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NUMERICAL SOLUTION OF 2-D TRANSONIC FLOW THROUGH AN AXIAL TURBINE STAGE

Abstract. This paper presents the solution of the steady and unsteady two-dimensional transonic flow throuqh an axial turbine stage. The rotational, adiabatic, inviscid flow of the perfect gas is considered. The analysis of the problem has been based on the time dependent Euler equations. In order to solve the problem, the finite volume method and a time-marching method have been applied. The results of numerical calculations for a chosen geometry of an axial turbine stage are presented.

INTRODUCTION

In modern heavy load stages of turbomachines, transonic and supersonic flows are often met. In recent years a great progress in computing transonic flow is noticeable. Particularly methods based on the time dependent equations for the conservation of mass, energy, and momentum have been developed. This lets maintain the hiperbolic type of differential equations in the whole computational domain for the whole velocity range. The main attraction of the time dependent Euler equations is the ability to compute mixed subsonic-supersonic flows with automatic capturing of shock waves. These equations are solved by using a time-marching technique. One of the first applications of the explicit time marching methods were done by Godunov [1] and McDonald [2]. These methods have found wide applications and are constantly being improved.

In the calculations of transonic flows, also the Denton method is successfully applied. This method has been widely used for solutions of the Euler equations through blade rows e.g. for flow in the meridional plane by Spurr [3], for unsteady flow by Mitchell [4], for wet steam flow by Bakhtar et al. [5] and for three-dimensional flow by Sarathy [6]. Another commonly used method is the MacCormack method. It is used for steady and unsteady (e.g. Ispas et al. [7]) flow calculations either in finite difference form in a rectangular computational domain obtained after a coorinate transformation or directly in finite volume formulation in the physical domain.

The corrected viscosity scheme, applied in this paper, is derived from the original Lax scheme. Some modifications of the Lax scheme, wich were introduced by Couston [8] and recently by Van Hove [9] and Arts [10], have caused that this method has become attractive to calculate two- and three-dimensional problems. The corrected viscosity scheme is a one-step, first order accuracy in time, explicit scheme.

Particular attention should be given to the unsteady effects of blage row interactions. It is important to predict these effects, especially when the blade row of interest is a rotor since measurement difficulties in rotating frame of reference.

One of the first attemps at predicting unsteady effects with using a time-marching method was made by Mitchell [4]. He considered a single blade passage for which he used in calculations the Denton scheme with the assumed unsteady condition at the inlet.

Ispas et al. [7] considered the similar problem and employed the MacCormack scheme for a single blade passage. They avoided the problem by solving the flow throughout the number of blade passages for which the flow repeated. They applied the calculation scheme with the second order accuracy in time but assumed the isotropic flow, which is quite problematic.

Hodson [11], on the basis of the Denton scheme, did the calculations of the flow through the blade passage with the application of modified boundary conditions, what lets consider a single blade passage. In comparision with the experiment, the applied method gives good results.

A similar problem was considered by Sokolowski et al. [12]. They took into account the two- and three-dimensional unsteady flow through a turbine stage using the Godunov scheme, and obtained interesting results.

It is worth noting that in the present considerations the models of inviscid, unsteady flows are used. It results from difficulty in the unsteady flow calculations which practically limited now the use of viscid models. The conformity of the results obtained from the inviscid flow calculations with the experiment is often good [11], which proves that the viscosity effects are not always dominant.

MATHEMATICAL FORMULATION OF THE PROBLEM

The problem is further considered with the following simplifications:

(a) flow is 2-D, plane,

(b) flow is inviscid and adiabatic,

(c) gas is perfect,

(d) no blade vibration.

