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DYNAMIC MODEL FOR THE SELECTION OF SERVOMOTORS IN SERIAL - PARALLEL MANIPULATOR

Summary. This paper presents the algorithm of dynamic analysis of manipulator with an arm of serial-parallel structure. A computer program performing such a task, assuring an easy and efficient use of calculation results for a driving system design is described.

1. Introduction

The interest in the parallel and serial-parallel manipulators results from the attempts to improve the dynamic properties and precision of industrial robots. Parallel manipulators are closed kinematic chains with one or more closed loops, where only some pairs are actuated. Compared with serial manipulators, which are indeed open kinematic chains, with all pairs actuated, they have some advantages like a stiffer mechanical structure and more precise positioning, while disadvantages are limited working space and reduced manoeuvrability of the wrist.

Lately the attempts have been made to construct a manipulator of parallel structure with an arm of three degrees of freedom and a wrist driven separately

An example of such manipulator with six degree of freedom and electric drive is described in [1]. The construction of an original arm of serial-parallel structure made it possible to achieve a bigger stiffness and payload capability than in typical serial manipulators with similar kinematic properties. The paper [1] consists of the description of the construction and the results of the first tests of the prototype as well as the algorithm of the kinematic analysis.

It is the purpose of this paper to present the algorithm of dynamic analysis of afore-mentioned manipulator and application of dynamic model for the selection of servomotors.

2. Structure of the manipulator

A general view of the manipulator prototype is shown in Fig. 1a. Fig. 1b presents its simplified kinematic diagram. Active (i.e. actuated) kinematic

1992

pairs are well-marked. Links 1+6 create the main part of its construction. Whith that, a rotatory column with a skew bracket 1, double joint 2 and the outrigger 3 are included in the arm. Links 4+6 constitute a spherical wrist.

The arm is driven by three electric actuators with the ball screw-nut



Fig. 1. Manipulator of serial-parallel structure: a) prototype, b) scheme

mechanism. One of them, designated M1 in Fig. 1a , using the lever mechanism, placed at the base designated 0, rotates the column. Two others, namely M2 and M3, form, with connecting them shafts, parallel drive system taking the shape of a triangle. The vertex of this triangle is articulated with the outrigger 3 by spherical joint designated P. Wrist is fixed at one of the ends of the outrigger. At the opposite end, three electric motors, i.e. M4, M5 and M6, are situated, producing wrist motion.

Drive transmission of the wrist includes two boxes of gear interconnected by parallel shafts placed inside the outrigger. First is mounted nearby the motors, second one, including differentials, is placed close to the wrist.

3. The inverse dynamics problem of manipulator

This problem consists in determination of moments developed by motors during execution of specified trajectory. A rigid body model is proposed, taking account of friction in driving gears, ignoring friction in joints of manipulator. For each link, a local body-fixed frame, orthogonal and dextral, is defined. In the figure 1b. these systems are represented by the axes x and z.

For links 1+6 local coordinate frames are choosen according to Denavit-hartenberg convention [2].

It is assumed that position r_i of the origin of link-i coordinate system and its orientation in an absolute reference frame $x_0 y_0 z_0$ is explicitly given at each moment as well as vectors \mathbf{r}_i , \mathbf{r}_i and \mathbf{w}_i , \mathbf{w}_i denoting linear and angular velocities and accelerations. The detailed algorithms of kinematic analysis of the manipulator for calculation of this quantities for given trajectory have been presented in [1].

Links 4+6 of the spherical wrist of the manipulator creates an open kinematic chain. Problem of inverse dynamics of this stucture is generally solved by means of a Newton-Euler recurrent formulation in such forms [8]:

$$\mathbf{F}_{i} = \mathbf{m}_{i} \mathbf{p}_{i} , \qquad (1)$$

$$\mathbf{N}_{i} = \mathbf{I}\mathbf{w}_{i}^{+} \mathbf{w}_{i} \times (\mathbf{I}\mathbf{w}_{i}) , \qquad (2)$$

where:

F. - total external force exerted on link i,

N - total external moment exerted on link i,

m, - mass of link i,

I - inertia tensor of link i,

$$\mathbf{p}_{i} = \mathbf{r}_{0} + \mathbf{r}_{i} + \mathbf{w}_{i} \times \mathbf{s}_{i} + \mathbf{w}_{i} \times (\mathbf{w}_{i} \times \mathbf{s}_{i}), \qquad (3)$$

where: s = -position of link i gravity center with respect to the frame 1.

