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Vojin DRENOVAC

The Faculty of Electrical and Mechanical Engineering
University of Tuzla, and
Mathematical Institute Belgrade

AN APPROACH TO STUDY OF IMPACT IN MULTIBODY SYSTEMS

Summary: Study of mechanical systems in which impacts occur is associated with forming two systems of equations: differential and algebraic. The first system of equations is used to describe the motion until an impact occurs, while the second one describes the impact itself.

This paper presents a method, the so-called Reduction Method, which enables analysis of these systems without forming the algebraic equations. They are substituted by a new set of differential equations which is easily derived from the equations of motion using reduction. Compared to the other methods, based on the classical theory of impact, the method mentioned above makes the computation of velocities after the impact easier, as well as the plastic impact studies, regardless of the number of degrees of freedom.

1. Introduction

The unilateral constraints in mechanical systems represent the environment for an impact occurance. They have local character and act in link elements such as joints, guides etc. Every disturbance of these constraints results in an impact.

Let us observe, as illustration, motion of a ball in vertical plane connected to a fixed point by a massless non-elastic string, fig. 1a. The ball motion is free until the string is not tight, i.e. until the unilateral constraint acts. String tightening corresponds to an impact and causes the change of the ball velocity. The system described can be represented in another way, by replacing the string by two massless rods.

By applying this way of modelling approach to mechanical systems with unilateral constraints equations of motion with semidefinite massmatrix are obtained. Singular massmatrix corresponds to the points of impact. Without restriction on the method used, numerical integration is interrupted in singular points.

The equations of motion for singular double pendulum are:

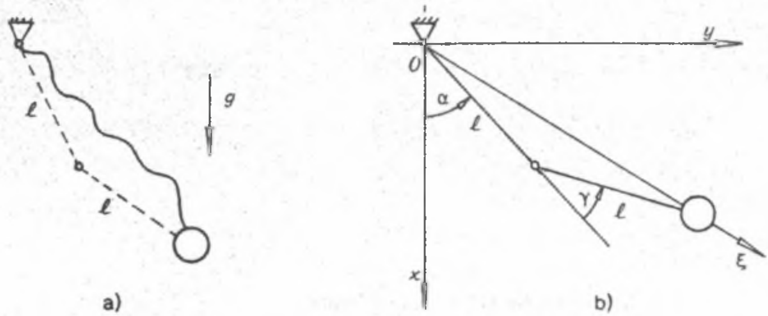

Fig. 1: a) Ball on the string - Replacing the string with two massless rods
b) Singular double pendulum

$$\begin{bmatrix} 2(1+\cos\gamma) & 1+\cos\gamma \\ 1+\cos\gamma & 1 \end{bmatrix}\cdot\begin{bmatrix}\ddot{\alpha}\\ \ddot{\gamma}\end{bmatrix}+\begin{bmatrix}-\sin\gamma(2\dot{\alpha}\dot{\gamma}+\dot{\gamma}^2)\\ \sin\gamma\dot{\alpha}^2\end{bmatrix} = -\frac{g}{\ell}\begin{bmatrix}\sin\alpha+\sin(\alpha+\gamma)\\ \sin(\alpha+\gamma)\end{bmatrix}$$

where ℓ is the length of the rods, and g Earth acceleration. As generalized coordinates are used α and γ, fig.1b. The interruption point is $\gamma=0$.

2. Equations of motion

Equations of motion of a mechanical system with f degrees of freedom in matrix form are:

$$\mathbf{M}(\mathbf{y},t)\ddot{\mathbf{y}} + \mathbf{k}(\mathbf{y},\dot{\mathbf{y}},t) = \mathbf{q}(\mathbf{y},\dot{\mathbf{y}},t) \tag{2.1}$$

where $\mathbf{y}$ is fx1 position vector comprising all generalized coordinates y^i (i=1(1)f). and $\mathbf{M}(\mathbf{y},t)$ is a fxf mass matrix. Vector $q(y,\dot{y},t)$, fx1, comprises all centrifugal and Coriolis forces, while all generalized forces are in vector $q(y,\dot{y},t)$, fx1.

The mass matrix of real mechanical systems subjected to bilateral constraints effects is always symmetric and positive definite. In case of unilateral constraints, using the way of modelling mentioned above, mass matrix is semi-definite. Points of impact in this case correspond to singular values of generalized coordinates.