Flows through the blade passage are governed by mass, momentum and energy conservation principles. The system of the conservation equations for volume V can be written in the integral form for the considered flow model as follows:

$$\frac{\partial}{\partial \tau} \int_V U \, dV + \int_V F \, dV = 0 \qquad (1)$$

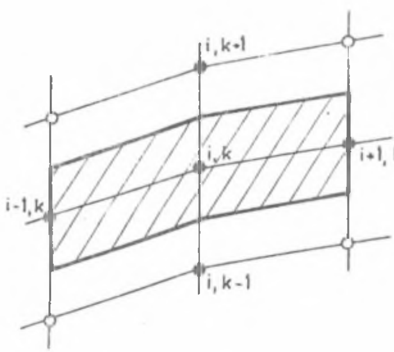
where:

$$U = \begin{Bmatrix} \rho \\ \rho \bar{w} \\ \rho p e \end{Bmatrix} \qquad F = \begin{Bmatrix} \rho \overline{wn} \\ \rho \bar{w}(\overline{wn}) + \bar{n} p \\ \rho(e + p/\rho)\overline{wn} \end{Bmatrix}$$

Using the perfect gas law and the definition of total absolute or relative energy, (1) is completed by the following relation:

$$e = \frac{1}{\gamma - 1} \frac{p}{\rho} + \frac{w^2}{2} + \frac{u^2}{2} \qquad (2)$$

The problem to be solved is an initial boundary value problem where an initial value distribution has to be know with all boundary conditions.



Rys. 1. Element bitrapezowy
Fig. 1. Bitrapezoidal element

DISCRETIZATION OF THE FLOW DOMAIN

The calculation domains of the stator and rotor are discretized by numerical grid. The numerical grid is composed of quasi-streamwise lines and pitchwise lines. The quasi-stream lines are equally spaced in the pitchwise direction. The pitchwise lines are variably spaced, they diminish from the inlet to the leading edge and increase from the trailing edge to the outlet. Within the blade passage the pitchwise lines are variably spaced, depenging on the required precision and the expected density gradients. On the numerical grid a bitrapezoidal element is defined according to [10]. The element is formed by the same pitchwise lines and quasi-stream lines (Fig. 1). The numerical domain for the steady flow calculation is shown in fig. 2a, while fig. 2b shows the domain for the unsteady flow calculation.

NUMERICAL PROCEDURE

Let us consider a small volume V over which the system of equations (1) is integrated. By the use of the Gauss-Ostrogradzky divergence theorem and averaged quantities (1) can be written:

$$\hat{U}^{n+1} = \hat{U}^{n} - \int_{n}^{\tau^{n+1}} \frac{1}{\Delta V} \sum_{m=1}^{M} (\hat{F} \Delta S)_m d\tau \tag{3}$$

where:

M - number of surfaces which limit a volume V

$\wedge$ - averaged quantities.

Assuming the finite volume method to determine a subintegral funkction $\hat{F}$ and the corrected viscosity scheme to integrate in the time partition $\Delta\tau$, equation (3) can be formulated as follows:

$$U^{n+1}_{i,k} = \frac{\Delta\tau}{V} \sum_{m=1}^{M} (\hat{F} \Delta S) + 0{,}25(U^{n+1}_{i-1,k} + U^{n}_{i,k+1} + U^{n}_{i+1,k} + U^{n}_{i,k-1}) + 0{,}25\alpha(U^{*}_{i-1,k} + U^{*}_{i,k+1} + U^{*}_{i+1,k} + U^{*}_{i,k-1} - 4U^{*}_{i,k}) \tag{4}$$

