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Synthesis of Mechanisms for a Given Motion of Two or More Links

Summary

In this paper the synthesis of planar, single-driven mechanisms which realize a given motion of more than one link is considered. The method based on the finite Burmester theory is presented and the results are illustrated by an example.

1. Introduction

In the analitycal synthesis of planar linkages two main methods found wide application. First consists in deriving the explicit synthesis equations including all the necessary mechanisms parameters (see for example [1, chapter IX, X]), while the second requires a search for the points which, during the motion, are placed on easily mechanized curves like circles and lines. This method was widely studied for a limited number of finitely separated positions. As the number of positions which should be realized by the mechanism increases, it is necessary to apply approximation methods. One of them, often used, is the least-square approximation [2,3].

In this work we have applied the latter method, the so called Burmester theory, to the synthesis of single-driven mechanisms for a given motion of two or more links.

2. Fixed and Moving Planes

The n planes $\pi^{(i)}$ (n - number of links (coupler planes) with given motion) undergo co-planar motion with respect to a fixed plane Π. Coordinate systems $0 - x^{(i)} - y^{(i)}$ and $0 - X - Y$ are rigidly attached to $\pi^{(i)}$ and Π respectively. Let $x^{(i)}$ and $y^{(i)}$ are the Cartesian coordinates of a point embedded in $\pi^{(i)}$ and $(X_k^{(i)}, Y_k^{(i)})$ its coordinates in the fixed system Π in the k-th position of $\pi^{(i)}$. The correspondence between these coordinate systems is given by the linear transformation which in matrix notation can be written:

$$\begin{bmatrix} X_k^{(i)} \\ Y_k^{(i)} \end{bmatrix} = \begin{bmatrix} X_{0k}^{(i)} \\ Y_{0k}^{(i)} \end{bmatrix} + \begin{bmatrix} \cos\Theta_k^{(i)} & -\sin\Theta_k^{(i)} \\ \sin\Theta_k^{(i)} & \cos\Theta_k^{(i)} \end{bmatrix} \begin{bmatrix} x^{(i)} \\ y^{(i)} \end{bmatrix} \tag{1}$$

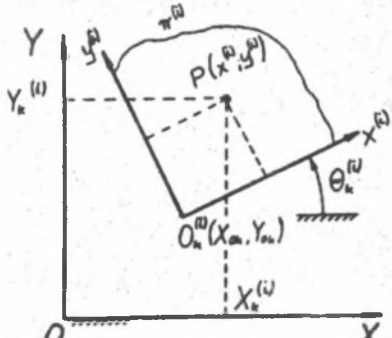

Fig. 1

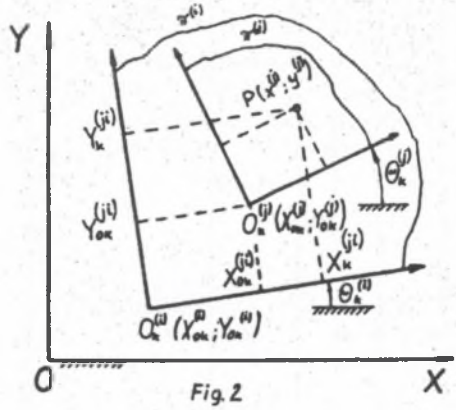

Fig. 2

Here we have used $(X_{0k}^{(i)}, Y_{0k}^{(i)})$ to denote the coordinates of the origin of the moving system $0 - x^{(i)} - y^{(i)}$ measured in the fixed system. As shown in Fig.1 $\Theta_k^{(i)}$ is the rotation angle. It is the angle measured from X axis to $x^{(i)}$ axis.

