ZESZYTY NAUKOWE POLITECHNIKI ŚLASKIEJ 2008
Seria: AUTOMATYKA z. 151 Nr kol. 1797

Lech JAMROŻ, Jerzy RASZKA
Politechnika Krakowska

THE USE OF A MAX-PLUS ALGEBRA IN SCHEDULING OF CYCLIC PROCESSES

Summary. This paper presents the use of Max-Plus algebra in modelling dynamic behaviour of a system with discrete cyclic processes. The job-shop system is considered, where scheduling is one of the most basic problems to be solved within manufacturing industry. The research is focused on a design and control at the lowest level of the system. The modelling and control on the operational level are developed. The considered here processes are sequential and cyclic. Timed Petri nets are used to specify the considered system. The MaxPlus algebraic technique to handle systems under consideration is proposed. As the performance evaluation measure the cycle time of the system is chosen. The MaxPlus algebra represents linear algebraic form of discrete systems

ZASTOSOWANIE MAX-PLUS ALGEBRY W SZEREGOWANIU PROCESÓW CYKLICZNYCH

Streszczenie. Artykuł przedstawia zastosowanie Max-Plus algebry w modelowaniu dynamicznych, cyklicznych procesów dyskretnych. Rozważany jest system równoległych procesów produkcyjnych. Każdy z procesów jest sekwencyjny i cykliczny. Sterowanie w systemie odbywa się na poziomie operacyjnym. Do specyfikacji procesów wykorzystano czasowe sieci Petriego. Ocenę wydajności pracy systemu przeprowadzono na gruncie teorii MaxPlus algebry.

1. Introduction

Discrete processes are represented by Flexible Manufacturing Systems. The paper considers the job-shop system, in which multiple-operation jobs are scheduled on multiple machines. Scheduling is one of the most basic problems to be solved within manufacturing industry. It is concerned with the sequence of operations on machines and is directly linked with productivity. The research is focused on a design and control at the lowest level of the system and on new developed modelling and controlling tools for the operational level. Considered here processes are sequential and cyclic. At the same time they can have a cooperative and competitive relationship. The access to each common resource is coordinated by a mutual exclusion protocol. It permits to determine an initial state ensuring the non-blocking collaboration of processes.

The performance evaluation of considered system is determined using timed Petri nets. The cycle time of the system is chosen as a performance evaluation measure. There is a great need for improving the scheduling operation within the manufacturing environment. Unfortunately, scheduling problems generally are considered within large scale systems and are difficult to solve in most practical situations. The paper provides the numerical solution which allows to evaluate the performance of the job-shop system.

Petri nets, a graph-oriented formalism, allow to model and analyze systems, which comprise properties such as concurrency and synchronization. A Petri net consists of transitions and places, which are connected by arcs. In the graphical representation, places are drawn as circles, transitions are drawn as thin bars or as rectangles, and arcs are drawn as arrows. The places and transitions are labeled with their names. Places may contain tokens, which are drawn as dots. The vector representing in every place the number of tokens is the state of the Petri net and is referred to as its marking. This marking can be changed by the firing of the transitions, which is determined by arcs.

More formally timed Petri nets (TPN) are 5-tuples [8]: TPN= (P,T,F,M_0,τ), where $P=(p_1,p_2,...,p_n)$, $|P|\neq 0$; $T=(t_1,t_2,...,t_m)$, $|T|\neq 0$ is a finite disjunct set of suitable places and transitions; M_0: $P\rightarrow N$ is the initial marking function which defines the initial number of tokens for every place. ($N=\{0,1,...\}$); τ: $T\rightarrow R^+$ is the firing time function, and $F\subset(PxT)\cup(TxP)$ is the set of arcs. The tools of the Petri net are well suited to the modelling of flexible manufacturing systems.

2. Algebraic representation

To handle considered systems the authors propose a MaxPlus algebraic technique. The MaxPlus or (max,+) [2] algebra represents linear algebraic form of discrete systems and supplies new tools to their modelling. The structure of the MaxPlus algebra by $R_{max} = (R \cup \{-\infty\}, \max, +)$ definition is equipped with a maximization and addition operations. Thus, it is the algebra over the real numbers with a maximum and addition as the two binary operations and $-\infty$ and 0 as the identity elements.