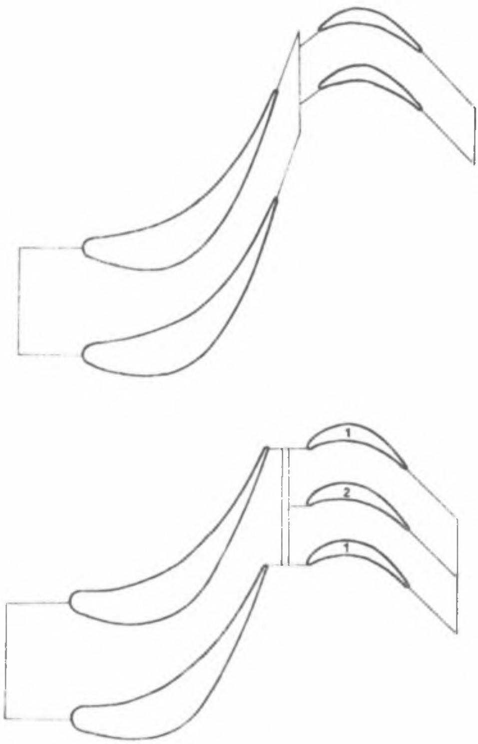
where:

i,k - spatial indices in the axial and pitchwise directions,

n - index of time,

α - numerical viscosity coefficient dependent on density gradient.

The terms superscripted by an asterisk are updated every Nv iterations, α is numerical coefficient close to 1, used to provide the effective correction. It is calculated as a function of the local density gradient in order to ensure the necessary damping near the flow discontinuity.



Rys. 2. Obszar obliczeniowy dla zagadnienia przepływu stacjonarnego (a) i niestacjonarnego (b)

Fig. 2. Calculation domain for steady (a) and unsteady (b) flow problem

Corrected viscosity scheme (4) is stable for the Courant - Friedrichs--Lewy condition, which limits the time step $\Delta\tau$.

The detailed description of this method can be found in [10].

BOUNDARY CONDITIONS

In order to have a well-posed problem, (1) must be completed with a set of boundary conditions.

The missing calculation points outside the computational domain are replaced by the corresponding points at the other periodic boundary. Along a solid walls an impermeability condition must be fulfilled. As the solution procedure explicitely uses the transport terms, this condition is satisfied by setting to zero the mass and energy fluxes and by considering only the static pressure acting on the section and pressure side in the calculation of the momentum fluxes.

The conditions applied on the inlet and outlet plans of a stage are the same as in most other time marching methods (where the axial velocity is subsonic). On the upstream boundary, the absolute stagnation density and stagnation pressure are specified together with the absolute flow direction. On the downstream boundary the static pressure is specified and held constant (the so called non-reflecting condition can also be used).

In calculating the boundary conditions on the outlet from the stator domain and on the inlet to the rotor domain are not set. Depending whether the problem of a steady or unsteady flow is considered, parameters on the connection of the stator and rotor numerical grids are updated in a two different ways.

1. Steady flow

In the problem of the steady flow through a stage, the connection of the stator and rotor numerical grids is realized by assuming an averaged parameters in the pitchwise direction on the lines IA=MX-1 of stator and IA=2 of rotor (Fig. 3a). The problem is considered in the two systems of reference, absolute for the stator and relative for the rotor. From the averaging parameters notated in the relative system of reference, values on line IA=MX of the stator (which overlaps with line IA=1 of the rotor) can be calculated using the one-dimensionsl corrected viscosity scheme. By updating the values from the absolute system of reference to the relative one, static parameters are determined due to the following equations.

$$H_a - c_y u = H_w - u^2/2 = \mathrm{const}$$

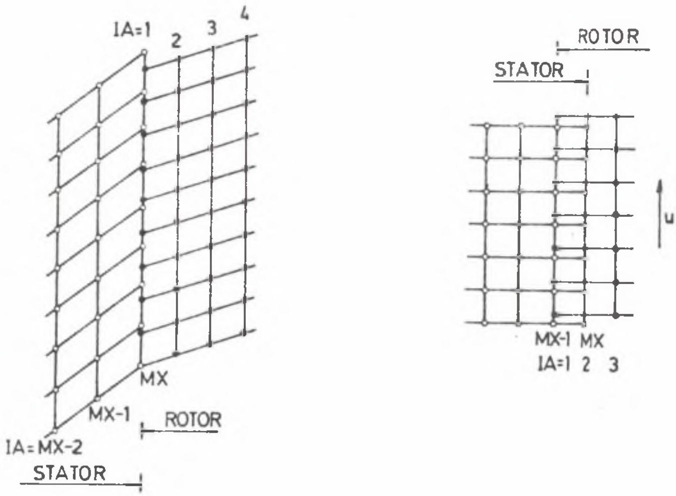
$$\frac{T_{ow}}{T_{oa}} = 1 - \frac{u^2}{2H_a}\,(2cy/u - 1) \qquad (5)$$

