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ON THE DYNAMIC AND MATHEMATICAL MODELLING OF SPACIAL VIBRATIONS IN FLEXIBLY INSTALLED MACHINES

Abstract. The present work examines the interaction between torsional and general vibrations in machine units based on a mathematical model submitted by the authors. These two oscillating motions are usually examined separately, but actually they are interacting. The general vibrations give rise to additional inertia forces to all mobile linkages of the unit which after being applied to the reduction link produce an additional moment exciting torsional vibrations. The latter produce additional inertia forces exciting vibrations of the unit as a whole. The results obtained both in separate examination of the vibrations and their mutual interaction have been analysed comparatively. The analysis in question proved availability of new resonance phenomena unprognosticable in the classic method.

The flexibly installed machine units perform two oscillating motions simultaneously:

- general vibrations of the units as a solid, excited by the unbalanced inertia forces,
- torsional vibrations excited by intermittently changing forces.

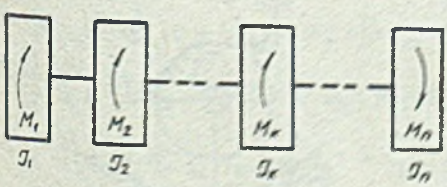

Fig. 1

These two oscillating motions are usually examined separately. The general vibrations are investigated using the model illustrated on Fig. 1. The inertia forces determined at constant angular velocity are reduced in the mass centre. The torsional vibrations are investigated by the model illustrated on Fig. 2. Actually these two motions are interact-

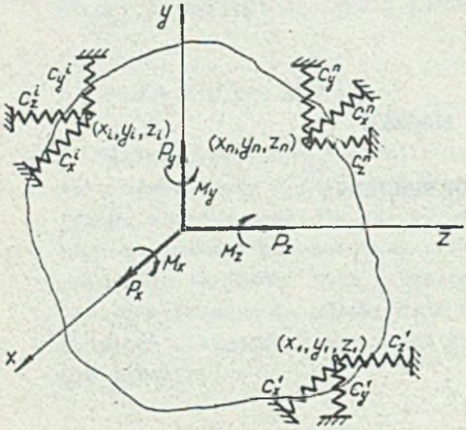

Fig. 2

ing. The general vibrations produce additional inertia forces to all the mobile linkages which after being reduced produce an additional moment exciting the unsteady rotation. On the other hand the unsteady rotation of the reduction linkage produces additional inertia forces exciting the vibrations of the unit as a whole. In this connection it is necessary to work out a mathematical model describing the above mentioned motions in their natural interaction. The way how to make it is given in (1) (2) and (3). In (1) mechanical..mathematical model describing the interaction between torsional vibrations and general spacial vibrations is given. In (2) and (3) the interaction between torsional and general vibrations when the general vibrations are in one direction is examined. We prove herewith that the oscillating power exchanged between the above mentioned oscillating motions is activated in certain stationary conditions and that the interaction between them needs to be considered because otherwise that would cause considerable errors.

The present work examines the interacting general spacial vibrations and the torsional vibrations in stationary conditions. On Fig. 1 a dynamic model

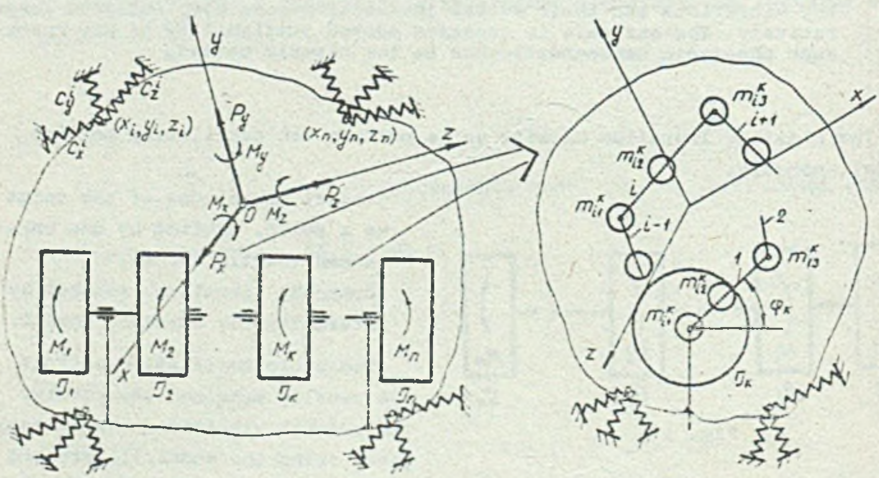

Fig. 3

is illustrated. You can also see a model of n discrete masses describing the torsional vibrations. All discrete masses are obtained after reduction of a mechanism having P_k $(k = 1,2,\dots,n)$ linkages.

