# International Conference on <br> COMPUTER INTEGRATED MANUFACTURING 

# Internationale Kr aferenz über <br> RECHNERINTEGRIERTE FERTIGUNGSSYSTEME 

Zakopane, March 24-27 1992

Krzysztof MIANOWSKI, Kazimierz NAZARCZUK, Tomasz SLOMKOWSKI

The Institute of Aeronautics and Applied Mechanics
Warsaw University of Technology, Warsaw, Poland

DYNAMIC MODEL FOR THE SELECTION OF SERVOMOTORS IN SERIAL - PARALLEL MANIPULATOR

Summary. This paper presents the algorithm of dynamic analysis of manipulator with an arm of serial-parallel structure. A computer program performing such a task, assuring an easy and efficient use of calculation results for a driving system design is described.

1. Introduction

The interest in the parallel and serial-parallel manipulators results from the attempts to improve the dynamic properties and precision of industrial robots. Parallel manipulators are closed kinematic chains with one or more closed loops, where only some pairs are actuated. Compared with serial manipulators, which are indeed open kinematic chains, with all pairs actuated, they have some advantages like a stiffer mechanical structure and more precise positioning, while disadvantages are limited working space and reduced manoeuvrability of the wrist.

Lately the attempts have been madt to construct a manipulator of parallel structure with an arm of three degrees of freedom and a wrist driven separately

An example of such manipulator with six degree of freedom and electric drive is described in \{1]. The construction of an original arm of serial-parallel structure made it possible to achieve a bigger stlffness and payload capabllity than in typical serlal manipulators with similar kinematic properties. The paper [1] consists of the description of the construction and the results of the first tests of the prototype as well as the algorithm of the kinematic analysis.

It is the purpose of this paper to present the algorithm of dynamic analysis of afore-mentioned manipulator and application of dynamic nodel for the selection of servomotors.

## 2. Structure of the manipulator

A general view of the manipulator prototype is shown in Fig. ia. Fig. ib presents its simplified kinematic diagram. Active (i.e. actuated) kinematic
pairs are well-marked. Links $1 \div 6$ create the main part of its construction. Whith that, a rotatory column with a skew bracket 1 , double joint 2 and the outrigger 3 are included in the arm. Links $4 \div 6$ constitute a spherical wrist.

The arm is driven by three electric actuators with the ball screw-nut

b)


Fig. 1. Manlpulator of serial-parallel structure: a) prototype, b) scheme
mechanism. One of them, designated Mi in Fig. la , using the lever mechanism, placed at the base designated 0 , rotates the column. Two others, namely $M 2$ and M3, form, with connecting them shafts, parallel drive system taking the shape of a triangle. The vertex of this triangle is articulated with the outrigger 3 by spherical Joint designated $P$. Wrist is fixed at one of the ends of the outrigger. At the opposite end, three electric motors, i.e. M4, MS and MG, are situated, producing wrist motion.

Drive transmission of the wrist includes two boxes of gear interconnected by parallel shafts placed inside the outrigger. First is mounted nearby the motors, second one, including differentials, is placed close to the wrist.

## 3 The inverse dynarics problem of manipulator

This probiem consists in determination of moments developed by motars during execution of specified trajectory. A rigid body model is proposed, taking account of friction in driving gears, ignoring friction in joints of manipulator. For each link, a local body-fixed frame, orthogonal and dextral, is defined. In the figure $1 b$. these systems are represented by the axes $x_{1}$ and $z_{1}$. For llnks $1+6$ local coordinate frames are choosen according to Lenavit-fiarienbers convention. [2].

It is assumed that position $r_{\text {, of }}$ of origin of $11 n k-1$ coordinate system and its orientation in an absolute reference frame $x_{0} y_{0} z_{0}$ is explicitly given at
each moment as well as vectors $\dot{r}_{1}, \ddot{r}_{1}$ and $w_{1}, \dot{w}_{1}$ denoting inear and angular velocities and accelerations. The detailed algorithms of kinematic analysis of the manipulator for calculation of this quantities for given trajectory have been presented in [1].