$$\frac{p_{ow}}{p_{oa}} = \left(\frac{T_{ow}}{T_{oa}}\right)^{\frac{\gamma}{\gamma-1}} \;; \quad \frac{p_{ow}}{p_{oa}} = \left(\frac{\varrho_{ow}}{\varrho_{oa}}\right)^{\gamma}$$

$$c_x = w_x \;; \quad c_y = w_y + u$$

where:

H - enthalpy; subscripts: o - stagnation, a - absolute, w - relative.



Rys. 3. Połączenie siatki kierownicy i wirnika dla zagadnienia przepływu stacjonarnego (a) i niestacjonarnego (b)

Fig. 3. Stator and rotor grid connection for steady (a) and unsteady (b) flow problem

2. Unsteady flow

The numerical grids, that discretize the computational domain of the stator and rotor, have a common field (Fig. 3b). Line IA=MX-1 of the stator overlaps line IA=2 of the rotor.

The solution procedure is based on a repeated interaction between two blade row calculations. The flow parameters on line IA=MX-1 of the stator are updated from the absolute system of reference into the relative one due to equations (5) at each node. It permits to determine parameters on line IA=1 of the rotor taking into account an interaction of blade rows at a given moment ot time. In the same way the values from the nodes on line IA=2 of the rotor are updated into the values on the nodes on line IA=MX of the stator.

STEADY FLOW PREDICTION

Calculations are performed on the hub section of a last stage of the large output steam turbine. The blade passages of a turbine stage are discretized by the numerical grid with 21x67 grid points on the rotor and 21x60 grid points on the stator (Fig. 2a). The described method lets perform calculations for any stage blade ratio.

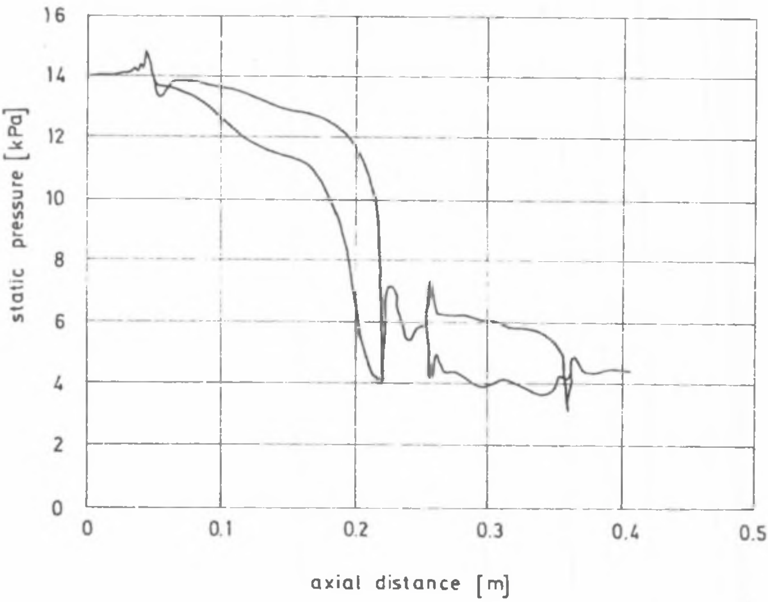
The rotor blade row moves, relatively to the stator blade row, at velocity u. The unterrow spacing / stator pitch ratio is equal 0,37.
Flow parameters are:

