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IMPRECISE INFORMATION IN COMPUTER AIDED DECISION: NEW GENERALIZED MODUS PONENS METHODS

Conclusion. The Generalized Modus Ponens according to ZADEH is a formulation of syllogistic reasoning the theoretical bases of which are attractive; but it is not correct because it doesn't verify the most elementary validity criteria. The method we elaborate allows the correction of this defect and permits the building of a coherente formulation, which could furthermore be applied to any existing model, so that we can increase the number of tools placed at diagnosis aided systems conceivers' disposal.

List of the mathematical symbols used in the paper

- $\forall$ for all
- $\in$ belongs to
- $\triangleq$ equal by definition
- $\hat{\wedge}$ T norm
- $\neg$ complementation (for proposition)
- $(\bar{X})$ not (X)
- $\cup$ union
- $\cap$ intersection
- $>$ greater than
- $\exists$ it exists
- $\Leftrightarrow$ equivalent to

1. Representation of a rule "If X is p, then Y is q" according to ZADEH

A variable X taking its values in an universe of discourse U is entirely characterized by a possibility distribution function named π_X, and defined a:

$$\pi_X : U \longrightarrow [0\ 1]$$

$$u \longrightarrow \mu_X(u)$$

This function restricts the possible values of X.

A proposition "X is p" means that "X variable satisfies p predicate", where p is a fuzzy subset defined by a membership function μ_p such as:

$$\mu_p : U \longrightarrow [0\ 1]$$

$$u \longrightarrow \mu_p(u)$$

"X is p" can be represented by $\pi_X = \mu_p$ [ZADEH 81].

In the same way, if μ_q is q membership function, "Y is q" can be written $\pi_Y = \mu_q$.

A rule such as "If X is p, then Y is q" is interpreted as a causal link between "X is p" and "Y is q", i.e. can be defined by a conditional possibility distribution function $\pi_{A/X}$.

ZADEH designed a representation for this function based on the implicative sum [ZADEH 77], i.e. if U and V stand for the respective universes of discourses of p and q fuzzy subsets:

$$\forall u \in U,\ \forall v \in V \quad \pi_{Y/X}(u,v) = \mathrm{Min}\left[1,\ 1 - \mu_p(u) + \mu_q(v)\right] \tag{1}$$

All these definitions permit us to define a general line of reasoning combining a rule and an assertion.

2. Generalized modus ponens according to ZADEH

The Generalized Modus Ponens is an attempt to modelize the basis reasoning which takes account of a causal rule P_1.

$P_1 \triangleq$ "if X is p, then Y is q"

and of an assertion P_2 upon the X variable

$P_2 \triangleq$ "X is p'"

The aim of such a reasoning is to obtain a predicate which restricts the Y variable, such as "Y is q'". For example, if $p' = p$, i.e. in the classic Modus Ponens case, the consequent predicate has to be "Y is q".

As we have already stated it, the propositions P_1 and P_2 may be translated in terms of possibility distribution function:

P_1 by $\pi_{Y/X}$

and

P_2 by π_X

As ZADEH said, the two functions $\pi_{Y/X}$ and π_X induce on V a function π_Y defined as:

$$\forall v \in V \quad \pi_Y(v) = \pi_X(u) \circ \pi_{Y/X}(u,v) \tag{2}$$

where o stands for sup min composition.

This possibility distribution function, in turn, implicity builds a proposition $P_3 \triangleq$ "Y is q'" where q' is defined by $\mu_{q'} = \pi_Y$.