Links $4+6$ of the spherical wrist of the manipulator creates an open kinematic chain. Problem of inverse dynamics of this stucture is generally solved by means of a Newton-Euler recurent formulation in such forms [8]:

$$
\begin{gather*}
F_{1}=m_{1} P_{i}  \tag{1}\\
w_{1}=I w_{2}+w_{1} \times\left(I w_{1}\right) \tag{2}
\end{gather*}
$$

where:
$F_{1}$ - total external force exerted on link 1 ,
$N_{3}$ - total external moment exerted on 1 ink 1 ,
$m_{1}$ - mass of link 1 ,
$I_{1}$ - inertia tensor of link 1 ,
$p_{1}$ - acceleration of link 1 gravity center, including acceleration due to gravity $\ddot{r}_{0}$, determined by the formula:

$$
\begin{equation*}
p_{1}=\ddot{r}_{0}+\ddot{r}_{1}+\ddot{w}_{1} \times s_{1}+w_{1} \times\left(w_{1} \times s_{1}\right) \tag{3}
\end{equation*}
$$

where: s - position of link $i$ gravity center with respect to the All vectors and tensors are refered to the absolute frame.

Following recurent equations can be used to calculate forces and moments exerted between separate links of open kinematic chain, begining from the last one:

$$
\begin{gather*}
f_{1}=F_{1}+f_{1+1}  \tag{4}\\
n_{i}=N_{1}+n_{i+1}+\left(r_{1}+s_{1}-r_{1-1}\right) \times F_{1}+\left(r_{1}-r_{1-1}\right) \times f_{1+1} \tag{5}
\end{gather*}
$$

where: $f_{1}$ and $n_{1}$ are force and moment exerted on link 1 by its antecedent, link ( $i-1$ ). In case of last link ( $i=n$ ) vectors $f_{n+1}$ and $n_{n+1}$ result fror the payload of end effector.

The value of driving moment actuating link 1 , articulated with link $1-1$ by rotational joint can be obtained as follows:

$$
\begin{equation*}
M_{1}=z_{1-1} \cdot n_{1} \tag{6}
\end{equation*}
$$

where $z_{i-1}$ is the unit vector of the axis of rotation.
According to the procedure presented by equations (1)+(6) one can evaluate moments ariving links 4,5 and 6 of the wrist.

In order to find forces develcped oy linear actuators of parallel drive systen, determinaion of reaction in spherlcal joint $P$, connecting thls assembly with the outrigger 3 , is required at first. Three components of this force desigriated $f_{p}$ can be evaluated usine three independent equations since the friction ir spherical palr is Enorea. Taking advartajes of the fact that revolute joints rotating about axes $z_{3}, z_{i}$ and $z_{11}$ are passive afore-mentioned equations can be formulated using expression (6) with venishing left side of it $\left\{\mathrm{M}_{\mathrm{i}}=\mathrm{Cl}\right.$.

Forces acting upon link 3 are $f_{3}, f_{4}$ and yet unknown force $f_{p}$ Therefore force and moment in joint rotating about $z_{2}$ axis are expressed by relations:

$$
\begin{gather*}
f_{3}=F_{3}+f_{4}+f_{p}  \tag{7}\\
n_{3}=N_{3}+n_{4}+\left(r_{3}+s_{3}-r_{2}\right) \times F_{3}+\left(r_{p}-r_{2}\right) \times f_{p} \tag{8}
\end{gather*}
$$

The inertial force of gyro coming from motion of electric drives rotors is considered during determination of $N_{3}$ vector.

On the right side of equation (8) force $f_{p}$ is still unknown. Thus this equation gives relation $n_{3}\left(f_{p}\right)$. Moment in the joint rotating about axis $z_{i}$ can be obtained from equation (5) for $1=2$. We substitute into it expression (7) for $f_{3}$. This yields to relation $n_{2}\left(f_{p}\right)$. Triangle, formed by two actuators of parallel drive assembly, can be considered as one link designated 12, connected with link 1 by rotational joint having motion axis $z_{11}$. Taking into account that upon innk 12 in $P$ force $-f_{p}$ is acting, total force $F_{12}$, equilibring all inertia forces of the assembly, can be obtained as well as total moment $\mathbf{N}_{12}$. Subsequently, one can evaluate the moment in joint rotating about $z_{11}$ axis, obtaining relation $n_{12}\left(f_{p}\right)$.

Considering that $z_{1-1} \cdot \mathbf{n}_{1}=0$ for $1=2,3$ and 12 , it is possible to formulate three scalar equations for three components of force $\mathbf{f}_{\mathbf{p}}$.