ρ_{oa} = 0,102 [kg/m^3]

p_{oa} = 0,0144 [MPa]

α_o = 0 [°]

p_2 = 0,005 [MPa]



Rys. 4. Rozkład ciśnienia statycznego dla zagadnienia przepływu stacjonarnego

Fig. 4. Blade surface static pressure distribution for steady flow calculations

The calculations are performed simultanceously for the stator and rotor. As the initial data, the stagnation parameters distribution in the whole computational domain of the stage is assumed. To eliminate numerical instability, the selection of the suitable reduction of the outlet static pressure and increase of angular velocity of the rotor blade row has been made. During the first 800 iterations, the static pressure at the outlet from the stage is droped till demanded value p_2 is reached. From 500th to about 1500th iteration the angular velocity of the rotor blade row is gradually increased. The calculations are carried on until the steady-state is reached in the whole computational domain of the stage. Changes of the relative internal energy $\Delta e/e$ was less then $2x10^{-5}$ and fluctuations of the outlet angle β_2 less then $0,02^{o}$. Using a IBM PC 386 computer the necessary CPU time per grid point and per time step was $2,85x10^{-3}s$. 8000 time steps were performed.

The average in time parameters distribution in the stage domain is the result of the calculations. Fig. 4 shows the static pressure distribution on the stator and rotor blade surfaces. In the stator and rotor blade passages are supersonic areas. In the stator maximal isentropic Mach number is equal 1,6 and in the rotor it is 1,4, Chosen average flow parameters are:

$$\alpha_1 = 68,1 \ [^{o}]$$

$$\beta_1 = 50,5 \ [^{o}]$$

$$p_1 = 5484 \ [Pa]$$

$$\beta_2 = 51,6 \ [^{o}]$$

$$p_{ow} = 8145 \ [Pa]$$

Where:

α, β - flow angles (absolute, relative), subscripts: 1 - inlet of rotor, 2 - outlet of stage.

In order to determine the variation range of the flow parameters in the stage, it is necessary to solve the problem of the unsteady flow through the stage.

UNSTEADY FLOW PREDICTION

Modelling unsteady effects in the blade passages of a stage is based on determining the interaction between the stator and rotor blade rows. The unsteady effects are only due to the rotation.

Usually, the number of blades in the stator blade row will not be an integral multiple of the number of blades in the rotor blade row. If it is assumed that the pitch of the stator blades and pitch of the rotor blades are, respectively t_1 , t_2 , then the periodical calculation domain for the whole stage is defined by the following period:

$$T = K_1 t_1 = K_2 t_2 \tag{6}$$

where:

K_1 , K_2 - prime numbers.

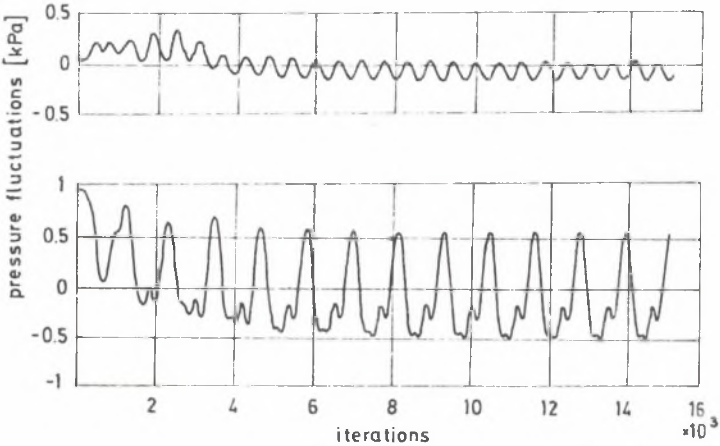
Equation (6) shows that the unsteady effects in the flow will be periodic when in calculations K_1 and K_2 blade passages respectively of the stator and of the rotor is considered. Some computational difficulties arise when the stage blade rotio has relatively big natural numbers K_1 , K_2 . This problem can be solved in two ways. Calculations can be done on the periodic domain of the whole stage (with period T) with a time-independent periodicity condition or on a single blade passage for the stator and rotor with using a time-dependent periodicity condition. The second method involves an application of time-lagged periodic boundary conditions to the computational domain. If we consider complex geometries of a stage, both the methods are computer time consuming and required supercomputers. Nowadays it is very costly.