In a more general way [DUBOIS 83], π_Y function can be determined with a Sup-Tnorm composition i.e. if $\wedge$ stands for a triangular norm [MENGER 42].

$$\forall v \in V \quad \pi_Y(v) = \sup_{u \in U} \left[\pi_X(u) \wedge \pi_{Y/X}(u,v)\right] \tag{3}$$

To put it short, we obtain an inference diagram named Generalized Modus Ponens according to ZADEH, i.e.:

If $P_1 \triangleq$ "If X is p then Y is q"

and

$P_2 \triangleq$ "X is p'"

Then $P_3 \triangleq$ "Y is q'"

with

$$\forall v \in V \quad \pi_Y(v) = \sup_{u \in U} \left(\mathrm{Min}\left[\mu_X(u),\ \pi_{Y/X}(u,v)\right]\right) \tag{4}$$

i.e. by using the corresponding membership functions

$$\forall v \in V \quad \mu_{q'}(v) = \sup_{u \in U} \left(\mathrm{Min}\left[\mu_{p'}(u),\ 1 - \mu_p(u) + \mu_q(v)\right]\right) \tag{5}$$

It is worth pointing out that equation (3) is the analogue of the classical probabilistic formula

$$\forall v \in V \qquad \mathrm{Prob}_Y(v) = \sum_{u \in U} \mathrm{Prob}_{Y/X}(u,v) \;.\; \mathrm{Prob}_X(u) \tag{6}$$

The formulation of the Generalized Modus Ponens according to ZADEH is attractive because of the logical bases of its building and because of the probabilistic analogy it presents.

However, any reasoning formulation, to be a good one, has to verify the following criteria:

- C_1: If P_2 is defined as "X is p", the inference result must clearly be $P_3 \triangleq$ "Y is q". This is the classical Modus Ponens.
- C_2: Classical Modus Tollens: the piece of knowledge "Y is non q", written "Y is $\neg$q", leads to the result $P_3 \triangleq$ "X is $\neg$p".

Now the Generalized Modus Ponens according to ZADEH doesn't verify those criteria, as we show it in the following example.

Application of Generalized Modus Ponens to Modus Ponens

An expert gives a rule such as "If X is p, then Y is q" and defines p and q through their membership functions, see Fig. 1 and Fig. 2.

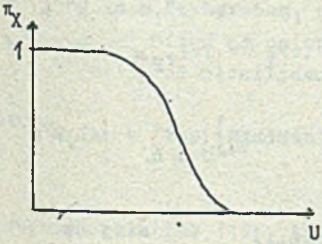

Fig. 1

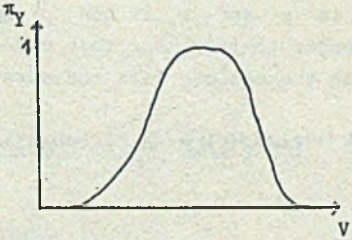

Fig. 2

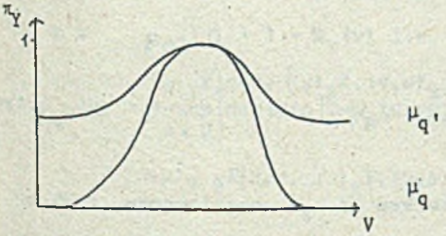

Fig. 3

If the facts base contains the piece of information "X is p", the expert immediately deduces "Y is q"; yet the calculation carried out by the Generalized Modus Ponens will engender "Y is q'" with $\mu_q \neq \mu_q$ (See Fig. 3).

Figure 3 shows the significant difference between the expected result q and the real one q'.

We notice that Generalized Modus Ponens according to ZADEH dist... knowledge introduced in each rule by the expert. In [MOREAU 8..] we shown how to obtain a good formulation from ZADEH's one. The step ... sists in looking on the causal rules such as "If X is p, then Y ... under the form If X is p and X is not non p then Y is q".

This model we called Modified Generalized Modus Ponens correspon... the following syllogistic diagram:

If $P_1 \triangleq$ "If X is p then Y is q"

and

$P_2 \triangleq$ "X is p'"

Then $P_3 \triangleq$ "Y is q'"

where Y is defined by

$$\forall v \in V \quad \pi_Y(x) = 2x \sup_{u \in U} \left[\mathrm{Min}(\pi_X(u),\ \pi_{Y/X}(u,v))\right] - 1$$

The definition of $\pi_{Y/X}$ is given by (1).