The linkages of the mechanism are modelled by the dynamical equivalence conditions and by three discrete masses m^k_{ij} $(i = 1,2,\dots,P_k,\ j = 1,2,3;\ k = 1,2,\dots,n)$. The φ_k coordinate determines the position of the k reduction link. The mechanisms are linked through massfree elastic linkages having the stiffness of C_{kk+1} $(k = 1,2,\dots,n-1)$. The unit is a solid body mounted on a foundation using P elastic elements. Each elastic element is connected with the unit in a point with x_i, y_i, z_i $(i = 1,2,\dots,p)$ coordinates, having C^i_x, C^i_y, C^i_z reciprocating stiffnesses and K^i_x, K^i_y, K^i_z torsion stiffnesses. We will investigate herewith the general vibrations by the coordinate system initiating from the mass centre and exes orientated on the main inertia axes. The position of the body is determined by the shifts (u, v and w) in direction of x, y and z axes and the rotations of the body to the same axes α, β and γ. With P_x, P_y, P_z, M_x, M_y, M_z are designated the components of the main vector and the resultant moment of the external forces exciting the general vibrations.

After applying the Langrange method we have the following differential equations (1):

$$
\left|\begin{array}{l}
m\ddot{u} + C_x u + U_z\beta - U_y\gamma = P_x + \Phi_u - f_u(\beta,\dot{\beta},\ddot{\beta},\gamma,\dot{\gamma},\ddot{\gamma},\ddot{\varphi}_k) \\
m\ddot{v} + C_y v + \gamma V_x - \alpha V_z = P_y + \Phi_v - f_v(\alpha,\dot{\alpha},\ddot{\alpha},\gamma,\dot{\gamma},\ddot{\gamma},\ddot{\varphi}_k) \\
m\ddot{w} + C_z w + \alpha W_x - \beta W_x = P_z + \Phi_w - f_w(\alpha,\dot{\alpha},\ddot{\alpha},\beta,\dot{\beta},\ddot{\beta},\ddot{\varphi}_k) \\
J_x\ddot{\alpha} + C_{xx}\alpha - vV_z + wW - \beta C_{xy} - \gamma C_{xz} = M_x + \Phi_\alpha - f_\alpha(\ddot{v},\ddot{w},\dot{\alpha},\ddot{\alpha},\dot{\gamma},\ddot{\gamma},\ddot{\varphi}_k) \\
J_y\ddot{\beta} + C_{yy}\beta - wW_x + uU_z - \gamma C_{yz} - \alpha C_{yx} = \\
= M_y + \Phi_\beta - f_\beta(\ddot{u},\ddot{w},\alpha,\dot{\alpha},\ddot{\alpha},\beta,\dot{\beta},\ddot{\beta},\gamma,\dot{\gamma},\ddot{\gamma}) \\
J_z\ddot{\gamma} + C_{zz}\gamma - uU_y + vV_x - \alpha C_{xz} - \beta C_{zy} = \\
= M_z + \Phi_\gamma - f_\gamma(\ddot{u},\ddot{v},\alpha,\dot{\alpha},\ddot{\alpha},\dot{\beta},\ddot{\beta},\dot{\gamma},\ddot{\gamma},\ddot{\varphi}_k) \\
J_k\ddot{\varphi}_k - C_{k-1k}(\varphi_{k-1} - \varphi_k) + C_{kk+1}(\varphi_k - \varphi_{k+1}) = \\
= M_k + \Phi_k - f_k(\ddot{u},\ddot{v},\ddot{w},\ddot{\alpha},\ddot{\beta},\ddot{\gamma}) \\
\forall k = 1,2,\dots,n
\end{array}\right. \qquad (1)
$$

where:

Φ_a; $(a = u,v,w,\alpha,\beta,\gamma)$ are components of the main vector and the main moment of the inertia forces resulting from the rotation with constant angular velocity:

$$\Phi_u = -\omega^2 \sum_{k=1}^{n} (S_x^k)'' ; \quad \Phi_v = -\omega^2 \sum_{k=1}^{n} (S_y^k)'' ; \quad \Phi_w = -\omega^2 \sum_{k=1}^{n} (S_z^k)''$$

$$\Phi_\alpha = -\omega^2 \sum_{k=1}^{n} (J_{yz''}^k - J_{zy''}^k) ; \quad \Phi_\beta = -\omega^2 \sum_{k=1}^{n} (J_{zx''}^k - J_{xz''}^k) ; \tag{2}$$

$$\Phi_\gamma = -\omega^2 \sum_{k=1}^{n} (J_{xy''}^k - J_{yx''}^k)$$

$\Phi_k = \frac{1}{2}\omega^2 \frac{dJ_k}{d\varphi_k}$ - reduced moment of the inertia forces resulting from rotation with constant angular velocity;