It can be used appropriately to determine the marking times within a given Petri net and at the beginning the vector filled with a marking state. The MaxPlus algebra tools are useful to investigate properties of a network. For the network which consists of n nodes and some arcs connecting these nodes the time activities of nodes can written as [1]:

$$x_i(k+1) = \max(A_{i,1} + x_1(k) , A_{i,2} + x_2(k), ... A_{i,n} + x_n(k)), i = 1,2,...,n \quad (2.1)$$

The MaxPlus algebra notation for maximization and addition operations has useful shorthand symbols, like ⊕ (pronounced 'o-plus') and ⊗ (pronounced 'o-times'). They represent two binary operations with the max as the sum (i.e. $a \oplus b = \max(a, b)$) and the ordinary sum as the product (i.e. $a \otimes b = a + b$). Using these symbols the (2.1) equation becomes as follows:

$$x_i(k+1) = \bigoplus_j A_{i,j} \otimes x_j(k), i = 1,2, ... n \quad (2.2)$$

where:

$\bigoplus_j a_j$ - refers to the maximum of elements a_j with respect to appropriate j,

$\otimes$ - refers to the addition,

Coefficients of the $A_{i,j}$ matrix represent the sum of the activity times of the j node and the traveling time from the j node to the i node of this network and $x_i(k)$ is the earliest epoch at which the i node becomes active for the k-th time. In a vector notation the (2.1) equation becomes as follows:

$$x(k+1) = A \otimes x(k) \tag{2.3}$$

In practical examples the autonomous timed Petri net, which models cyclic processes (with feedback arcs) is determined by the following formula:

$$x(k+1) = A_0 \otimes x(k+1) \;\oplus\; A_1 \otimes x(k),\; k = 0,1,2,3 \tag{2.4}$$

The solution is given by [1]:

$$x(k+1) = A \otimes x(k),\; k = 0,1,2,3\,... \tag{2.5}$$

where:

$$A = A_0^{*} \otimes A_1$$

$$A_0^{*} = I \oplus A_0^{1} \oplus A_0^{2} \oplus A_0^{3} ... \oplus A_0^{l} \tag{2.6}$$

I - refers to the identity matrix in R_{max}; zeros on diagonal and $-\infty$ elsewhere (0 and $-\infty$ are neutral elements for MaxPlus algebra operations).

The A_0 and A_1 matrices can be determined from the incidence matrix of the net for its actual initial marking and times operations.

3. Computational examples and experimental results

In the presented here example three job types $\{r1, r2, r3\}$ with eight operations are to be scheduled on three not identical machines $\{m1, m2, m3\}$. Every one of these three jobs has two or three operations and each operation may be scheduled on up to three different machines. A Petri net model structure of job-shop for the following marking vector: ${}^{1}M_0 = \{1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0\}$ is given in Fig. 1.

Table 1 presents the times of operations.

Table 1

	Machines		
Proc.	*m1*	*m2*	*m3*
r1	$\tau_1=1$	$\tau_2=5$	$\tau_3=1$
r2	$\tau_4=3$	-	$\tau_5=2$
r3	$\tau_6=2$	$\tau_8=4$	$\tau_7=3$

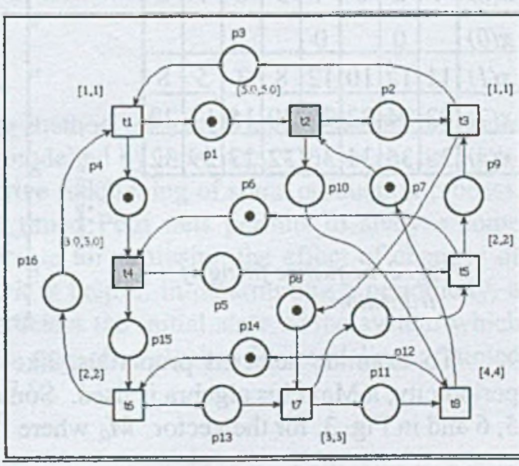

Fig. 1. Petri net of the processes

Equation (2.4) for marking vector ${}^{1}M_0$ is given by:

$$
\begin{array}{ll}
x_1(k+1) = 1 \otimes x_3(k+1) \oplus 2 \otimes x_6(k+1) & x_5(k+1) = 3 \otimes x_4(k) \oplus 3 \otimes x_7(k+1) \\
x_2(k+1) = 1 \otimes x_1(k+1) \oplus 4 \otimes x_8(k+1) & x_6(k+1) = 3 \otimes x_3(k) \oplus 4 \otimes x_8(k) \\
x_3(k+1) = 5 \otimes x_2(k) \oplus 2 \otimes x_5(k+1) & x_7(k+1) = 1 \otimes x_3(k \oplus 2 \otimes x_6(k+1) \\
x_4(k+1) = 1 \otimes x_1(k+1) \oplus 2 \otimes x_5(k+1) & x_8(k+1) = 5 \otimes x_2(k \oplus 3 \otimes x_7(k+1)
\end{array}
\quad (3.1)
$$

The equation (3.1), which has the A_0 nilpotent matrix achieves a convergence for $l = 5$, i.e. $A_0^5 = 0$ (all coefficients equal $-\infty$). The obtained (2.6) result is introduced in Table 2. Table 3 represents the final transient matrix A in equation (2.5), where the space denotes the $-\infty$ value.