To interprete a causal rule such as "If X is p , then Y is ... under form "If X is p and X is not non p , then Y is q" suits to reality ... an expert when defining a rule between X and Y evaluates the in... of X is p and X is non p upon Y is q and Y is non q ...

In order to draw near this reality, we are going to build ... based on the analogy with the more general probabilistic formula:

$$\forall v \in V \quad \mathrm{Prob}_Y(v) = \sum_{u \in U} \left[\mathrm{Prob}_{Y/X}(u,v) . \mathrm{Prob}_X(u) + \mathrm{Prob}_{Y/X}(u,v) ... \mathrm{Prob}_{\bar{X}} ...\right.$$

3. Generalization of the generalized modus ponens

Analogically to the formula (8) we may write two possibili... which, in a total manner, allow to take into account X and ... Y and $\bar{Y}$ (non Y). [MOREAU 86 a]

$$\forall v \in V \quad \pi_Y(v) = \sup_{u \in U} \left[\mathrm{Max}(\mathrm{Min}\left[\pi_{Y/X}(u,v), \pi_X(u)\right], \mathrm{Min}\left[\pi_{Y/\bar{X}}(u, ...\right. \right.$$

$$\forall v \in V \quad \pi_{\bar{Y}}(v) = \sup_{u \in U} \left[\mathrm{Max}(\mathrm{Min}\left[\pi_{\bar{Y}/X}(u,v), \pi_X(u)\right], \mathrm{Min}\left[\pi_{\bar{Y}/\bar{X}}(u,v) ...\right. \right.$$

The different conditional possibility distributions are b... implicative sum, which, in the case of the rule:

"If X is p then Y is q"

give the following equations, $\rceil p$ being defined by $\mu_{\rceil p} = 1 - \mu_p$

$\forall u \in U \qquad \forall v \in V$

$$\pi_{Y/X}(u,v) = \text{Min}\left[1,\ 1 - \mu_p(u) + \mu_q(v)\right] \tag{11}$$

$$\pi_{Y/\bar{X}}(u,v) = \text{Min}\left[1,\ \mu_p(u) + \mu_q(v)\right] \tag{12}$$

$$\pi_{\bar{Y}/X}(u,v) = \text{Min}\left[1,\ 2 - \mu_p(u) + \mu_q(v)\right] \tag{13}$$

$$\pi_{\bar{Y}/\bar{X}}(u,v) = \text{Min}\left[1,\ 1 + \mu_p(u) - \mu_q(v)\right] \tag{14}$$

By applying formulas (9) and (10) to the usual diagram

$P_1 \triangleq$ "If X is p then Y is q"

$P_2 \triangleq$ "X is p"

we obtain two results.

* Through relation (9), the resulting assertion of the inference is $P_3 \triangleq$ "Y is q_1" where q_1 is defined by

$\forall v \in V \quad \mu_{q1}(v) = \pi_Y(v)$

$$\forall v \in V\ \mu_{q_1}(v) = \sup_{u \in U}\left[\text{Max}\left(\text{Min}\left[1 - \mu_p(u) + \mu_q(v), \mu_{p'}(u)\right], \text{Min}\left[\mu_p(u) + \mu_q(v), 1 - \mu_{p'}(u)\right]\right)\right] \tag{15}$$

* Through relation (10), this result becomes

$P_4 \triangleq$ "Y is q_2" with

$\forall v \in V \quad \mu_{\rceil q2}(v) = \pi_Y(v)$ i.e.

