów gaussowskich i impulsowych występujących w silnie zdegradowanych barwnych obrazach cyfrowych.