S_x^k, S_y^k, S_z^k - mass statical moment of the relatively mobile linkages (with m_{ij}^k masses):

$$S_x^k = \sum_{i=1}^{p_k} \sum_{j=1}^{3} m_{ij}^k x_{ij}^k ; \quad S_y^k = \sum_{i=1}^{p_k} \sum_{j=1}^{3} m_{ij}^k y_{ij}^k ; \quad S_z^k = \sum_{i=1}^{p_k} \sum_{j=1}^{3} m_{ij}^k z_{ij}^k$$

The statistical moments divide into constant (S_{ao}^k) and variable part:

$$S_a^k = S_{ao}^k + S_{a1}^k(\ _k) ; \quad a = x,\ y,\ z; \tag{3}$$

The mass characteristics $J_{ab'}$ and $J_{ab''}$; $(a,\ b = x,\ y,\ z)$ are derivatives of the centrifugal mass inertia moments J_{ab}:

$$J_{a,b} = \sum_{k=1}^{n} J_{a,b}^k ; \quad J_{a,b}^k = \sum_{i=1}^{p_k} \sum_{j=1}^{3} m_{ij} a_{ij}^k b_{ij}^k ; \quad a,\ b = x,\ y,\ z;$$

$$J_{ab'} = \sum_{k=1}^{n} J_{ab'} ; \quad J_{ab''} = \sum_{k=1}^{n} J_{ab''}^k ; \quad J_{ab'} = \sum_{i=1}^{p_k} \sum_{j=1}^{3} a_{ij}^k (b_{ij}^k)' ;$$

$$J^k_{ab''} = \sum_{i=1}^{p_k} \sum_{j=1}^{3} a^k_{ij} (b^k_{ij})'' .$$

Symbols ( )' and ( )'' designated the derivatives of the φ_k geometrical parameter:

f_k - raduced moment of the inertia forces of the m^k_{ij} masses resulting from the general vibrations of the mechanisms:

$$f_k = \ddot{u}(S^k_x)' + \ddot{v}(S^k_y) + \ddot{w}(S^k_z)' + \ddot{\alpha}(J^k_{yz'} - J^k_{zy'}) + \ddot{\beta}(J^k_{zx'} - J^k_{xz'}) +$$

$$+ \ddot{\gamma}(J^k_{xy'} + J^k_{yx'}) + \Delta J_k \ddot{\varphi}_k \tag{4}$$

here ΔJ_k signifies the variable part of the mass moment of inertia J_k.

f_a; $(a = u, v, w, \alpha, \beta, \gamma)$ - are components of the main vector and the main moment of the forces of inertia of the m^k_{ij} masses resulting from the general vibrations of the mechanisms and the irregular rotation from the torsional vibrations:

$$f_a = \sum_{k=1}^{n} f^k_a$$

$$f^k_u = \ddot{\beta} S^k_z + 2\dot{\beta}\omega(S^k_z)' + \beta\omega^2 (S^k_z)'' - \ddot{\gamma} S^k_y - 2\dot{\gamma}\omega(S^k_y)' -$$

$$-\gamma\omega^2 (S^k_y)'' + \ddot{\varphi}_k (S^k_k)'$$

$$f^k_v = \ddot{\gamma} S^k_x + 2\dot{\gamma}\omega(S^k_x)' + \gamma\omega^2 (S^k_x)'' - \ddot{\alpha} S^k_z - 2\dot{\alpha}\omega(S^k_z)' -$$

$$-\alpha\omega^2 (S^k_k)'' + \ddot{\varphi}_k (S^k_y)'$$

$$(5)$$

$$f^k_w = \ddot{\alpha} S^k_y + 2\dot{\alpha}\omega(S^k_y)' + \alpha\omega^2 (S^k_y)' - \ddot{\beta} S^k_x - 2\dot{\beta}\omega(S^k_x)' -$$

$$-\beta\omega^2 (S^k_x)'' + \ddot{\varphi}_k (S^k_z)'$$

$$f^k_\alpha = \dot{\beta}\omega (J_x)' - \ddot{v} S^k_z + \ddot{w} S^k_y - \ddot{\beta} J^k_{xy} - 2\dot{\beta}\omega J^k_{xy} - \ddot{\gamma} J^k_{xz} -$$