$\forall v \in V \quad \mu_{q2}(v) = 1 - \pi_Y(v)$ i.e.

$$\forall v \in V\ \mu_{q2}(v) = 1 - \sup_{u \in U}\left[\text{Max}\left(\text{Min}\left[2 - \mu_p(u) - \mu_q(v), \mu_{p'}(u)\right], \text{Min}\left[1 + \mu_p(u) - \mu_q(v), 1 - \mu_{p'}(u)\right]\right)\right] \tag{16}$$

In fact, proposition P_4 is equivalent to "Y is q_2"

We obtain two formulations. We may wonder if they are identical, and of course if criteria C_1 and C_2 are verified. Let's apply first to an example of classical Modus Ponens.

Example:

Let's take the previous example again. We remind that the rule "If X is p then Y is q" has been defined by the membership functions of p and q, see Fig. 4 and 5.

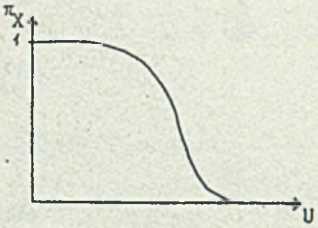

Fig. 4

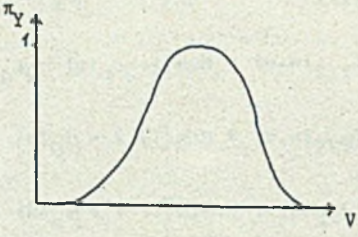

Fig. 5

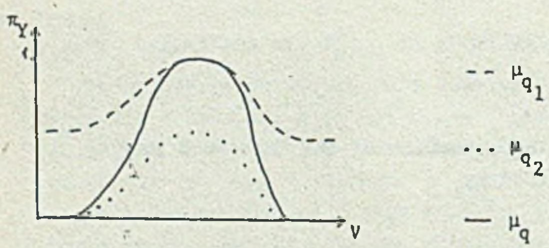

Fig. 6

The facts base contains the piece of information "X is p" and the expert deduces from it "Y is q". Yet the calculation with (15) and (16) will engender two propositions "Y is q_1" and "Y is q_2" defined figure 6.

In this example we notice in one hand that the formulations issued from (15) and (16) don't respect criterion C_1 and in an another hand that they are not identical. As we have done in [MOREAU 86b], we are going to find, from these models, two "good" formulations. To do that, we study theoretically the application of these methods to Modus Ponens.

4. Application to Modus Ponens and Modus Tollens

The syllogistic diagram of Modus Ponens encounters when assertion "X is p" meets the rule "If X is p then Y is q". In those conditions, formula (15) and (16) become respectively:

$$\forall v \in V \quad \mu_{q1}(v) = \operatorname*{Sup}_{u \in U}\left[\operatorname{Max}\left(\operatorname{Min}\left[1-\mu_p(u)+\mu_q(v), \mu_p(u)\right], \operatorname{Min}\left[\mu_p(u)+\mu_q(v), 1-\mu_p(u)\right]\right)\right] \tag{17}$$

$$\forall v \in V \quad \mu_{q2}(v) = 1-\operatorname*{Sup}_{u \in U}\left[\operatorname{Max}\left(\operatorname{Min}\left[2-\mu_p(u)-\mu_q(v), \mu_p(u)\right], \operatorname{Min}\left[1+\mu_p(u)-\mu_q(v), 1-\mu_p(u)\right]\right)\right] \tag{18}$$

We suppose that all the membership functions are continuous on $[0\ 1]$ and that at least one point of the universe of discourse exists for which the function takes the value 1 and at least one point for which the function takes the value 0. In these conditions we show that:

$$\forall v \in V \qquad \mu_{q1}(v) = \frac{1 + \mu_q(v)}{2} \tag{19}$$

$$\forall v \in V \qquad \mu_{q2}(v) = \frac{\mu_q(v)}{2} \tag{20}$$

Démonstration

We can write (17) under the form

$$\forall v \in V \quad \mu_{q1}(v) = \mathrm{Max}\left[\sup_{u \in U}(\mathrm{Min}\left[1-\mu_p(u)+\mu_q(v), \mu_p(u)\right], \sup_{u \in U}(\mathrm{Min}\left[\mu_p(u)+\mu_q(v), 1-\mu_p(u)\right])\right] \tag{21}$$

