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František PALČÁK

Department of Technical Mechanics of Faculty of Mechanical Engineering of Slovak Technical University
Gottwaldovo nam. 17 CS-812 31 Bratislava

THE INFLUENCE OF ACTUAL LOCAL MOBILITY OF LINKS IN THE CLOSED ROLLING JOINT ON THE ACTUAL MOBILITY OF PLANAR MECHANISMS

Summary. Due to the application of general Freudenstein mobility criterion for mechanisms with global position coordinates of the links it is proved that the links in closed rolling joint are mutually in permanent singular position from which yields that those links have increased actual local mobility.

In the proof the method of determining a total number of global position coordinates of the links is exploited. The use of the derived property is illustrated in the process of finding actual mobility of a compound planetary train.

1. The statement of the problem

When we want to find the actual mobility n^s of the mechanism we use frequently mobility criteria (see [1]) as in Eq. (1)

$$n_K = n_V(u - 1) - \sum_{t=1}^{n_V-1} ts_t \tag{1}$$

where $n_K = n^s$ is the actual mobility of a correct mechanism n_V is the global mobility of a free link (in planar motion $n_V = 3$),

u is the number of links,

t is the class of joint connecting two links in which a local mobility n_t of a link to the adjacent one is

$$n_t = n_v - t \tag{2}$$

s_t is the total number of joints of the class t connecting two links

$$s_t = \sum_{v=2}^{v_m} s_{tv}(v - 1) \tag{3}$$

where v is the number of links in the joint of the class t, v_m is the maximum number of links in joints of the mechanism, s_{tv} is the number of joints of the class t connecting v links.

Let us denote additionally s the number of pairs connected by joints in the mechanism

$$s = \sum_{t=1}^{n_v - 1} s_t \tag{4}$$

Paul [2] showed that links in the rolling joint form so called permanent critical form which essentially removes first from the two constraints: no slip at the point of contact and fixed center distance. For this reason he has classified rolling joint into class $t = 1$. Then we obtain from Eq. (1) formally right results for the actual mobility of the mechanism.

Above interpretation of rolling joint properties is not fully correct for two reasons:

a) We assumed that all links are rigid with frictionless contact surfaces, but we know that rolling surfaces cannot move mutually without existence of contact surface with a corresponding tangential friction force, which in the statically equivalent model of rolling joint represents two unknown components, so rolling joints belong into class $t = 2$.

b) Equation (1) is valid for correct, nonsingular mechanisms, so in this form we cannot exploite it for obtaining the actuall mobility of a mechanism, although we classify the rolling joint as a joint of the class $t = 1$ or $t = 2$. For correct evaluation of actual mobility we need the mobility criterion respecting the actual properties of rolling joints.

2. Actual local mobility of links in the rolling joint

Let us consider the set of interconnected links (further SIL) RVR (revolute-rolling-revolute) from Fig. 1. The links 2, 3 with contact point C have general position in Fig. 3. The instanteneous magnitude of relative velocity of point $C \in 3$ in respect to the link 2 shall be

$$v_{C23} = \overline{P_{23}C}\,\dot{\varphi}_{23} = \overline{P_{23}C}\,(\dot{\varphi}_{13} - \dot{\varphi}_{12}) \tag{5}$$

If

$$\overline{P_{12}P_{13}} \neq 0 \tag{6}$$

then from nonslip condition

$$v_{C23} = 0 \tag{7}$$

for links 2, 3 we obtain

$$\overline{P_{23}C} = 0 \tag{8}$$

so the contact point C lies on the pole line p

$$C \equiv P_{23} \in p \tag{9}$$

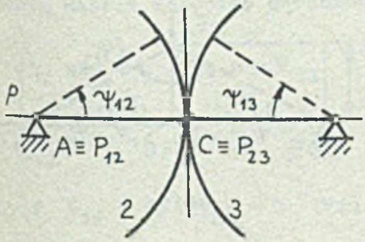

Fig. 1

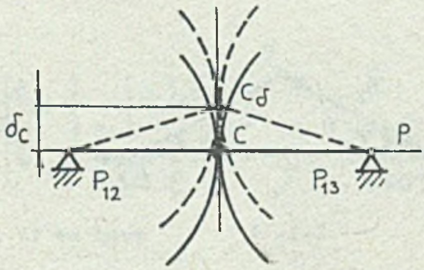

Fig. 2

After Eq. (9) links 2, 3 in SIL: RVR from Fig. 1 satisfy geometical condition for rolling because the contact point C lies on the pole line p given after Kennedy-Aronhold by instanteneous rotation centers P_{12}, P_{23}, P_{13}.

2.1. Application of general mobility criterion

Let us first compute the actual mobility of SIL: RRR from Fig. 4 and Fig. 5, too, because links 2, 3 have mutual position geometrically analogous to the links 2, 3 of SIL:RVR from Fig. 1.