For further investigations it is also necessary to introduce a transformation equation for a relative movement of two planes (Fig.2). In this case one of them (say i-th) may be considered as the reference plane. If an arbitrary point P in the second plane (say j-th) has the coordinates $x^{(j)}$ and $y^{(j)}$ its coordinates in the reference system are given by the equations:

$$\begin{bmatrix} X_k^{(ji)} \\ Y_k^{(ji)} \end{bmatrix} = \begin{bmatrix} X_{0k}^{(ji)} \\ Y_{0k}^{(ji)} \end{bmatrix} + \begin{bmatrix} \cos\Theta_k^{(ji)} & -\sin\Theta_k^{(ji)} \\ \sin\Theta_k^{(ji)} & \cos\Theta_k^{(ji)} \end{bmatrix} \begin{bmatrix} x^{(j)} \\ y^{(j)} \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} X_{0k}^{(ji)} \\ Y_{0k}^{(ji)} \end{bmatrix} = \begin{bmatrix} \cos\Theta_k^{(i)} & \sin\Theta_k^{(i)} \\ -\sin\Theta_k^{(i)} & \cos\Theta_k^{(i)} \end{bmatrix} \begin{bmatrix} X_{0k}^{(j)} - X_{0k}^{(i)} \\ Y_{0k}^{(j)} - Y_{0k}^{(i)} \end{bmatrix} \tag{3}$$

where $\Theta_k^{(ji)} = \Theta_k^{(j)} - \Theta_k^{(i)}$.

Then the parameters of the relative motion will be $X_{0k}^{(ji)}$, $Y_{0k}^{(ji)}$ and $\Theta_k^{(ji)}$.

3. Basical Concepts

It is well known that for three arbitrary co-planar positions of a rigid body each point of the body determines a unique circle with its center at a corresponding point in the fixed plane II. As the number of required positions increases to four, a freely choosen point of the body will not be placed on the arc in all four positions. This feature will be satisfied only by points of the body which lie on a special curve - the circlepoint curve - which is in one-to-one correspondence with the centerpoint curve embedded in the fixed (reference) plane II. For five positions there are at most four circlepoints (called Burmester points) in the moving body (plane).

In singular cases of the Burmester theory the centerpoints or circlepoints may be in infinity (while the corresponding circlepoints or centerpoints are finite) and they are named *straight-line* and *concurrency points* respectively [4].

4. The Synthesis Algorithm

After the choice of a structural scheme (example schemes with links with a given motion drawn with heavy line are shown in Fig.3 to 9) of the $n_r = (4n - 1)$-link mechanism[1] the necessary number of circlepoints (in the moving planes) with corresponding centerpoints (in the fixed

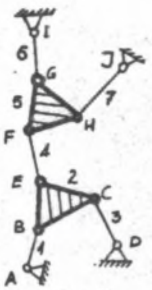

Fig. 3

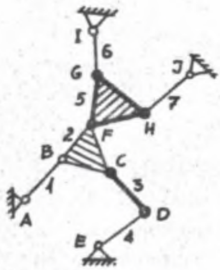

Fig. 4

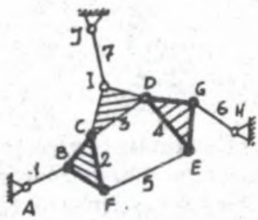

Fig. 5

plane) should be found for all the links with a given motion. This can be done applying one of the graphical or numerical methods given for example in [1, pp 757-760], [4], [5].
The points determined in this way can be used to attach the links with a given motion (coupler planes) to the frame by means of binary links.