Let's have

$$A = \sup_{u \in U}(\mathrm{Min}\left[1 - \mu_p(u) + \mu_q(v), \mu_p(u)\right]) \tag{22}$$

and

$$B = \sup_{u \in U}(\mathrm{Min}\left[\mu_p(u) + \mu_q(v), 1 - \mu_p(u)\right]) \tag{23}$$

From now on we suppose that v is given.

We notice that A is the expression of Generalized Modus Ponens according to ZADEH applied to Modus Ponens. We already know that [MOREAU 86b]

$$A = \frac{1 + \mu_q(v)}{2} \tag{24}$$

Let's study the value of B. Noticing that

$$\mu_p(u) + \mu_q(v) < 1 - \mu_p(u) \Longleftrightarrow \mu_p(u) < \frac{1 - \mu_q(v)}{2}$$

we decompose U is two subsets U_1 and U_2 defined by:

$$U_1 = \left\{u \in U / \mu_p(u) < \frac{1 - \mu_q(v)}{2}\right\}$$

$$U_2 = \left\{ u \in U / \mu_p(u) > \frac{1 - \mu_q(v)}{2} \right\}$$

These definitions allow us to write

$$U = U_1 \cup U_2$$

and

$$U_1 \cap U_2 = \emptyset$$

In these conditions, formula (23) becomes

$$B = \operatorname*{Sup}_{u \in U_1 \cup U_2} \left(\operatorname{Min}\left[\mu_p(u) + \mu_q(v), 1 - \mu_p(u)\right]\right) \tag{25}$$

i.e. after decomposing

$$B = \operatorname{Max}\left[\operatorname*{Sup}_{u \in U_1} \left(\operatorname{Min}\left[\mu_p(u) + \mu_q(v), 1 - \mu_p(u)\right]\right), \operatorname*{Sup}_{u \in U_2} \left(\operatorname{Min}\left[\mu_p(u) + \mu_q(v), 1 - \mu_p(u)\right]\right)\right] \tag{26}$$

We exploite the facts:

$$u \in U_1 \Longleftrightarrow \mu_p(u) < \frac{1 - \mu_q(v)}{2} \Longleftrightarrow \mu_p(u) + \mu_q(v) < 1 - \mu_p(u)$$

$$u \in U_2 \Longleftrightarrow \mu_p(u) > \frac{1 - \mu_q(v)}{2} \Longleftrightarrow \mu_p(u) + \mu_q(v) > 1 - \mu_p(u)$$

and we obtain

$$B = \operatorname{Max}\left[\operatorname*{Sup}_{u \in U_1} \left(\mu_p(u) + \mu_q(v)\right), \operatorname*{Sup}_{u \in U_2} \left(1 - \mu_p(u)\right)\right] \tag{27}$$

Let's have

$$B_1 = \operatorname*{Sup}_{u \in U_1} \left(\mu_p(u) + \mu_q(v)\right) \tag{28}$$

and

$$B_2 = \operatorname*{Sup}_{u \in U_2} \left(1 - \mu_p(u)\right) \tag{29}$$

We use the definitions of U_1 and U_2 and the fact that v is given to deduce from (28) and (29)

$$B_1 = \sup_{u/\mu_p(u) < \frac{1-\mu_q(v)}{2}} (\mu_p(u) + \mu_q(v)) \tag{30}$$

$$B_2 = \sup_{u/\mu_p(u)) > \frac{1-\mu_q(v)}{2}} (1 - \mu_p(u)) \tag{31}$$

The membership function μ_p is continuous on $[0\ 1]$ by hypotheses, and it reaches the values 0 and 1. Therefore

$$\forall \alpha \in [0\ 1] \quad \exists u_o \in u/\mu_p(u_o) = \alpha$$

For any v it is sure that $\frac{1 - \mu_q(v)}{2}$ belongs to $[0\ 1]$.
Then an element u_1 of U exists for which

$$\mu_p(u_1) = \frac{1 - \mu_q(v)}{2} \tag{32}$$

We can assert then

$$B_1 = \sup_{u/\mu_p(u) < \mu_p(u_1)} (\mu_p(u) + \mu_q(v)) \tag{33}$$

i.e.