$$- 2\omega\dot{\gamma} J^k_{x'z} + \ddot{\varphi}_k (J_{yz'} - J_{zy'}) + J^k_x \ddot{\alpha}$$

$$f^k_\beta = \dot{\beta}\omega(J_y)' - \ddot{v}S^k_x + \ddot{u}S^k_z - \ddot{\gamma}J^k_{yz} - \dot{\gamma}2\omega J^k_{zy'} - \gamma\omega^2(J^k_{zy''} - J^k_{yz''}) -$$

$$- \ddot{\alpha}J^k_{yx} - 2\dot{\alpha}\omega J^k_{xy'} - \alpha\omega^2(J^k_{xy''} + J^k_{yx''}) +$$

$$+ \ddot{\varphi}_k(J^k_{zx'} - J^k_{xz'}) + J^k_y\ddot{\beta}$$

$$f^k_\gamma = \dot{\gamma}\omega(J_z)'' - \ddot{u}S^k_y + \ddot{v}S^k_k - \ddot{\alpha}J^k_{zx} - \dot{\alpha}2\omega J^k_{xz'} - \alpha\omega^2(J^k_{xz''} - J^k_{zx''}) -$$

$$- \ddot{\beta}J^k_{zy} - 2\dot{\beta}\omega J^x_{yz'} + \ddot{\varphi}_k(J^k_{xy'} - J^k_{yx'}) + J^k_z\ddot{\gamma}$$

In (6) J^k_x, J^k_y and J^k_z signify the mass inertia moments of the mobile linkages to the main inertia axes:

$$J^k_x = \sum_i \sum_j m^k_{ij}\left[(y^k_{ij})^2 + (z^k_{ij})^2\right]; \quad J^k_y = \sum_i \sum_j m^k_{ij}\left[(x^k_{ij})^2 + (z^k_{ij})^2\right];$$

$$J^k_z = \sum_i \sum_j m^k_{ij}\left[(x^k_{ij})^2 + (y^k_{ij})^2\right]$$

The unit parameters are as follows:

$$m = m_o + \sum_k \sum_i \sum_j m^k_{ij}; \quad C_x = \sum_{i=1} C^i_x; \quad C_y = \sum_i C^i_y; \quad C_z = \sum_i C^i_z$$

$$U_z = \sum_i C^i_x z_i; \quad U_y = \sum_i C^i_x y_i; \quad V_x = \sum_i C^i_y x_i; \quad V_z = \sum_i C^i_y z_i$$

$$W_x = \sum_i C^i_z x_i; \quad W_y = \sum_i C^i_z y_i \tag{7}$$

$$C_{xx} = \sum_i (K^i_x + C^i_y z^2_i + C^i_z y^2_i); \quad C_{yy} = \sum_i (K^i_y + C^i_z x^2_i + C^i_x z^2_i)$$

$$C_{zz} = \sum_i (K^i_z + C^i_x y^2_i + C^i_y x^2_i); \quad C_{xy} = \sum_i C^i_z x_i y_i; \quad C_{xz} = \sum_i C^i_y x_i z_i$$

$$C_{yz} = \sum_i C^i_x y_i z_i.$$

Here m_o signifies the mass of the linkages which are relatively immobile.

We will substitute the variables in equation (1) by the following substitution:

$$\varphi_k = \sum_{j=0}^{n-1} \alpha_{kj} q_j \tag{8}$$

Here α_{kj} are modes of the free torsional vibrations defined by the following equations:

$$\begin{cases} J_k\ddot{\varphi}_k - C_{k-1,k}(\varphi_{k-1} - \varphi_k) + C_{kk+1}(\varphi_k - \varphi_{k+1}) = 0 \\ \forall K = 1,2,\ldots,n \end{cases} \tag{9}$$

After substituting (8) in (1) and transformations based on the orthogonality conditions we have the equations describing the torsional vibrations

$$J_{ej}(\ddot{q}_j + \omega_j^2 q_j) = M_{ej} + \dot{f}_{ej} \tag{10}$$

where:

$$M_{ej} = \sum_{k=1}^{n} (M_k + \bar{\phi}_k)\alpha_{kj}; \quad f_{ej} = \sum_{k=1}^{n} f_k\alpha_{kj}; \quad J_{ej} = \sum_{k=1}^{n} J_{ko}\alpha_{kj}$$

ω_j; $(j = 0,1,2,\ldots,n-1)$ - frequencies of the free torsional vibrations defined by (9).