All vectors and tensors are refered to the absolute frame.

Following recurrent equations can be used to calculate forces and moments exerted between separate links of open kinematic chain, begining from the last one:

$$f_{1} = F_{1} + f_{1+1}, \qquad (4)$$

$$\mathbf{n}_{i} = \mathbf{N}_{i} + \mathbf{n}_{i+1} + (\mathbf{r}_{i} + \mathbf{s}_{i} - \mathbf{r}_{i-1}) \times \mathbf{F}_{i} + (\mathbf{r}_{i} - \mathbf{r}_{i-1}) \times \mathbf{f}_{i+1}$$
(5)

where: f_i and n_i are force and moment exerted on link i by its antecedent, link (i-1). In case of last link (i=n) vectors f_{n+1} and n_{n+1} result from the payload of end effector.

The value of driving moment actuating link i, articulated with link i-1 by rotational joint can be obtained as follows:

$$M_{i} = z_{i-1} \cdot n_{i}$$
 (6)

where z_{i-1} is the unit vector of the axis of rotation.

According to the procedure presented by equations (1)+(6) one can evaluate moments driving links 4, 5 and 6 of the wrist.

In order to find forces developed by linear actuators of parallel drive system, determination of reaction in spherical joint P, connecting this assembly with the outrigger 3, is required at first. Three components of this force designated $f_{\rm c}$ can be evaluated using three independent equations since the

friction in spherical pair is ignored. Taking advantages of the fact that revolute joints rotating about axes z_2 , z_1 and z_{11} are passive, afore-mentioned equations can be formulated using expression (6) with vanishing left side of it (M = C).

Forces acting upon link 3 are f_3 , f_4 and yet unknown force f_p . Therefore force and moment in joint rotating about z_3 axis are expressed by relations:

n

$$f_{3} = F_{3} + f_{4} + f_{p}$$
, (7)

$$= N_{3} + n_{4} + (r_{3} + s_{3} - r_{2}) \times F_{3} + (r_{p} - r_{2}) \times f_{p}$$
(8)

The inertial force of gyro coming from motion of electric drives rotors is considered during determination of N vector.

On the right side of equation (S) force \mathbf{f}_p is still unknown. Thus this equation gives relation $\mathbf{n}_3(\mathbf{f}_p)$. Moment in the joint rotating about axis \mathbf{z}_1 can be obtained from equation (S) for i=2. We substitute into it expression (7) for \mathbf{f}_3 . This yields to relation $\mathbf{n}_2(\mathbf{f}_p)$. Triangle, formed by two actuators of parallel drive assembly, can be considered as one link designated 12, connected with link 1 by rotational joint having motion axis \mathbf{z}_{11} . Taking into account that upon link 12 in P force $-\mathbf{f}_p$ is acting, total force \mathbf{F}_{12} , equilibring all inertia forces of the assembly, can be obtained as well as total moment \mathbf{N}_{12} . Subsequently, one can evaluate the moment in joint rotating about \mathbf{z}_{11} axis, obtaining relation $\mathbf{n}_{12}(\mathbf{f}_p)$.

Considering that $\mathbf{z}_{i-1} \cdot \mathbf{n} = 0$ for i = 2,3 and 12, it is possible to formulate three scalar equations for three components of force \mathbf{f}_p .