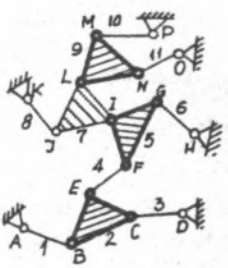

Fig. 6

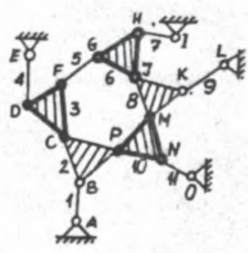

Fig. 7

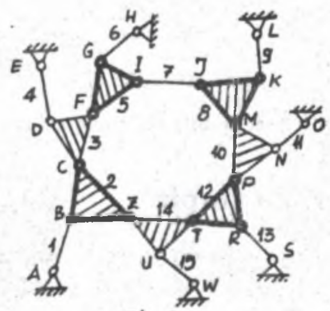

Fig. 8

The next step is essentially the same. It consists also in determining the circlepoints and centerpoints but in the relative movement of each necessary pair of moving links (coupler planes) that is a pair of links which should be coupled together with a binary link according to the structural scheme of the mechanism. (e.g. Fig.6 pairs of links 2,5 and 5,9). The same method to obtain the circlepoints and centerpoints as in the previous step can be applied, but instead of the displacement parameters the relative displacement parameters $(X_{0k}^{(j_1)}, Y_{0k}^{(j_1)}, \Theta_k^{(j_1)})$ obtained from (3) should be substituted.

[1]see Appendix

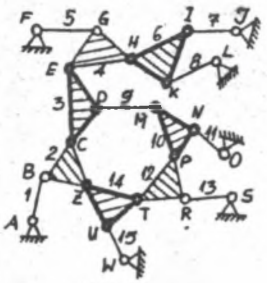

Fig. 9

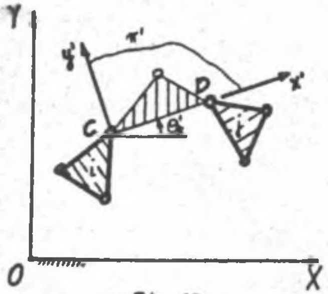

Fig. 10

In some particular cases one more step of the algorithm is necessary. Some of the structural schemes require determining of circlepoints and centerpoints connected with the link coupling the links with a given motion (see fo example link No.3 in Fig.5). The motion of the plane π' attached to the link may be regarded as given because the motion of both kinematic pairs C, D (see Fig.10) is known.

Point C embedded in the i-th plane is the centerpoint in the relative motion of the planes i and j. Point D, the corresponding circlepoint of C, is embedded in the j-th plane. Their coordinates X_C, Y_C, X_D, Y_D in the fixed plane can be easily obtained from transformation (1). Choosing, say, the x-axis along the link from C to D the position parameters of the plane π' are

$$\begin{aligned} X'_{0k} &= X_C \\ Y'_{0k} &= Y_C \\ \tan\Theta'_k &= \frac{Y_D - Y_C}{X_D - X_C} \end{aligned} \tag{4}$$

With these results the circlepoint with corresponding centerpoint can be found.

5. Example

Design a single-driven mechanism for four positions of two links. Two sets of positions are given in Table 1. Let's accept the structural scheme of the mechanism as shown in Fig.3. According

Table 1:

	Plane $i = 1$				Plane $i = 2$			
k	1	2	3	4	1	2	3	4
$X_{0k}^{(i)}$	0.380	0.470	0.573	0.585	0.565	0.720	0.798	0.840
$Y_{0k}^{(i)}$	0.205	0.250	0.283	0.263	0.680	0.745	0.842	1.010
$\Theta_k^{(i)}$[deg]	63.0	62.5	71.5	81.0	10.0	16.0	22.1	32.5

to it two circlepoints should be found for each moving plane and one circlepoint for the relative movement of them.

The parameters of the relative positions obtained from (3) are listed in Table 2. For the designed mechanism the chosen circlepoints and centerpoints are listed in Table 3 (all coordinates are given for the fixed system for the first position of the mechanism). The resulting mechanism

Table 2:

k	1	2	3	4
$X_{0k}^{(21)}$	0.507	0.555	0.602	0.778
$Y_{0k}^{(21)}$	0.051	0.007	−0.036	−0.135
$\Theta_k^{(21)}$[deg]	−53.0	−46.5	−49.4	−48.5

Table 3:

		circlepoint		centerpoint		kinem. pairs
		X	Y	X	Y	
Plane 1	1	-0.082	0.102	0.030	0.004	B — A
	2	0.572	0.173	∞	∞	C — D
Plane 2	1	-0.170	0.905	-0.077	1.193	G — I
	2	0.413	1.040	0.417	1.204	H — J
Rel. movement		0.544	0.612	0.380	1.279	F — H

in its two extreme design positions is shown in Fig.11. The circlepoint-centerpoint pair CD represents the singular case of Burmester theory i.e. the case when the centerpoint D is in infinity and the circlepoint C becomes a straight-line point [4].