We shall use general mobility criterion derived by Freudenstein [3]

$$n_F = m - h_m \tag{10}$$

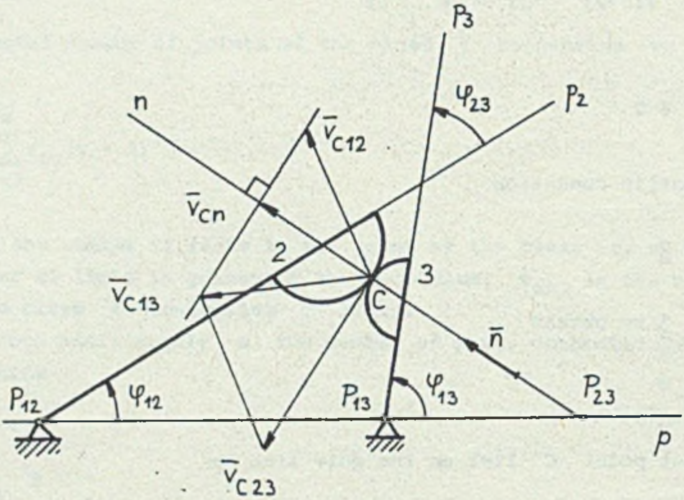

Fig. 3

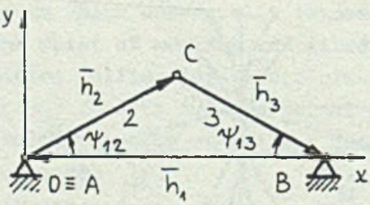

Fig. 4

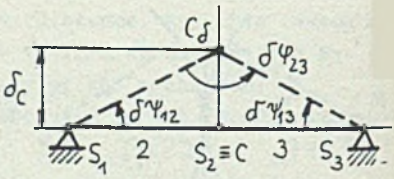

Fig. 5

where $n_F = n^s$ is the actual mobility of SIL, m is the total number of global position coordinates (variables angles or lenghts) of moving links in respect to the base frame and can be stated by Palčák [4] from equation

$$m = c - k + s_1 \tag{11}$$

where c is the total number of local position coordinates (variable angles or lenghts) of a link in the joint to the adjacent one

$$c = \sum_{t=1}^{n_v - 1} n_t s_t \tag{12}$$

k is the number of independent simple loops by Euler

$$k = s - u + 1 \tag{13}$$

t_1 is the number of joints of the class $t = 1$, h_m is the rank of rectangular Jacobian matrix J_{dm} (of order $d \times m$)

$$J_{dm} = \left[\frac{\partial f_i}{\partial \phi_j}\right] \qquad \begin{array}{l} i = 1,2,\dots,d \\ j = 1,2,\dots,m \end{array} \tag{14}$$

of explicit constraint equations

$$f_i(\phi_1,\dots,\phi_m) = 0, \quad i = 1,2,\dots,d \tag{15}$$

d is the number of dependent global position coordinates for correct SIL

$$d = 2k + s_1 \tag{16}$$

From vector loop position equation of links for SIL:RRR (Fig. 4)

$$\overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BA} = \overline{0} \tag{17}$$

we obtain matrix velocity equation

$$\begin{bmatrix} h_2 \sin\phi_{12} & h_3 \sin\phi_{13} \\ H_2 \cos\phi_{12} & -h_3 \cos\phi_{13} \end{bmatrix} \begin{bmatrix} \dot\phi_{12} \\ \dot\phi_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{18}$$

Then, if $\phi_{12} \neq 0$, $\phi_{13} \neq 0$ (Fig. 4) we have

$$\det J_{dm} \neq 0 \tag{19}$$

so SIL:RRR is in nonsingular state, next

$$h_m = 2 \tag{20}$$

and after Eq. (10) for $m = 2$ is

$$n_F = n^S = 0 \tag{21}$$

For angles $\phi_{12} = \phi_{13} = 0$ (Fig. 5) we obtain

$$\det J_{dm} = 0 \tag{22}$$

so SIL:RRR is in singular state and

$$h_m = 1 \tag{2?}$$

then after Eq. (10) for $m = 2$ is

$$n_F = n^s = 1$$

The singularity of mutual position of links 2, 3 makes revolute joint S_2 (Fig. 5) particulary passive and then there exists a virtual local angular displacement $\delta\varphi_{23}$.

2.2. The mobility analysis of SIL:RVR

The instanteneous relative position of links 2, 3 in SIL:RVR (Fig. 1) is characterized by the position of contact point C. After analogous example from Fig. 5 is SIL on Fig. 1 in instanteneous singular state and because this property is valid for all contact points of mating surfaces of links 2, 3 they are in permanent singular position. Then the contact point C can perform the virtual displacement into position $C\delta \in p$ (Fig. 2). So links 2, 3 shall be in general position after Fig. 3 in which they do not obey nonslip condition Eq. (7), then we can say that the rolling joint is kinematically equivalent to the slip joint. The permanent singularity of mutual position of links 2, 3 on Fig. 1 causes that the rolling joint is permanently particulary passive.

Let us denote n_N the number of not destroyed local freedoms of links in joints of SIL

$$n_N = \sum_{i=1}^{s} n_{Ni} \tag{25}$$

where n_{Ni}, $i = 1,2,\ldots,s$ is the number of freedoms of the link in the joint S_i, which the link will obtain due the passivity of the joint.