Due to our computer equipment it was necessary to simplify the stage geometry. The rotor blades pitch was corrected to the value which enabled the considerations of the stage blade rows with blade ratio 1:2 (Fig. 2b), instead of 25:47. Calculations could be considered now for a one stator blade passage and two rotor blade passages.

The computational domain of the stator is discretized by the numerical grid with 25x82 points, while the rotor domain, consisting of two passages, is discretized by the grid with 26x74 points. In the calculations of the unsteady flow through a stage, the initial condition is assumed from the solution of the steady flow for each of blade passages separately. This kind of an initial condition does not require initaial calculations which are done in the steady flow case.

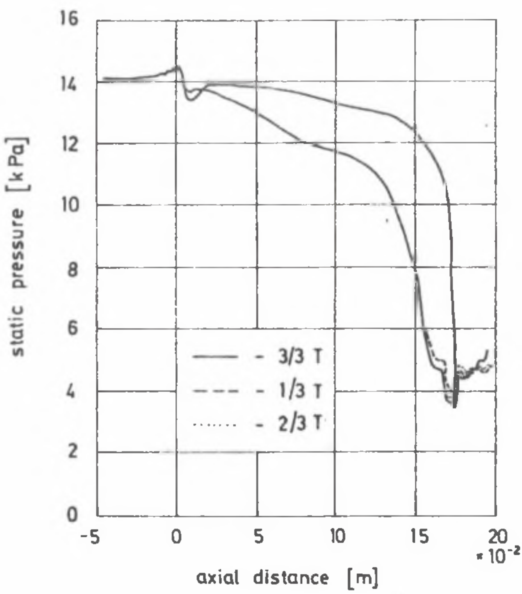
Using a IBM PC 386 computer, 15000 time steps were performed. The calculations are carried on till the periodic changes of parameters in each grid point of the stator and rotor are achieved, The periodicity condition was controled by the calculation for each point the relative parameters fluctuation in each time step. The maximal periodicity error was 0,25%.

Fig. 5a shows the example of the course of pressure fluctuations in one of the numerical grid points of the stator; fig. 5b shows the course of these fluctuations in one of the grid points of the rotor.Bacause of the blades ratio 1:2, the period of the parameters changes in the stator domain is etwice longer than the period in the rotor domain.



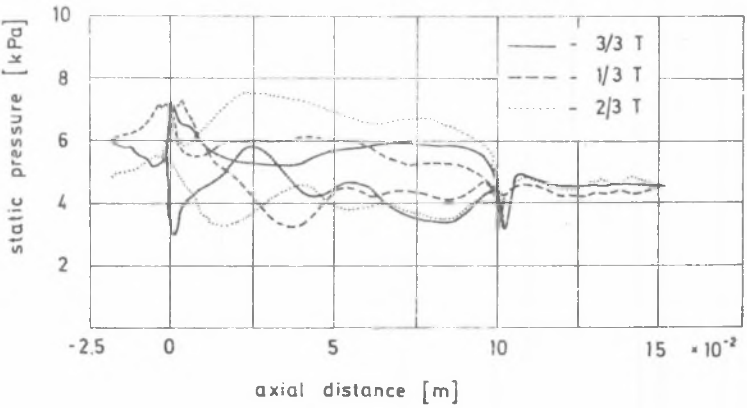
Rys. 5. Zbieżność rozwiązania do stanu okresowego w kierownicy (a) i wirniku (b)