When $j = 0$ we have $\omega_o = 0$; $\alpha_{10} = \alpha_{20} = \alpha_{30} = \ldots \alpha_{no} = 1$.

According to the principle of the basic main movements (4, 5) for machine units we can admit that motion law is formed principally based on the motion of the unit as a solid (q_o) and the vibrations q_1 and q_2 from the initial two natural frequences ω_1 and ω_2:

$$\varphi_k \approx q_o + \alpha_{k1}q_1 + \alpha_{k2}q_2 \tag{11}$$

After substituting (11) and (10) in (1) we obtain a simplified mathematical model:

$$\begin{cases} m\ddot{u} + C_x u + U_z\beta - U_y\gamma = P_x + \Phi_u - f_u \\ m\ddot{v} + C_y v + \gamma V_x - \alpha V_z = P_y + \Phi_v - f_v \\ m\ddot{w} + C_z w + \alpha W_y - \beta W_x = P_z + \Phi_w - f_w \\ J_x\ddot{\alpha} + C_{xx}\alpha - vV_z + wW_y - \beta C_{xy} - \gamma C_{xz} = M_x + \Phi_\alpha - f_\alpha \\ J_y\ddot{\beta} + C_{yy}\beta - wW_x + uU_z - \gamma C_{yz} - \alpha C_{yx} = M_y + \Phi_\beta - f_\beta \\ J_z\ddot{\gamma} + C_{zz}\gamma - uU_y + vV_x - \alpha C_{xz} - \beta C_{zy} = M_z + \Phi_\gamma - f_\gamma \end{cases} \quad (12\text{-a})$$

$$J_{ej}(\ddot{q}_j + \omega_j^2 q_j) = M_{ej} + f_{ej} \qquad j = 0, 1, 2 \quad (12\text{-b})$$

To solve (12) we will apply paralelly the method of the small parameter and the method of the harmonious balance. Before doing this we will have to solve the problem about the choice of a small parameter. The inertia forces

$$f_a; \quad (a = u, v, w, \alpha, \gamma), \quad f_{ej}; \quad (j = 0, 1, 2)$$

are small compared to the rest of the forces. This allows us to use a small formal parameter ε, in equation (12) i.e.:

$$f_a = \varepsilon \tilde{f}_a; \quad a = u, v, w, \alpha, \beta, \gamma; \quad f_{ej} = \varepsilon \tilde{f}_{ej}; \quad j = 0, 1, 2. \quad (13)$$

After replacing of $\varepsilon = 0$ the initial system (12) disintegrates into two separate subsystems. One of them (12-a) corresponds to the proposed by Papcovich mathematical model describing the general vibrations of the unit as a solid (Fig. 1). The second subsystem (12-b) describing the torsional vibrations in their separate investigation using the model illustrated on Fig. 2. It is quite evident that when the small parameter is introduced under resonance, the vibration power of the exciters of interacting torsional and general vibrations is distributed in the coordinates the way it is when there is no interaction between the mentioned vibrations. That is so, since the modes of the free vibrations of models (12-a) and (12-b) in separate investigation will be the same as the modes of the free vibrations of the dynamically connected systems. It proves that using the small parameter method in this case leads to iteration process when the n-solution does not tend to zero.

There is also another way of introducing a small parameter to the system of differential equations (12). Functions f_a and f_{ej}; $(a = u, v, w, \alpha, \beta, \gamma)$

are linear with regard to the generalised coordinates and their derivatives, but having coefficients representing periodical functions of φ_k. Some of these coefficients are having a consttant constituent part too. This is the reason for us to separate the members with constant coefficients in f_a and f_{ej} and represent them as follows:

$$f_a = f_{ao} + \varepsilon f_{a1}; \quad f_{ej} = f_{ejo} + \varepsilon f_{ej1} \tag{13}$$

Based on (5), functions f_{ao} and f_{ej} are as follows:

$$f_{ao} = \frac{1}{T}\int_0^T \sum_{k=1}^{n} f_a^k d\varphi_k; \quad f_{ejo} = \frac{1}{T}\int_0^T \sum_{k=1}^{n} f_k d_{kj} d\varphi_k \tag{14}$$

where T is the period of the established vibrations, while functions εf_{a1} and εf_{ej1} consist of members with variable coefficients only.

With the small parameter introduced as described above where $\varepsilon = 0$ we have the free vibrations of the two subsystems interconnected. The new modes of vibrations differ from the modes of vibrations in separate investigation of the torsional and the general vibrations.