Table 2

A_0	1	2	3	4	5	6	7	8
1	0		1		3	8	6	
2	1	0	2		4	9	7	4
3			0		2	7	5	
4	1		2	0	4	9	7	
5					0	5	3	
6						0		
7						2	0	
8						5	3	0

Table 3

A	1	2	3	4	5	6	7	8
1		6	7	11				12
2		9	8	12				13
3		5	6	10				11
4		7	8	12				13
5			4	8				9
6				3				4
7			1	5				6
8		5	4	8				9

The results of iterative calculation $x(k)$ for $k = 0, 1, 2...$ according to (2.5) dependencies are introduced in Table 4. These values in the following k cycles represent the times of the starting activity of transitions in the considered here Petri net. These results find a confirmation in the passed simulation using some other application [7]. Fig 2 shows firing sequence of a transition for the initial marking vector ${}^{1}M_0$.

Table 4

$x(k)$	1	2	3	4	5	6	7	8
$x(0)$		0		0				
$x(1)$	11	12	10	12	8	3	5	8
$x(2)$	23	24	22	24	20	15	17	20
$x(3)$	35	36	34	36	32	27	29	32

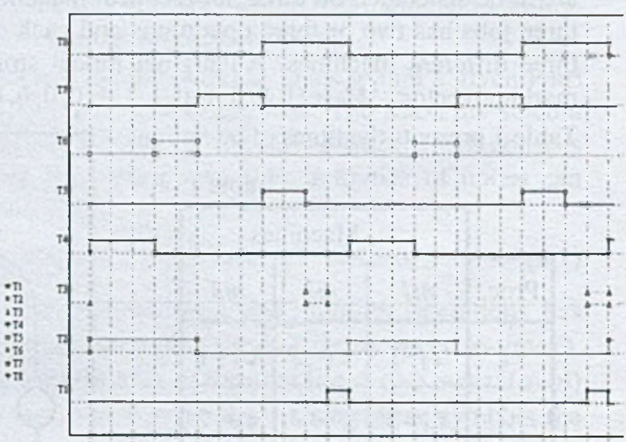

Fig. 2. Sequence firing of transition

To examine selected properties, like safety, liveliness, deadlock freeness and periodicity, a MaxPlus algebra is used. Some of the properties are presented on Tables 5, 6 and in Fig. 3, for the vector ${}^{2}M_0$, where ${}^{2}M_0 = \{0,0,0,1,0,1,1,1,0,0,0,0,0,1,0,0\}$.

Table 5

A_0^{100}	1	2	3	4	5	6	7	8
1			232			233		
2	232						231	
3		236			233		.	
4							231	
5	232							
6								
7			199			200		
8					233			

Table 6.

$x(k)$	1	2	3	4	5	6	7	8
$x(0)$		0		0				
$x(1)$	230	231	235	1	8	3	5	235
$x(2)$	469	470	471	470	244	239	241	471

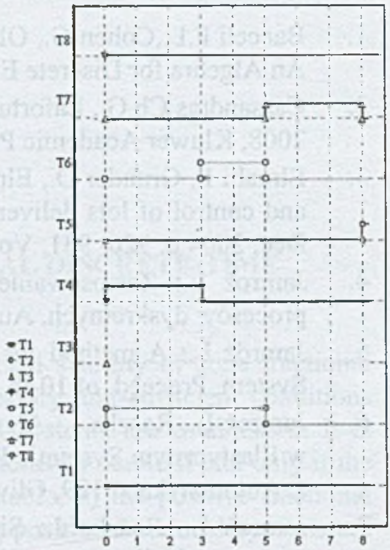

Fig. 3. A firing sequence for the marking vector ${}^2M_0$

Table 7

ν	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	λ
1	1	0	0	1	0	1	1	1	0	0	0	0	0	1	0	0	12.0
2	1	0	0	1	0	1	1	1	0	0	0	0	0	1	0	0	
3	1	0	0	0	0	1	1	0	1	0	0	0	0	1	0	1	19.0

It is not possible to obtain a convergence in the iterative $A_0{}^*$ calculation as a deadlock has occurred within the processes. The other very essential Petri net proprieties - are a periodicity and a maximum cycle mean (λ). The obtained results (presented in Table 7) show the λ values for the three ν sequences of values for the ${}^{\nu}M_0$ marking vector, where ${}^{\nu}M_0 = \{p_1, p_2, p_3, \dots p_{16}\}$.