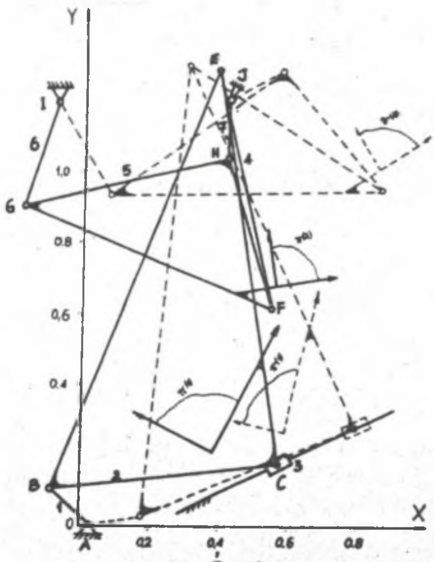

Fig. 11

6. Conclusion

As it has been shown in the paper it is possible to design a single-driven mechanism for a given motion of more than one link based on the proposed method. The algorithm can easily be used also for the case of more than five design positions by application of the least–square approximation [2,3]. For the determination of kinematics of the designed mechanism there is a possibility of using the theory of infinitesimally separated positions. Its application and also the case of spatial mechanisms are hoped to be shortly expanded.

References

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Appendix

The number of degrees of freedom of planar mechanisms with lower pairs is expressed by the equation:

$$W' = 3n_r - 2p_5$$

where: W' - number of degrees of freedom,
n_r - number of movable links,
p_5 - number of lower pairs.

A single-driven mechanism has $W' = 1$ whence after rearranging the terms

$$n_r = \frac{1 + 2p_5}{3}.$$

We remark, that the last equation imposes the odd number of movable links (see the table).

p_5	1	4	7	10	13	16	19	22	...
n_r	1	3	5	7	9	11	13	15	...
n		1		2		3		4	...

If the number of planes with a given motion equals one ($n = 1$), the number of movable links is 3 (four-bar mechanism) and the equation $n_r = 4n - 1$ is satisfied. In the following we shall prove, that the addition of any succesive plane with a given motion requires the addition of four movable links.

Increasing by one the number of planes with a given motion, we must also increase the number of links. This number cannot be increased by two because the link rigidly attached to the plane with a given motion can be coupled together neither with another coupler plane nor with a frame (their relative motion would be restricted only to rotation with respect to the coupling pivot). So it is necessary to add the next "intermediary" link and one more link to satisfy the mobility criterion. This makes together 4 extra links.

Synteza mechanizmów realizujących dany ruch dwóch lub więcej ogniw

Streszczenie

W pracy przedstawiono zagadnienie syntezy płaskich, jednobieżnych mechanizmów dźwigniowych, które realizują dany ruch dwóch lub więcej ogniw. Zastosowana metoda jest oparta na skończonej teorii Burmestra,a wyniki zilustrowane zostały przykładem.

СИНТЕЗ МЕХАНИЗМОВ, ОСУЩЕСТВЛЯЮЩИХ ДАННОЕ ДВИЖЕНИЕ ДВУХ ИЛИ БОЛЬШЕ ЗВЕНЬЕВ

Резюме

В работе представлена проблема синтеза плоских односторонних рычажных механизмов, которые осуществляют данное движение двух или больше звеньев. Примененный метод опирается на завершённой теории Бурместра, а результаты проиллюстрированы примерами.

Recenzent: prof. dr hab. inż. J. Wojnarowski

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