$$B_1 = \mu_p(u_1) + \mu_q(v) \tag{34}$$

We use (32) again

$$B_1 = \frac{1 - \mu_q(v)}{2} + \mu_q(v)$$

$$B_1 = \frac{1 + \mu_q(v)}{2} \tag{35}$$

As for B_2, we can write from (31) that

$$B_2 = 1 - \inf_{u/\mu_p(u) > \frac{1 - \mu_q(v)}{2}} (\mu_p(u)) \tag{36}$$

i.e. using (32)

$$B_2 = 1 - \operatorname*{Inf}_{u/\mu_p(u_1) < \mu_p(u)} (\mu_p(u)) \qquad (37)$$

$$B_2 = 1 - \mu_p(u_1)$$

We use again (32) and finaly obtain

$$B_2 = 1 - \frac{1 - \mu_q(v)}{2}$$

$$B_2 = \frac{1 + \mu_q(v)}{2} \qquad (38)$$

Injecting (35) and (38) in (27) permits us to calculate B since $B = \text{Max}\left[B_1, B_2\right]$

i.e. $B = \text{Max}\left[\frac{1 + \mu_q(v)}{2}, \frac{1 + \mu_q(v)}{2}\right]$

$$B = \frac{1 + \mu_q(v)}{2} \qquad (39)$$

The formulas (39) and (24) were obtained for any v of V.
So we affirm that

$$\forall v \in V \quad A = \frac{1 + \mu_q(v)}{2} \quad \text{and} \quad B = \frac{1 + \mu_q(v)}{2}$$

i.e. from (21)

$$\forall v \in V \quad \mu_{q1}(v) = \text{Max}\left[\frac{1 + \mu_q(v)}{2}, \frac{1 - \mu_q(v)}{2}\right]$$

$$\forall v \in V \quad \mu_{q1}(v) \; \frac{1 + \mu_q(v)}{2}$$

We can also write (18) under the form

$$\forall v \in V \; \mu_{q2}(v) = 1 - \text{Max}\left[\operatorname*{Sup}_{u \in U}(\text{Min}\left[2 - \mu_p(u) - \mu_q(v), \mu_p(u)\right]), \operatorname*{Sup}_{u \in U}(\text{Min}\left[1 + \mu_p(u) - \mu_q(v), 1 - \mu_p(u)\right])\right] \qquad (40)$$

Let's C and D be respectively

$$V = \operatorname*{Sup}_{u \in U} \left(\mathrm{Min}\left[2 - \mu_p(u) - \mu_q(v), \mu_p(u)\right]\right) \tag{41}$$

and

$$D = \operatorname*{Sup}_{u \in U} \left(\mathrm{Min}\left[1 + \mu_p(u) - \mu_q(v), 1 - \mu_p(u)\right]\right) \tag{42}$$

We notice in one hand that

$$2 - \mu_p(u) - \mu_q(v) < \mu_p(u) \Longleftrightarrow \mu_p(u) > 1 - \frac{\mu_q(v)}{2}$$

and in an another hand that

$$1 + \mu_p(u) - \mu_q(v) < 1 - \mu_p(u) \Longleftrightarrow \mu_p(u) < \frac{\mu_q(v)}{2}$$

and we decompose U by

$$U = \left\{u \in U / \mu_p(u) < 1 - \frac{\mu_q(v)}{2}\right\} \cup \left\{u \quad U / \mu_p(u) > 1 - \frac{\mu_q(v)}{2}\right\}$$

for the calculation of C, and by

$$U = \left\{u \in U / \mu_p(u) < \frac{\mu_q(v)}{2}\right\} \cup \left\{u \in U / \mu_p(u) > \frac{\mu_q(v)}{2}\right\}$$

for the calculation of D.