Fig. 5. Convergence of the solution to a periodic state in stator (a) and rotor (b)



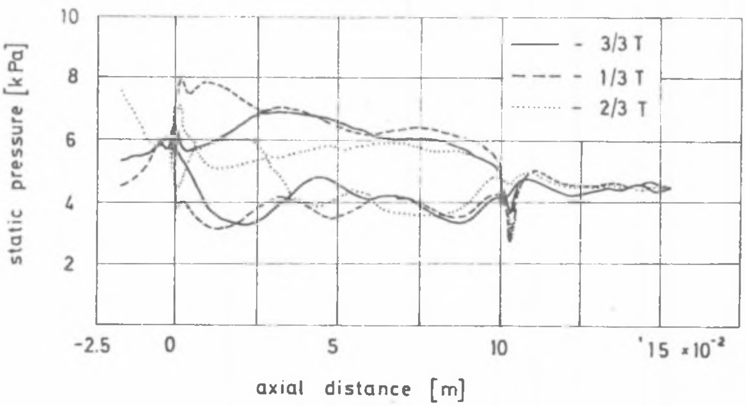
Rys. 6. Rozkład niestacjonarnego ciśnienia statycznego na powierzchni łopatki kierowniczej

Fig. 6. Stator blade surfaces unsteady static pressure distribution



Rys. 7. Rozkład niestacjonarnego ciśnienia statycznego na powierzchni łopatki wirnikowej - łopatka nr 1

Fig. 7. Rotor blade surfaces unsteady static pressure distribution - blade No 1



Rys. 8. Rozkład niestacjonarnego ciśnienia statycznego na powierzchni łopatki wirnikowej - łopatka nr 2

Fig. 8. Rotor blade surfaces unsteady static pressure distribution - blade Nr 2

Fig. 6 shows the static pressure distribution on the stator blade surfaces for example for three different positions of the blade rows. The first position corresponds to the possition shown in fig.2(3/3T). The second and third positions result from the displacement of the blade rows, respectivity of 1/3T and 2/3T (T is in this case a stator blades pitch). According to fig. 6, the pressure changes in the stator, except a small outlet region at the suction side, are not significant. With respect to the transonic flow in the stator, upstream propagation of the flow perturbations, arised during the blade rows interaction, in the stator blade passage are not observed, At a given time in the rotor row, flows are different in different blade-to blade passages. Fig. 7 shows the static pressure distribution on the surfaces of rotor blade No. 1, and fig. 8 on the surfaces of rotor blade No. 2.

Pressure changes on rotor blades are significant. Particularly big changes occur in the inlet part of the rotor blade passage. Chosen average flow parameters changed within the range:

$$\alpha_1 = 66{,}1 - 69{,}2\ [^\circ]$$

$$\beta_1 = 48{,}5 - 52{,}8\ [^\circ]$$

$$p_1 = 5295 - 5670\ [\mathrm{Pa}]$$

$$\beta_2 = -51{,}9 - 52{,}5\ [^\circ]$$

$$p_{ov} = 8100 - 8117\ [\mathrm{Pa}]$$

The obtained values approximately correspond to the values obtained from steady flow calculations.

On the ground of the static pressure fluctuations on the stator and rotor blade surfaces, the dynamic load fluctuations can be determined [13].

CONCLUSIONS

The paper presents two methods to the model two-dimensional transonic flow through an axial turbine stage. These methods make possible a better knowledge of the two blade-row interaction. The steady-state can be obtained by using the first method. It lets determine the averaging in time parameters in the whole computational domain by the averaging procedure in the interrow spacing. The second method allows to consider the flow perturbation during the stator and rotor interaction. Modelling instationary effects, occuring in the flow through a stage, is necessary bacause they considerably influence on the parameters distribution. The qualifications of the obtained results should be done, as all other numerical calculations, on the basis of measurements.