If we want to exploite the mobility criterion Eq. (1) then the actual mobility n^s of mechanism with rolling joints shall be

$$n^s = n_K + n_N \tag{26}$$

Actual value d^s of parameter d from Eq. (16) valid for all cases (singular or nonsingular state of SIL) can be obtained from equation

$$d^s = m - n^s \tag{27}$$

It is convenient to designate the global position coordinates

$$\phi_i, \quad i = 1,2,\dots,m \tag{28}$$

independent

$$\phi_{ni}, \quad i = 1,2,\dots,n^s \tag{29}$$

dependent

$$\phi_{zi}, \quad i = 1,2,\dots,d^s, \tag{30}$$

global variable angles ϕ_{ij} and global variable lenghts p_{ij}.

The values computed for SIL:RVR from Fig. 1

$n_v = 3$, $u = 3$, $s_{12} = 0$, $s_{22} = s_2 = s = 3$, $k = 1$, $c = 3$,

$n_K = 0$, $n_{N2} = n_N = 1$, $n^s = 1$, $m = 2$, $d^s = 1$ (Fig. 7).

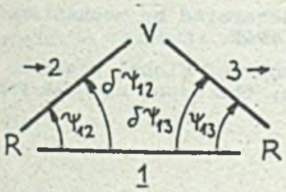

Fig. 6

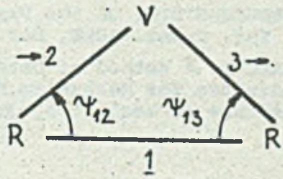

Fig. 7

The structural scheme of SIL:RVR with global position coordinates if rolling joint is in the class t = 1 (Fig. 6), resp. t = 2 (Fig. 7).

If the link 2 is the input link, then the global angular displacements of wheels 2, 3 (see Fig. 1) are $\phi_{n1} = \phi_{12}$, $\phi_{z1} = \phi_{13}$.

3. The actual mobility of compound planetary train (Fig. 8)

After Fig. 9 we have $u = 5$, $s_{22} = s_2 = s = 6$, $t = 2$, $k = 2$, $n_2 = 1$, $n_K = 0$, $n_{N1} = 0$, $n_{N2} = n_{N3} = 1$, $n_{N4} = n_{N5} = n_{N6} = 0$, $n_N = 2$, $c = 6$, $m = 4$, $d^s = 2$, $n^s = 2$. If the links 2, 4 are the input links the $\phi_{n1} = \phi_{12}$, $\phi_{n2} = \phi_{14}$, $\phi_{z1} = \phi_{13}$, $\phi_{z2} = \phi_{15}$ and link 5 may be the output link.

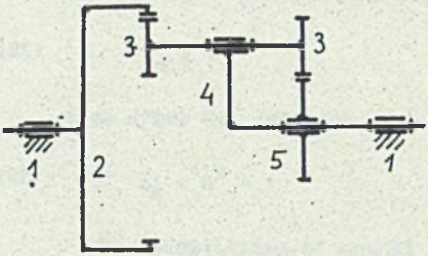

Fig. 8

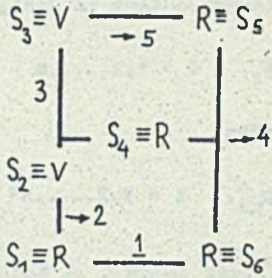

Fig. 9

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WPŁYW LOKALNEJ AKTUALNEJ MOBILNOŚCI POŁĄCZEŃ W ZAMKNIĘTYCH PRZEGUBACH NA AKTUALNĄ MOBILNOŚĆ MECHANIZMÓW PLANETARNYCH

S t r e s z c z e n i e

Dzięki zastosowaniu kryterium ogólnej mobilności Freudensteina dla mechanizmów z pozycją globalnych współczynników połączeń udowodniono, że połączenia te są wzajemne w stałej pojedynczej pozycji, z czego wynika, że zwiększyły one lokalną mobilność.

W dowodzie wykorzystana jest metoda determinowania całkowitej liczby pozycji globalnych współczynników połączeń. Użycie otrzymanej własności przedstawione jest w procesie odnajdywania aktualnej mobilności pociągu planetarnego.

ВЛИЯНИЕ ЛОКАЛЬНОЙ АКТУАЛЬНОЙ МОБИЛЬНОСТИ СОЕДИНЕНИЙ
ЗАМКНУТЫХ ШАРНИРОВ НА АКТУАЛЬНУЮ МОБИЛЬНОСТЬ
ПЛАНЕТАРНЫХ МЕХАНИЗМОВ

Р е з ю м е

Благодаря применению критерия общей мобильности Freudensteina для механизмов с общей позицией коэффициентов соединений доказано, что соединения эти взаимны в постоянной единичной позиции, из чего вытекает, что они увеличили локальную мобильность.

В доказательстве использован метод детерминирования целого числа позиции глобальных коэффициентов соединений. Использование полученной характеристики представлено в процессе поиска актуальной мобильности планетарного поезда.

Recenzent: Dr inż. Andrzej Nowak

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