We operate in a way similar to the first demonstration and obtain

$$\forall v \in V \quad C = 1 - \frac{\mu_q(v)}{2} \tag{43}$$

$$\forall v \in V \quad D = 1 - \frac{\mu_q(v)}{2} \tag{44}$$

which immediately give us μ_{q2}

$$\forall v \in V \quad \mu_{q2}(v) = 1 - \mathrm{Max}\left[1 - \frac{\mu_q(v)}{2}, 1 - \frac{\mu_q(v)}{2}\right]$$

$$\forall v \in V \quad \mu_{q2}(v) = \frac{\mu_q(v)}{2}.$$

So we have

$$\forall v \in V \quad \mu_{q1}(v) = \frac{1 + \mu_q(v)}{2} \tag{19}$$

$$\forall v \in V \quad \mu_{q2}(v) = \frac{\mu_q(v)}{2} \tag{20}$$

These formulas can be rewritten to give

$$\forall v \in V \quad \mu_q(v) = 2\mu_{q1}(v) - 1 \tag{45}$$

$$\forall v \in V \quad \mu_q(v) = 2\mu_{q2}(v) \tag{46}$$

which indisputably shows that the two following definitions of Generalized Modus Ponens verify criterion C_1.

$$\forall v \in V \ \pi_Y(v) = 2\times\sup_{u \in U}\left[\mathrm{Max}(\mathrm{Min}[\pi_{Y/X}(u,v), \pi_X(u)], \mathrm{Min}[\pi_{Y/X}(u,v), \pi_X(u)])\right] - 1 \tag{47}$$

$$\forall v \in V \ \pi_Y(v) = 2\times(1 - \sup_{u \in U}\left[\mathrm{Max}(\mathrm{Min}[\pi_{Y/X}(u,v), \pi_X(u)], \mathrm{Min}[\pi_{Y/X}(u,v), \pi_X(u)])\right]) \tag{48}$$

All the conditional possibility distributions are defined with the implicative sum.

We have now to study the behaviour of these new definitions towards the criterion C_2. The Modus Tollens corresponds to the inference diagram based on the contrapositive symetry of inference rules i.e. the equivalence

"If X is p then Y is q" $\Longleftrightarrow$ "If Y is $\neg$q then X is $\neg$p"

and allows from propositions

"If X is p then Y is q"

and

"Y is $\neg$q"

the deduction of the assertion

"X is $\neg$p"

The Modus Tollens for a rule is nothing but the Modus Ponens for the symetric of this rule, in sense of contraposition. From general formulas (47) and (48), we transcribe the Modus Tollens by

$$\forall u \in U \quad \pi_{1x}(u) = 2 \times \mu_{p1}(u) - 1 \tag{49}$$

$$\forall u \in U \quad \pi_{2x}(u) = 2 \times \mu_{p2}(u) \tag{50}$$

where p_1 and p_2 are respectively defined by (15) and (16), i.e.

$$\forall u \in U \quad \mu_{p1}(u) = \sup_{v \in V}\left[\mathrm{Max}\left(\mathrm{Min}\left[1-\mu_{\neg q}(v)+\mu_{\neg p}(u),\ \mu_q(v)\right],\ \mathrm{Min}\left[\mu_{\neg q}(v)+\mu_{\neg p}(u),\ 1-\mu_{\neg q}(v)\right]\right)\right] \tag{51}$$

$$\forall u \in U \quad \mu_{p2}(u) = 1-\sup_{v \in V}\left[\mathrm{Max}\left(\mathrm{Min}\left[2-\mu_{\neg q}(v)-\mu_{\neg p}(u),\ \mu_{\neg q}(v)\right],\ \mathrm{Min}\left[1+\mu_{\neg q}(v)-\mu_{\neg p}(u),\ 1-\mu_{\neg q}(v)\right]\right)\right] \tag{52}$$