4. Concluding remarks

The obtained results confirm that the method presented here is useful for some classes of discrete systems, which can be modelled by Petri nets. This might be a job-shop system under deterministic and repetitive functioning of some production process. An algebraic description of processes of timed Petri nets permits to analyse some properties of systems. This method is suitable for analysing the effect of changes of the initial state of a system. In particular, it is helpful in determining a periodicity, a time of a cycle (a production rate) and to detect the initial state of the system which carries to deadlock. The technique of the max-plus algebra for modeling of timed discrete and hybrid systems is given in [2,3,9].

BIBLIOGRAPHY

1. Bacceli F.L.,Cohen G., Olsder G.J., Quadrat J.P.: Synchronization and Linearity. An Algebra for Discrete Event Systems, London, John Wiley & Sons Ltd, 1992.
2. Cassandras Ch.G., Lafortune S.: Introduction to Discrete Event Systems Springer 2008, Kluwer Academic Publishers 2007.
3. Elmahi I., Grunder O., Elmoudni A.: A max plus algebra approach for modeling and control of lots delivery. Industrial Technology, 2004. IEEE ICIT '04. 8-10 Dec. 2004 p. 926- 931, Vol. 2.
4. Jamroż L.: Zastosowanie metod ewolucyjnych w optymalizacji cyklicznych procesów dyskretnych. Automatyka, t.5, z.1, Wyd. AGH, Kraków 2001.
5. Jamroż L.: A method for work-in-process reduction in Flexible Manufacturing System. Proceed. of 10-th Intern. Symp. on SMC, Zakopane 2001, p. 221-226.
6. Jamroż L., Raszka J.: Algorytm Genetyczny do równoważenia obciążeń maszyn w Elastycznym Systemie Produkcyjnym. Zeszyty Naukowe Politechniki Śląskiej, s. Automatyka z. 129, Gliwice 2000, s. 135-144.
7. Jamroż L., Raszka J.: Simulation method for the performance evaluation of system of discrete cyclic processes. Proceedings of the 16-th IASTED International Conference on Modelling, Identification and Control, Innsbruck, Austria, 17-19.02.97, p. 190-193.
8. Murata T.: Petri nets: properties, analysis and applications. Proceedings of the IEEE, vol. 77, no. 4, p.541-580, 1989.
9. Villani E., Paulo E., Valette R.: Modelling and Analysis of Hybrid Supervisory Systems Series: Advances in Industrial Control Springer 2008.

Recenzent: Prof. dr hab. inż. Andrzej Świerniak

Omówienie

Elastyczny System Produkcyjny z punktu widzenia sterowania wykonywaniem operacji widziany jest jako system procesów dyskretnych. Procesy są cykliczne i przebiegają współbieżnie. Pojedynczy proces związany jest określoną sekwencją operacji, wykonywanych według zadanego programu produkcyjnego. Współbieżny przebieg procesów wymaga koordynacji w zakresie dostępu do wspólnych zasobów. Do specyfikacji systemu procesów wykorzystano czasowe sieci Petriego. Mechanizm synchronizacji procesów, zapewniający bezblokadowy ich przebieg, oparty jest na protokole „wzajemnego wykluczania”. Protokół ten w odniesieniu do poszczególnych zasobów gwarantuje zachowanie żywotności sieci Petriego. Do oceny wydajności pracy systemu wykorzystano własności Max-Plus algebry. Model rozważanego systemu, sformułowany w terminach Max-Plus algebry, umożliwia w sposób analityczny badanie własności jego zachowania. Synteza równań stanu systemu pozwala określić minimalny cykl pracy systemu oraz dyskretne chwile rozpoczęcia i zakończenia wykonywania operacji. Połączenie algebraicznego podejścia z teorią czasowych sieci Petriego umożliwia tworzenie ogólnego modelu systemu uwzględniającego jego dynamikę. Umożliwia też wyznaczenie początkowego znakowania sieci Petriego, dla którego zachowana jest własność żywotności sieci.