Equations (2.1) can be transformed into a form suitable for numerical integration

$$\dot{\mathbf{x}} = \Phi(\mathbf{x},t) \tag{2.2}$$

where $\mathbf{x}$ is 2fx1 vector $\mathbf{x}=[\mathbf{y}^T,\dot{\mathbf{y}}^T]^T$, and $\Phi(\mathbf{x},t)$ is 2fx1 vector function

$$\Phi(\mathbf{x},t) = \begin{bmatrix}\dot{\mathbf{y}}\\ \mathbf{M}^{-1}(\mathbf{q}-\mathbf{k})\end{bmatrix}.$$

3. Reduction Method

Numerical integration of equations (2.2) cannot be performed in case of a semidefinite mass matrix. In singular points the determinant of the mass matrix M is zero and the elements of M^{-1} cannot be determined.

In the neighbourhood of singular points equations (2.2) can be observed as a differential equations with a small parameter at derivatives. In this case the mass matrix determinante value is very small and can be treated as a small parameter.

When the small parameters converge to zero and applying the Tichonow's theory a degenerative system of differential equations (2.2) can be obtained [3], [4], [5], [7]. This system can be used in the neighbourhood of the singular point and corresponds to system of algebraic equations. The velocities after the impact can be determined by the numerical integration of the degenerative system.

By eliminating respective columns and rows which correspond to singular coordinates, in equations (2.1), degenerative system is obtained in the other way. The form of the new system of equations is

$$\underset{\sim}{M}\,\ddot{\underset{\sim}{y}} + \underset{\sim}{k} = \underset{\sim}{q}. \tag{3.1}$$

Matrix $\underset{\sim}{M}$ is regular hier and reduced vector $\underset{\sim}{y}$ comprises all nonsingular generalized coordinates.

Reduction of equations (2.1) represents a direct application of Tichonow's theory to the second order differential equations.

In our case of singular double pendulum the new reduced system is

$$2(1 + \cos\gamma)\ddot{\alpha} - \sin\gamma(2\dot{\alpha}\dot{\gamma} + \dot{\gamma}^2) = -g/\ell\,(\sin\alpha + \sin(\alpha + \gamma)). \tag{3.2}$$

4. Numerical Integration

Numerical integration of equation (2.1) with the semidefinite mass matrix requires a few explanations:

1° When does the interruption of numerical integration occur ?

2° How large is the neighbourhood of the singular point in which the numerical integration cannot be performed ?

3° How are the new initial conditions for the integration defined after interruption ?

The answers are as follows:

1° Criterium for the interruption of integration can be derived from the determinant of the mass matrix. The usual dependency of the mass matrix determinant on time is shown on fig. 2.

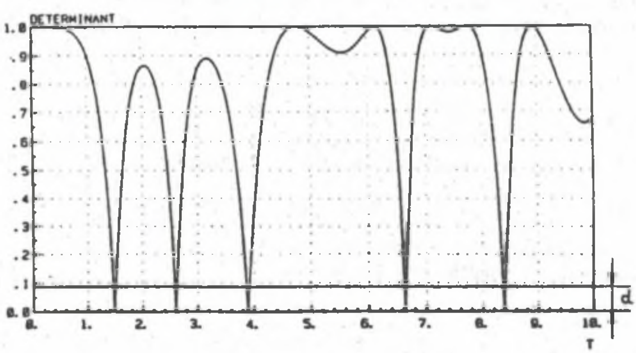

Fig. 2: Interruption of integration using small parameter d

The points of integration interruption are m(t)=detM=0. These points must be excluded by the condition: $m(t) \geq d$ ($d \approx 0$). This condition must be examined after each integration step.

2° The feature of conservative systems with respect to the volume integral in phase space (Liouville's theorem [2]) is used to determine the neighbourhood of the singular point. According to this theorem a product of singular coordinate, for instance $y^1 = y$, and its velocity in the neighbourhood of the singular point is constant, [1],[6].

$$y \cdot \dot{y} = \text{const.} \tag{4.1}$$

The other, nonsingular coordinates have no effect. Starting with relation (4.1) we get for the interval Δt

$$\Delta t = \left| \frac{y^*}{\dot{y}^*} \right| \tag{4.2}$$

where (*) denotes values of variables at the interruption point. Here is made the assumption that the singularity is at the middle of Δt interval.