Two actuators assembly of parallel drive forms planar closed loop actuated at P by force \mathbf{f}_p applied to link 8. Kinetostatic analysis of this chain can be carried out in analogical way as it was last-made, by virtually cutting it in joint connecting links 7 and 8 and putting into equations unknown force \mathbf{f}_g and moment \mathbf{n}_g , components of which can be calculated from six independent equations. Analysis is obstructed by the fact that considered chain is hyperstatic. However, since its aim is the determination of driving moments of actuators motors, analysis can be simplified. It was find after experiments that due to provided preloads, friction at actuators rod sliders is independent of forces perpendicular to rod axes. In this context driving moment of each actuator depends only on axial component of force Q_i

of this components are:

$$Q_{\mathbf{g}} = (-\mathbf{f}_{\mathbf{p}} + \mathbf{f}_{\mathbf{g}}) \cdot \mathbf{x}_{\mathbf{g}} = (-\mathbf{f}_{\mathbf{p}}' + \mathbf{f}_{\mathbf{g}}') \cdot \mathbf{x}_{\mathbf{g}}$$
(9)

$$\mathbf{Q}_{\mathbf{7}} = -\mathbf{f}_{\mathbf{8}} \cdot \mathbf{x}_{\mathbf{7}} = -\mathbf{f}_{\mathbf{8}}^{\prime} \cdot \mathbf{x}_{\mathbf{7}}$$
(10)

where:

x, i x - unit vectors of rods axis,

 f_p^* i f_g^* - projections of forces f_p^* and f_g^* on x_7, x_8^* plane performing following relation:

$$\mathbf{f}_{1}' = \mathbf{f}_{1} - (\mathbf{f}_{1} \cdot \mathbf{z}_{7})\mathbf{z}_{7} \tag{11}$$

Both unit vectors \mathbf{x}_{7} and \mathbf{x}_{8} are perpendicular to \mathbf{z}_{7} , so:

$$(\mathbf{f}_{1}, \mathbf{z}_{7})\mathbf{z}_{7}, \mathbf{x}_{1} = 0$$
 (1 = 7.8) (12)

and it is possible to substitute into formulae (9) and (10) \mathbf{f}_i for \mathbf{f}'_i . It means that for evaluation of forces \mathbf{Q}_i and \mathbf{Q}_i determination of all components of force \mathbf{f}_i and moment \mathbf{n}_i is not necessary. Calculation of force \mathbf{f}'_i is sufficient.

It can be shown in similar way, that vector f_n^* is determined by expressions

 $M = n \cdot z$ and $M = n \cdot z$, where n and n denote moments transmitted by joints A and B having rotational axes z, and z,

Two formulae for two components of force f can be obtained regarding that $M_{g}=0$ and $M_{g}=0$ since joints A and B are passive. Third expression is $f'_{g} \cdot z_{g} = 0$. Relations $n_{A}(f'_{B})$ and $n_{B}(f'_{B})$ can be formulated in analogical way as in the case of $n_2(f_p)$, considering actuator as one segment. After determination of force f'_2 , it is possible to calculate axial forces exerted on actuators rods following formulae (9) and (10).

The identical actuator was applied to produce rotatory column motion by the use of lever mechanism presented in figure 2. Determination of driving force developed by this actuator is made in two steps. First, by application of dynamic model of manipulator, moment $M_1 = n_1 \cdot z_0$ which should supply rotatory column to perform specified motion must be calculated. All moments and forces

acting on manipulator are taken into account while determination of moment n. Second step consists in kinetostatic analysis of driving lever mechanism, assuming that on its last link moment -M z is exerted. Analysis is carried out

using algorithms presented previously for spatial closed loops despite the fact that the mentioned mechanism is planar. That way, obtained forces in joints, perpandicular to the plane formed by four bar mechanism, may be useful for resistance analysis.

Determination of force Q_{\star} developed by actuator finishes examined problem.

4. Determination of motors driving moments considering friction

At every joint of electricaly driven actuators with a ball screw-nut mechanism, backlash have been eliminated by the preload. Regarding this it was assumed that the value of moment needed to cover the friction forces in every kinematic pair of actuator is independent of the load but depends on the direction of developed speed.