REFERENCES

1. GODUNOV S.K.: Differential method for the numerical calculation of hydrodynamic shocks, Math. Coll., Vol 47, No. 3, 1959 (in Russian).
2 McDONALD P.W.: The Computation of Transonic Flow Through Two Dimensional Gas Turbine Cascades, ASME Paper 71-GT-89.
3. SPURR A.: The Prediction of Three-Dimensional Transonic Flow in Turbomachinery Using a Combined Throughflow and Blade-to-Blade Time Marching Method. Int. J. Heat & Fluid Flow. Vol. 2. Dec. 1980.
4. MITCHEL N.A.: A Time Marching Method for Unsteady Two-Dimensional Flow in a Blade Passage. Int. J. Heat & Fluid Flow. Vol. 2. No. 4, 1980.
5. BAKHTAR F., and TOCHAI M.: An Investigation of Two-Dimensional Flows of Nucleating and Wet Steam by the Time-Marching Method. Int. J. Head & Fluid Flow. Vol.2. No. 1, 1980.
6. SARATHY K.P.: Computation of Three-Dimensional Flow Fields Through Rotating Blade Rows and Comparasion with Experiment. ASME Paper No. 81--GT-121, 1981.
7. ISPAS I., GROLLIUS H., GALLUS H.E.: Uber den Einfluβ von Nachlaufdellen auf die instationare Druckverteilung an den nachfolgenden Schaufelreihen in Axialverdichtern und Axialturbinen. VDI-Berichte Nr 361, 1980.
8. COUSTON M.: Time Marching Finite Area Method, in Transonic Flows in Axial Turbomachinery, VKI LS 84, 1976.
9. Van HOVE W.: Calculation of Three-Dimensional. Inviscid. Rotational, Flow in Axial Turbine Blade Rows. ASME Journal of Engineering for Gas Turbines and Power. Vol. 106, April 1984.
10. ARTS T.: Cascade Flow Calculations Using a Finite Volume Method, in Numerical Methods for Flows in Turbomachinery Bladings, VKI LS 1982-05.
11. HODSON H.P.: An Inviscid Blade-to Blade Prediction of Wake-Generated Unsteady Flow, ASME Journal of Engineering for Gas Turbines and Power, Vol. 107, April 1985, 337.
12. SOKOLOWSKY G.A., GNIESIN W.I.: Unsteady Transonic and Viscous Turbomachinery Flows. Naukowa dumka, Kiev 1986 (in Russian).
13. WRÓBLEWSKI W.: Rozwiązanie zadania transonicznego przepływu przez układy łopatkowe stopnia. Praca doktorska, Gliwice 1988.

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NUMERYCZNE ROZWIĄZANIE DWUWYMIAROWEGO PRZEPŁYWU TRANSONICZNEGO PRZEZ STOPIEŃ TURBINY OSIOWEJ

S t r e s z c z e n i e

Artykuł przedstawia rozwiązanie stacjonarnego i niestacjonarnego dwuwymiarowego przepływu transonicznego przez stopień turbiny osiowej. Rozważany jest wirowy, adiabatyczny, nielepki przepływ gazu idealnego.

Analiza zagadnienia prowadzona jest na bazie zależnych od czasu równań Eulera.

W celu rozwiązania problemu zastosowano metodę kroków czasowych i metodę objętości skończonych. Przedstawiono rezultaty obliczeń numerycznych dla wybranej geometrii stopnia turbiny osiowej.

НУМЕРИЧЕСКОЕ РЕШЕНИЕ ПЛОСКОГО ТРАНСЗВУКОГО ТЕЧЕНИЯ
В СТУПЕНИ ОСЕВОЙ ТУРБИНЫ

Р е з ю м е

Представлено решение стационарного и нестационарного трансзвукового течения в решетках ступени осевой турбины. Развешиваеться невязкие, вихревое, адиабатические течение газа. Уравнения Эйлера решаеться методом установливания в связи с методом конечных обемов. Приведены результаты решения задачи.