3° When defining new initial conditions two cases must be distinguished: a case of singular and a case of nonsingular coordinates. As singular coordinates are linear functions of time in time interval Δt (it follows from Tichonow's theory), a mean velocity $\bar{\dot{y}}$

$$\bar{\dot{y}} = 2y^*/\Delta t = 2y^*\dot{y}^*/y^* = 2\dot{y}^* \tag{4.3}$$

can be used, fig. 3.

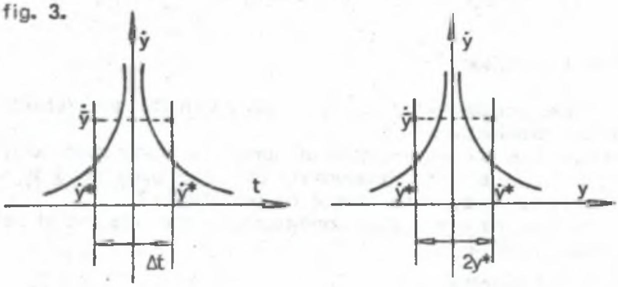

Fig. 3: Meaning of the mean velocity

In the other case the values of nonsingular coordinates and velocities at the end of interval Δt are obtained by integration of degenerative system (3.1). Singular coordinates, which appear as parameters in equation (3.1) are assumed to be linear functions.

5. Plastic impacts analysis

The Reduction method in the neighbourhood of the singular point is related to conservative systems, where impacts are elastic; impact coefficient $k=1$. In case of plastic impacts k falls in interval $[0,1)$, [8], following questions arise:

1° How can be the impact coefficient introduced in the computation ?
2° How can numerical integration be performed ?

The answers are the following:

1° Impact coefficient k ($k=|V''/V'|$) refers to the projection of the velocity on the impact line, where (') and ('') denote the values of variables before and after the impact. This coefficient and the generalized velocities that appear by numerical integration should be related in some way. It can be performed when multiplying generalized velocities by impact coefficient

$$\hat{\dot{y}} = k\,\dot{y}'' ,$$

where $\hat{\dot{y}}$ denotes a new initial value for the next integration and $\dot{y}''$ denotes the value at the end of the time interval Δt. This relation can easily be prooved, see fig. 1b.

As we are interested in impact line direction only, ξ, it follows that

$$\dot{\xi} = -\ell\dot{\gamma}^2\sin(\gamma/2).$$

By introducing the impact coefficient k, relation $\hat{\dot{\xi}}=k\dot{\xi}''$ is obtained. Here is assumed that coordinates vαlues are constant.

2° In case when the impact coefficient is in interval $[0,1)$ numerical integration slightly differs from the case when k=1. Criterium $d \geq m(t)$ used for mass matrix determinant examination must be sharpend after every pass through a singular point. This is the consequence of the fact that the numerical value of the determinant decreases fast when passing through singular points. If this criterium were constant the numerical value of the determinant would be inside the criterium very fast. Factor used for criterium sharpening should be choosen in such away that tangent of the singular coordinate in the interruption point makes an angle of 89°-90° or 90°-91° degrees.

The results of numerical integration using reduction method and classical impact theory are compared on fig. 4 for the impact coefficient values k=0, k=0.5, k 1 and α = 30°, γ = 90°, d=0.0001 and t=5.

6. Conclusion

This paper is a contribution to the study of mechanical systems with impact. In such systems the mass matrix is semidefinite as a result of special way of modelling. Numerical integration of equations of motion requires a special treatment, because an interrupt occur in singular points. Independently of the method used, according to the classical impact theory , transition through each of the singular points demands an algebraic equations system. The proposed Reduction Method suggests so-called reduced system instead. This system can be easily obtained by elimination rows and columns which correspond to singular coordinates from the equations of motion.

Reduction Method extended to plastic impacts enables their analysis in various points of a mechanical system and can be combined with common integration methods.