This leads to difference in the vibration power distribution in case of resonance.

To define the forced vibrations we expand the functions εf_{a1} and εf_{e1} in order of Fourier:

$$f_{a1} = \sum_{k=1}^{\infty} f_{a1k}^{s} \sin k\omega t + \sum_{k=1}^{\infty} f_{a1k}^{c} \cos k\omega t \tag{15}$$

$$f_{a1k}^{s} = \frac{1}{\pi}\int_0^{2\pi} \sum_{p=1}^{n} f_a^p \sin k\varphi_p \ \varphi_p; \quad f_{a1k}^{c} = \frac{1}{\pi}\int_0^{2\pi} \sum_{p=1}^{n} f_a^p \cos k\varphi_p d\varphi_p$$

$$f_{ej1} = \sum_{k=1}^{\infty} f_{ej1k}^{s} \sin k\omega t + \sum_{k=1}^{\infty} f_{ej1k}^{c} \cos k\omega t \tag{15}$$

$$f_{ej1k}^{s} = \frac{1}{\pi}\int_0^{2\pi} \sum_{p=1}^{n} f_p \alpha_{pj} \sin k\varphi_p d\varphi_p; \quad f_{ej1k}^{c} = \frac{1}{\pi}\int_0^{2\pi} \sum_{p=1}^{n} f_p \alpha_{pj} \cos k\varphi_p \ \varphi_p$$

Coefficients f_{a1k}^{s}, f_{a2k}^{c}, f_{ej1k}^{s}, f_{ej1k}^{c} in (15) are linear functions having constant coefficients in regard to the generalised coordinates and their derivatives:

$$f^r_{a1k} = f^r_{a1k}(u,v,\ldots,\gamma,\ddot{u},\ddot{v},\ldots,\ddot{\gamma},\ddot{q}_o,\ddot{q}_1,\ddot{q}_2)$$
$$f^r_{ej1k} = f^r_{ej1k}(u,v,\ldots,\gamma,\ddot{u},\ddot{v},\ldots,\ddot{\gamma},\ddot{q}_o,\ddot{q}_1,\ddot{q}_2); \quad \forall r = c,s \tag{16}$$

After substituting (13) and (15) in (12) we have for solution as follows:

$$a = \sum_{k=1}^{\infty}(A_{ak}\sin k\omega t + B_{ak}\cos\omega t) = \sum_{k=1} E_{ak}\sin(k\omega t + \delta_{ak}) \tag{17}$$

$$q_p = \sum_{k=1}^{\infty}(C_{pk}\sin k\omega t + D_{pk}\cos\omega t) = \sum_{k=1}^{\infty} G_{pk}\sin(k\omega t + \delta_{pk})$$

We substitute (17) in (12). After carrying out a harmonic balance we obtain the unknown vibration amplitudes. It proves that the vibration interaction leads to the following new resonance conditions:

- resonances of general vibrations at the following ratio of the orders of harmonic constituents:

$$K_M\omega = \omega_c; \quad |K_s \pm K_M|\,\omega = \omega_c; \quad |K_J \pm K_M|\,\omega = \omega_c$$
$$|K_S \pm K_{\Delta J}|\,\omega = \omega_c; \quad |K_J \pm K_{\Delta J}|\,\omega = \omega_c \tag{18}$$
$$pK_s\omega = \omega_c, \quad p = 2,\ 3$$

- resonances of torsional vibrations:

$$|K_s \pm K_J|\,\omega = \omega_c; \quad pK_J\omega = \omega_c, \quad p = 1,\ 2,\ 3 \tag{19}$$

In (18) and (19) the following symbols are introduced:

ω_c - frequency of the free vibrations,

K_s - orders in expansion S^k_a of static moments in Fourier order,

K_J - orders in expansion of centrifugal mass inertia moments $J_{a,b}$, $J_{ab'}$, $J_{ab''}$; (a, b = x, y, z) in Fourier order,

K_M - orders of harmonic constituents of the external forces exciting torsional vibrations,

$K_{\Delta J}$ - orders of harmonic constituents of the variable part from the produced mass inertia moments ΔJ_k.

(18) and (19) do not use the whole variety of possible resonances.

The investigation carried out shows that the exciters of the torsional vibrations in traditional model can excite general resonance vibrations while the exciters of general vibrations excite torsional vibrations.