Driving moment M producing the motion of rod with the speed q and acceleration q, loaded by axial force Q, can be find as follows:

$$M = J k^{-1} q + k Q + M$$
(13)

where:

J - moment of inertia of both, rotor and screw, k - gear reduction rate $[m \cdot rad^{-1}],$

M - equivalent friction torque determined by relations:

$$M_{t} = M_{t0} \operatorname{sign} q \quad \text{for } q \neq 0 ,$$

$$M_{t} = M_{t0} \operatorname{sign} q \quad \text{for } q = 0 \quad i \quad q \neq 0$$

$$M_{t} \in \langle -M_{t0}, M_{t0} \rangle \quad \text{for } q = 0 \quad i \quad q = 0.$$

$$(14)$$

Method used to calculate M, which is the absolute value of equivalent, limiting friction torque, is outlined in [3]. Using equations (13) and (14) driving moments of electric motors causing actuators motion can be evaluated. Wrist actuating motors moments are obtained from matrix equation:

$$M_{=} J K^{-1} q + K^{1} Q + M$$
(15)

where:

 $M_m = [M_{m4}, M_{m5}, M_{m6}]^T$ - vector of driving moments of electric motors,

 $\mathbf{q} = \left[\Theta_{\mathbf{a}}, \Theta_{\mathbf{b}}, \Theta_{\mathbf{b}}\right]^{\mathrm{T}}$ - vector of joint angles of the wrist,

- J 3x3 diagonal inertia matrix of motor rotors and gear driving shafts,
- $Q = [M_4, M_5, M_6]^T$ vector of output moments, directly actuating wrist links, determined from equations (1)+(6),
- M 3x1 vector of equivalent friction torques acting on driving shafts

K - 3x3 matrix of gear reduction rate making good equation: $q \ = \ K \ q \ , \label{eq:q}$

$$= \mathbf{K} \mathbf{q}_{\mathbf{m}}, \tag{16}$$

where:

 $\mathbf{q}_{\mathbf{m}} = [\mathbf{\theta}_{\mathbf{m}4}, \mathbf{\theta}_{\mathbf{m}5}, \mathbf{\theta}_{\mathbf{m}5}]^{\mathrm{T}}$ - vector of driving motors rotor angles.

In equation (15) inertia of gear wheels and friction forces in differential gears are neglected.

5. Application examples

The algorithm presented above for solving the inverse dynamics problem of manipulator was used in RNT2 program written in Pascal language. It can also solve direct and inverse kinematics task. Dynamic analysis of manipulator performed with the aid of this program is one of the most foundamental steps in a driving motors (designated M5 in Fig. 1a) are shown in form of diagrams in Fig. 2. Diagrams present torque-speed relations during rectilinear translation of the gripper, on the same distance, with trapesoidal (Fig. 2a) and triangular (Fig. 2b) velocity profile, with identical constant acceleration (deceleration) Numbers given on the curves denote time passed from the start. Comparing such diagrams, determined for standard manipulator tasks, with motors torque-speed characteristics, one can easly estimate if proposed motors are correctly solveloads.



Fig. 2. Torque-speed diagrams for M5 motor corresponding to typical motion of manipulator with different velocity profile: a) trapezoidal b) triangular

By the use of RNT2 program and with the aid of CSSP simulation program, simulation investigations are performed. The aim of tests is elementary estimation of different control systems. In this case RNT2 program is used for calculation of generalized accelerations vector by means of an Orin-Walker method [8] consisting in solving inverse dynamics task for determination of several column of manipulator inertia matrix.

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DYNAMISCHE ANALYSE IN DER PROJEKTIERUNG VON MANIPULATORANTRIEBEN MIT REICHEN/

/PARALLELSTRUCTUR

Zusammenfassung

In der Arbeit wird ein Algorithmus zur dynamischen Analyse eines neuen Manipulatortyps mit Reihen/Parallelstructur vorgestellt. Außerdem wird ein Computerprogramm beschrieben, das in der Lage ist, diese Analyse durchzufüren und die unproblematische Anwendung ihrer Ergebnisse während der Projektierung des Antriebssystems garantiert. ANALIZA DYNAMICZNA W PROJEKTOWANIU NAPĘDU MANIPULATORA O STRUKTURZE SZEREGOWO--RÓWNOLEGŁEJ

Streszczenie

W pracy przedstawiono algorytm analizy dynamicznej nowego typu manipulatora z ramieniem o strukturze szeregowo-równoległej. Opisano program komputerowy dokonujący takiej analizy i zapewniający łatwe wykorzystanie jej wyników podczas projektowania układu napędowego.

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