Two actuators assembly of parallel drive forms planar closed loop actuated at $P$ by force $f_{p}$ applied to link 8 . Kinetostatic analysis of this chain can be carried out in analogical way as it was last-made, by virtualiy cutting it in joint connecting links 7 and 8 and putting into equations unknown force $f_{8}$ and moment $n_{B}$, components of which can be calculated from six independent equations. Analysis is obstructed by the fact that considered chain is hyperstatic. However, since its alm is the determination of driving moments of actuators motors, analysis can be simplified. It was find after experiments that due to provided preloads, friction at actuators rod sliders is independent of forces perpendicular to rod axes. In this context driving moment of each actuator depends only on axial component of force $Q_{i}$ exerted on rod at its end $P$. Values of this components are:

$$
\begin{gather*}
Q_{8}=\left(-f_{P}+f_{8}\right) \cdot x_{8}=\left(-f_{P}^{\prime}+f_{8}^{\prime}\right) \cdot x_{8}  \tag{9}\\
Q_{7}=-f_{8} \cdot x_{7}=-f_{8}^{\prime} \cdot x_{7} \tag{10}
\end{gather*}
$$

where

$$
\begin{aligned}
& \mathbf{x}_{7} \text { i } x_{8}-\text { unit vectors of rods axis, } \\
& f_{p}^{\prime} f_{8}^{\prime}- \\
& \text { projections of forces } f_{p} \text { and } f_{B} \text { on } x_{7}, x_{B} \text { plane performing }
\end{aligned}
$$

$$
\begin{equation*}
f_{1}^{\prime}=f_{1}-\left(f_{1} \cdot z_{7}\right) z_{7} \tag{11}
\end{equation*}
$$

Both unit vectors $x_{7}$ and $x_{8}$ are perpendicular to $z_{7}$, so:

$$
\begin{equation*}
\left(f_{1} \cdot z_{7}\right) z_{7} \cdot x_{1}=0 \quad(1=7.8) \tag{12}
\end{equation*}
$$

and it is possible to substitute into formulae (9) and (10) $f_{i}$ for $f_{i}^{\prime}$. It means that for evaluation of forces $Q_{7}$ and $Q_{B}$ determination of all components of force $f_{8}$ and moment $n_{8}$ is not necessary. Calculation of force $f_{8}^{\prime}$ is sufficient.

It can be shown in similar way, that vector $f_{8}^{\prime}$ is determined by expressions
$M_{A}=n_{A} \cdot z_{A}$ and $M_{B}=n_{B} \cdot z_{B}$ where $n_{A}$ and $n_{B}$ denote moments transmitted by joints $A$ and $B$ having rotational axes $z_{A}$ and $z_{B}$.

Two formulae for two components of force $f_{B}^{\prime}$ can be obtained regarding that $M_{A}=0$ and $M_{B}=0$ since joints $A$ and $B$ are passive. Third expression is $f_{B}^{\prime} z_{7}=0$. Relations $n_{A}\left(f_{8}^{\prime}\right)$ and $n_{B}\left(f_{B}^{\prime}\right)$ can be formulated in analogical way as in the case of $n_{3}\left(f_{P}\right)$, considering actuator as one segment After determination of force $f_{B}^{\prime}$, it is possible to calculate axial forces exerted on actuators rods following formulae (9) and (10).

The identical actuator was applied to produce rotatory column motion by the use of lever mechanism presented in figure 2. Determination of driving force developed by thls actuator is made in two steps. First, by application of dynamic model of manipulator, moment $M_{1}=n_{1} \cdot z_{0}$ which should. supply rotatory column to perform specified motion must be calculated. All moments and forces acting on manipulator are taken into account while determination of moment $n_{1}$.
Second step consists in kinetostatic analysis of driving lever mechanism, assuming that on its last link moment $-M_{1} z_{0}$ is exerted. Analysis is carried out using algorithms presented previously for spatial closed loops despite the fact that the mentioned mechanism is planar. That way, obtained forces in joints, perpandicular to the plane formed by four bar mechanism, may be useful for resistance analysis.

Determination of force $Q_{1}$ developed by actuator finishes examined problem.
4. Determination of motors driving moments considering friction

At every jolnt of electricaly driven actuators with a ball screw-nut mechanism, backlash have been eliminated by the preload. Regarding this it was assumed that the value of moment needed to cover the friction forces in every kinematic pair of actuator is independent of the load but depends on the direction of developed speed.