There is a numerical example to check the effectiveness of the submitted mechanical mathematical model and the quantitative estimation of the new results evolving from the vibration interaction. We have tested an elastically mounted diesel generator consisting of four-stroke three-cylinder diesel engine 3AL-25/38 and alternating current generator. Torsional vibrations are examined using five - mass dynamic model. The natural frequences of the torsional vibrations excluding their interaction with the general ones are as follows:

$$\omega_1 = 1190\ s^{-1},\quad \omega_2 = 1897\ s^{-1},\quad \omega_3 = 3277\ s^{-1},\quad \omega_4 = 4686\ s^{-1}. \qquad (20)$$

The natural frequences of the general spacial vibrations, excluding their interaction with the torsional vibrations are as follows:

$$\omega_{10} = 19{,}2,\quad \omega_{20} = 20{,}48,\quad \omega_{30} = 27{,}1 \qquad (21)$$

$$\omega_{40} = 52{,}87,\quad \omega_{50} = 63{,}44,\quad \omega_{60} = 76{,}79\ s^{-1}$$

The frequencies of the interacting torsional and general vibrations are:

$$\omega_{c1} = 18{,}37,\quad \omega_{c2} = 20{,}4,\quad \omega_{c3} = 28{,}4,\quad \omega_{c4} = 50{,}9$$

$$\omega_{c5} = 67{,}8,\quad \omega_{c6} = 86{,}98,\quad \omega_{c7} = 1190{,}9\ s^{-1},\quad \omega_{c8} = 1897{,}9\ s^{-1} \qquad (22)$$

Frequencies ω_{c7} and ω_{c8} correspond to the natural frequences ω_1 and ω_2 of torsional vibrations (20) determined by the traditional methods.

The natural frequencies ω_{ck} $(k = 1,2,\dots,6)$ correspond to the natural frequences ω_{ko} $(k = 1,2,\dots,6)$ of spacial general vibrations (21) while the difference between them is up to 13%.

Table 1 gives the modes of the general vibrations defined both in the traditional and the proposed method. To check the correctness of the obtained modes of the free interacting vibrations we have made an orthogonality verification.

Table 2 gives the results from calculating the resonance vibration where $\omega = 56{,}8\ s^{-1}$. In that table E^{o}_{ak} designate the numerical values of the general vibration amplitudes defined by the traditional method, while E_{ak} designate the general vibration amplitudes determined by considering their interaction with the torsional ones. G^{o}_{pk} and G_{pk} designate the torsional vibration amplitudes in their separate investigation and by considering their interaction with the general vibrations respectively.

Table 1

ω_c s^{-1}	u	v	w	α	β	γ
ω_{60} = 76,79	0	0,056	-0,21	0,976	-0,015	-10^{-3}
ω_{c6} = 86,98	$2,4.10^{-3}$	0,098	-0,177	0,976	-0,0144	0,026
ω_{50} = 63,44	-0,087	8.10^{-3}	0,026	0,087	0,99	-0,02
ω_{c5} = 67,8	-0,141	0,064	0,45	0,454	0,99	-0,047
ω_{40} = 52,87	5.10^{-3}	0,09	0,714	0,69	-0,036	-2.10^{-3}
ω_{c4} = 50,9	0,067	0,128	0,714	0,6	-0,304	-0,0117
ω_{30} = 27,1	0,237	-0,018	2.10^{-3}	0,014	0,042	0,97
ω_{c3} = 28,4	0,238	-0,33	0,02	0,105	0,045	0,97
ω_{20} = 20,48	0,955	0,04	-5.10^{-3}	-0,015	0,09	-0,28
ω_{c2} = 20,4	0,955	0,296	-0,023	-0,1	0,094	-0,218
ω_{10} = 19,2	-0,032	0,95	-0,058	-0,31	-2.10^{-3}	0,032
c1 = 18,37	-0,211	0,95	-0,06	-0,326	-0,014	0,267

Table 2 shows that when $\omega = 56,8\ s^{-1}$ the following resonances of general vibrations exist which can not be prognosticated by the traditional methods:

(i) resonance with natural frequency ω_{c3}:

$$\omega = \frac{\omega_{c3}}{0,5} = 56,8\ s^{-1}$$

(ii) resonance with natural frequency ω_{c6}:

$$\omega = \frac{\omega_{c6}}{1,5} = 58\ s^{-1}$$

In both cases the resonance vibration modes coincides with the free vibration modes. Vibrations in the order of K = 0,5 and K = 1,5 are not existing in the traditional model.

The results from the numerical example prove the quantitative appearance of the interaction between torsional and spacial general vibrations.