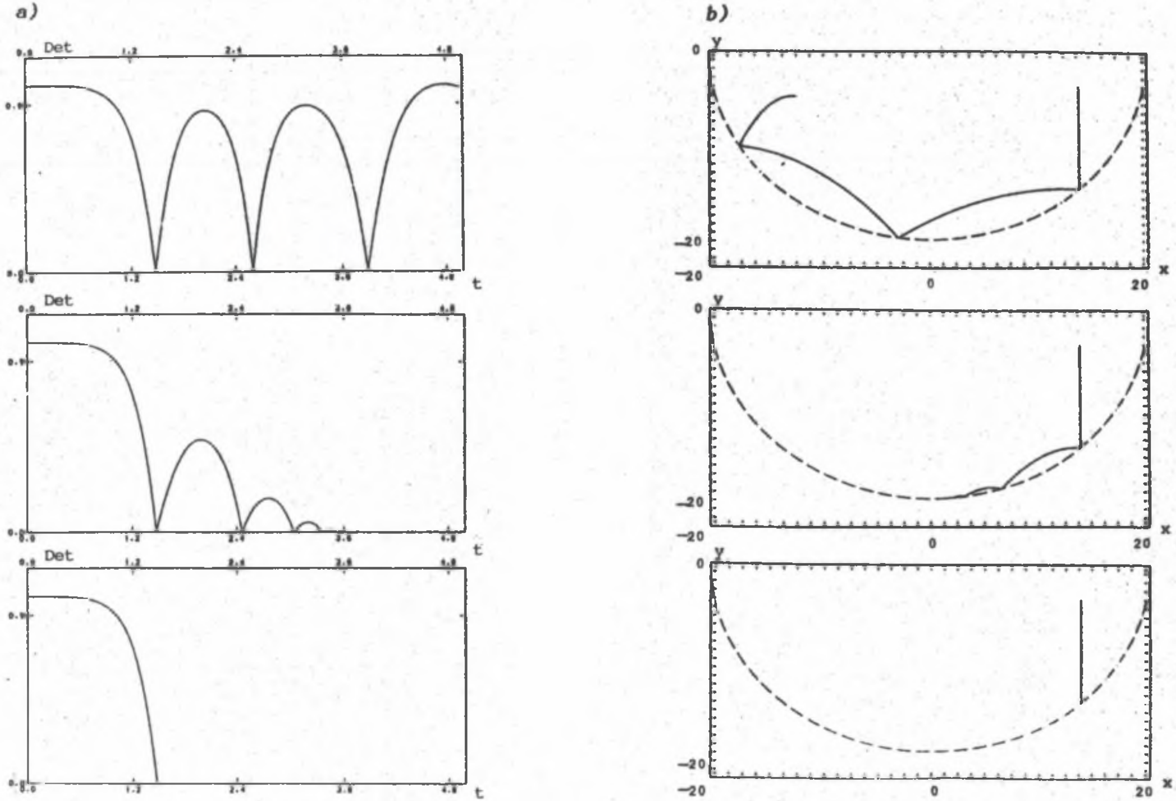

Fig. 4: Comparative results of integration using classical impact theory and the Reduction Method for the impact coefficient values k=1, k=0.5 and k=0

a) determinant of the mass matrix

B) trajectories

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ПОДХОД К ИССЛЕДОВАНИЯМ УДАРА В МНОГОЗВЕННЫХ СИСТЕМАХ

Р е з ю м е

Исследования механических систем, в которых выступает удар, связаны с образованием двух систем уравнений: дифференциального и алгебраического. Первая система уравнений применяется для описания движения до момента появления удара, при чём другую описывает сам удар.

Статья представляет так называемый редуктивный метод, который позволяет анализировать эти системы без образования алгебраических уравнений. Они замещаются новой системой дифференциальных уравнений с применением редукции. По сравнению в другими методами, опирающимися на классическую теорию удара, настоящий метод облегчает вычисление скорости после удара, а также исследование пластического удара независимо от количества степеней свободы.

PODEJŚCIE DO BADAŃ UDARU W UKŁADACH WIELOCZŁONOWYCH

S t r e s z c z e n i e

Badania układów mechanicznych, w których występuje udar, są powiązane z utworzeniem dwóch układów równań: różniczkowego i algebraicznego. Pierwszy układ równań stosuje się do opisania ruchu do momentu wystąpienia udaru, natomiast drugi opisuje sam udar.

Artykuł przedstawia tzw. metodę redukcyjną, która pozwala na analizę tych układów bez tworzenia równań algebraicznych. Podstawia się do nich nowy układ równań różniczkowych, wychodząc od równań, stosując redukcję. W porównaniu z innymi metodami opartymi na klasycznej teorii udaru powyższa metoda ułatwia obliczenie prędkości po udarze oraz badanie plastycznego udaru niezależnie od ilości stopni swobody.

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