Now, the formulas (19) and (20) allow to write here

$$\forall u \in U \quad \mu_{p1}(u) = \frac{1+\mu_{\neg p}(u)}{2} \tag{53}$$

$$\forall u \in U \quad \mu_{p2}(u) = \frac{\mu_{\neg p}(u)}{2}\ . \tag{54}$$

which finaly give, after being injected in (49) and (50)

$$\forall u \in U \quad \pi_{1x}(u) = 2 \times \frac{1+\mu_{\neg p}(u)}{2} - 1$$

i.e.

$$\forall u \in U \quad \pi_{1x}(u) = \mu_{\neg p}(u) \tag{55}$$

and

$$\forall u \in U \quad \pi_{2x}(u) = 2 \times \frac{\mu_{\neg p}(u)}{2}$$

i.e.

$$\forall u \in U \quad \pi_{2x}(u) = \mu_{\neg p}(u) \tag{56}$$

So, the criterion C_2 is respected by the both methods.

We have built, from the Generalized Modus Ponens according to ZADEH which does not verify the criteria C_1 and C_2, two formulations which verify these Modus Ponens and Modus Tollens criteria.

5. General case

Taking into account the analogy between (2) and (6), we have deduced from (8) the formulas (9) and (10) which are the bases of the elaboration of our models.

Now, if we start from the analogy between (3) and (6), (3) generalizes (2), we obtain generalizations of (9) and (10), i.e.

$$\forall v \in V \quad \pi_Y(v) = \sup_{u \in U}\left[\operatorname{Max}(\pi_{Y/X}(u,v) \wedge \pi_X(u), \pi_{Y/X}(u,v) \wedge \pi_{\bar{X}}(u))\right] \tag{57}$$

$$\forall v \in V \quad \pi_{\bar{Y}}(v) = \sup_{u \in U}\left[\operatorname{Max}(\pi_{\bar{Y}/X}(u,v) \wedge \pi_X(u), \pi_{\bar{Y}/\bar{X}}(u,v) \wedge \pi_{\bar{X}}(u))\right] \tag{58}$$

where $\wedge$ stands for a T-norm. If $\wedge$ = Min, and if the conditional possibility distributions are defined with the implicative sum, we find (9) and (10).

The formulas (57) and (58) permit then a generalization of all the existing formulations, and can help to the building of new ones.

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NIEPRECYZYJNA INFORMACJA W DECYZJACH WSPOMAGANYCH KOMPUTEROWO:
NOWE UOGÓLNIONE METODY "MODUS PONENS"

S t r e s z c z e n i e

W artykule tym interesujemy się systemami zdolnymi zająć się przybliżoną informacją, a także kodowaniem nieprecyzyjnej wiedzy podanej w formie reguły "Jeśli... to...". Posługujemy się tu teorią ZADEHA tworząc formułę zachowującą "Modus Ponens" i "Modus Tollens".

НЕТОЧНАЯ ИНФОРМАЦИЯ ПРИ РЕШЕНИЯХ С КОМПЬЮТЕРНОЙ ПОДДЕРЖКОЙ:
НОВЫЕ ОБОБЩЁННЫЕ МЕТОДЫ "MODUS PONENS"

Р е з ю м е

В настоящей статье нас интересуют системы способные заняться приближенной информацией, а также кодирование неточных знаний, представленных в виде правила "Если... то...". Пользуемся здесь теорией ZADEHA, образуя формулу сохраняющую "Modus Ponens" и "Modus Tollens".

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