Table 2

	U, m	V, m	W, m	, rad	, rad	, rad
$E^{o}_{a0,5}$	0	0	0	0	0	0
$E_{a0,5}$	$3,6.10^{-3}$	$5,13.10^{-3}$	$2,65.10^{-4}$	$1,4.10^{-3}$	$6,7.10^{-4}$	0,0146
E^{o}_{a1}	$1,66.10^{-4}$	$2,3.10^{-5}$	$1,46.10^{-4}$	2.10^{-4}	$1,46.10^{-3}$	$4,2.10^{-5}$
E_{a1}	$1,1.10^{-4}$	$1,4.10^{-4}$	$8,3.10^{-4}$	$7,5.10^{-4}$	$6,1.10^{-4}$	$2,1.10^{-5}$
$E^{o}_{a1,5}$	0	0	0	0	0	0
$E_{a1,5}$	$7,7.10^{-6}$	$2,2.10^{-4}$	$4,1.10^{-4}$	$2,08.10^{-3}$	5.10^{-5}	$6,8.10^{-5}$
E^{o}_{a2}	$8,4.10^{-7}$	$7,6.10^{-9}$	$2,5.10^{-7}$	3.10^{-7}	$3,3.10^{-5}$	$1,7.10^{-7}$
E_{a2}	$3,2.10^{-6}$	$1,1.10^{-6}$	$7,8.10^{-6}$	$1,4.10^{-5}$	$3,7.10^{-5}$	$6,8.10^{-7}$

K=0,5	q_o	q_1	q_2
$G^{o}_{p0,5}$	0,018	$2,4.10^{-4}$	7.10^{-7}
$G_{p0,5}$	0,024	3.10^{-4}	$2,2.10^{-6}$
K=1,5	q_o	q_1	q_2
$G^{o}_{p1,5}$	$1,4.10^{-3}$	5.10^{-5}	$1,5.10^{-6}$
$G_{p1,5}$	$3,6.10^{-3}$	$7,5.10^{-5}$	2.10^{-6}

K=1	q_o	q_1	q_2
G^{o}_{p1}	$2,4.10^{-4}$	10^{-4}	$2,2.10^{-5}$
G_{p1}	$1,6.10^{-3}$	$1,2.10^{-4}$	$1,5.10^{-5}$
K=2	q_o	q_1	q_2
G^{o}_{p2}	6.10^{-4}	4.10^{-5}	$1,1.10^{-5}$
G_{p2}	7.10^{-5}	11.10^{-5}	$1,7.10^{-5}$

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O DYNAMICZNYM I MATEMATYCZNYM MODELOWANIU DRGAŃ PRZESTRZENNYCH W URZĄDZENIACH O POSADOWIENIU PODATNYM

S t r e s z c z e n i e

Praca przedstawia współzależności pomiędzy skrętnymi i ogólnymi drganiami w elementach maszyn w oparciu o zaproponowany przez autora model matematyczny. Te dwa typy drgań są zazwyczaj badane oddzielnie choć właściwie współoddziałują. Ogólne drgania powodują dodatkowe siły inercyjne we wszystkich mobilnych połączeniach urządzenia, które po uwzględnieniu w procesie redukcji wytwarzają dodatkowy moment wywołujący drgania skrętne. Drgania te powodują powstanie dodatkowych sił inercyjnych, które z kolei wywołują drgania całości urządzenia. Uzyskane rezultaty zostały przeanalizowane zarówno podczas osobnych badań drgań jak i podczas ich współoddziaływania. Powyższa analiza udowodniła możliwość wystąpienia nowego zjawiska rezonansu nie dającego się przewidzieć metodą klasyczną.

О ДИНАМИЧЕСКОМ И МАТЕМАТИЧЕСКОМ МОДЕЛИРОВАНИИ ПРОСТРАНСТВЕННЫХ КОЛЕБАНИЙ УПРУГО МОНТИРОВАННЫХ МАШИН

Р е з ю м е

В настоящей работе, на основании предложений авторами математической модели, изучается взаимодействие между крутильными и общими колебаниями в машинных агрегатах. Эти два колебательных процесса обычно изучают независимо друг от друга. В действительности они взаимодействуют между собой. Общие колебания создают дополнительные инерционные силы, действующие на все подвижные звенья агрегата, которые приведенные к реперному звену, создают дополнительный момент, возбуждающий крутильные колебания. Они создают дополнительные силы, которые возбуждают колебания агрегата в целом.

В работе сделан сравнительный анализ результатов, полученных при раздельном изучении взаимносвязанных колебаний. Доказано также сущеуствование новых непрогнозируемых класическими методами резонансных явлений.

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