Driving moment $M_{m}$ producing the motion of rod with the speed $\dot{q}$ and acceleration $\ddot{q}$, loaded by axial force $Q$, can be find as follows:

$$
\begin{equation*}
M_{m}=J k^{-1} \ddot{q}+k Q+M_{t} \tag{13}
\end{equation*}
$$

where:
J - moment of inertia of both, rotor and screw,
k - gear reduction rate $\left[\mathrm{m} \cdot \mathrm{rad}^{-1}\right]$,
$M$ - equivalent friction torque determined by relations:

$$
\left.\begin{array}{l}
M_{t}=M_{t 0} \operatorname{sign} \dot{q} \text { for } \dot{q}=0,  \tag{14}\\
M_{t}=M_{t 0} \operatorname{sign} \ddot{q} \text { for } \dot{q}=0 \text { i } \ddot{q} \neq 0 \\
M_{t} \in\left\langle M_{t 0}, M_{t 0}\right\rangle \text { for } \dot{q}=0 \text { i } \ddot{q}=0 .
\end{array}\right\}
$$

Method used to calculate $M_{t o}$ which is the absolute value of equivalent, imiting friction torque, is outlined in [3]. Using equations (13) and il4) driving moments of electric motors causing actuators motion can be evaluated.

Wrist actuating motors moments are obcained from matrix equation:

$$
\begin{equation*}
M_{m}=J K^{-1} \ddot{q}+K^{T} Q+M_{i} \tag{15}
\end{equation*}
$$

where:
$M_{m}=\left[M_{m A}, M_{m S}, M_{M G}\right]^{T}$ - vector of driving moments of electric motors,
$q=\left[\theta_{4}, \theta_{5}, \theta_{6}\right]^{T}$ - vector of joint angles of the wrist,
J - $3 \times 3$ diagonal inertia matrix of motor rotors and gear driving shafts,
$Q=\left[M_{4}, M_{5}, M_{6}\right]^{T}$ - vector of output moments, directly actuating wrist
IInks, determined from equations (1)+(6),
$M_{t}-3 \times 1$ vector of equivalent friction torques acting on driving shafts
K - 3x3 matrix of gear reduction rate making good equation:

$$
\begin{equation*}
\mathbf{q}=\mathbf{R} q_{\mathbf{m}} . \tag{16}
\end{equation*}
$$

where:
$q_{m}=\left\{\theta_{m 4}, \theta_{m 5}, \theta_{m 6}\right\}^{T}$ - vector of driving motors rotor angles.
In equation (15) inertia of gear wheels anc friction forces in differential gears are neglected.

## 5. Application examples

The algorithm presented above for solving the inverse dynamics problem of manipulator was used in RNT2 program written in Pascal language. It can also solve direct and inverse kinematics task. Dynamic analysis of manipulator performed with the ald of this program is one of the most foundamental steps in a driving system design. Examples of calculation results for one of the wrist driving motors (designated $M$ in Fig. 1a) are shown in form of diagrams in Fig. 2. Diagrams present torque-speed relations during rectilinear translation of the gripper: on the same distance, with trapesoidal (Fig. 2a) and triangular (Fig. 2b) velocity profile, with identical constant acceleration (deceleration) Numbers given on the curves denote time passed from the start. Comparing such diagrams, determined for standard manipulator tasks: with motors torque-speed characteristics, one can easly estimate if proposed motors are correctly selected. It is also possible to verify requirements concerning short time overloads.


Fig. 2. Torque-speed diagrams for MS motor corresponding to typical motion of mailpulator with different velocity profile:
a) trapezoidal b) triangular

By the use of RNT2 program and with the aid of CSSP simulation program, simulation investigations are performed. The alm of tests is elementary estimation of different control systems. In this case RNT2 program is used for calculation of generalized accelerations vector by means of an Orin-Walker method [8] consisting in solving inverse dynamics task for determination of several column of manipulator inertia matrix.

## REFERENCES

[1] Bidzinski, J., Mianowski, K., Nazarczuk, K., Słomkowski, T.: A Manipulator with an Arm of Serial-Parallel Structure.
To appear in The Archive of Mechanical Engineering, Harsaw 1991.
[2] Denavit, J., Hartenberg, R.S.: A Kinematic Notation for Lower-Pair Mechanisms Based on Matrices, ASME Journal of Aplied Mechanics, June 1955, pp. 730+736.
[3] Hiller, M., Kecskemethy, A.: A Computer-oriented Approach for the Automatic Generation and Solution of the Equations of Motion for Complex Mechanisms. Proc. 7th World Congress th. Mach. Mech. Sevilla 1967, pp. $42+430$.
[4] Innocenti, C. Parenti-Castelli, V.: Direct Position Analysis of the Stewart Platform Mechanism, Mech. Mach. Theory Vol.25, No. 6, pp. 611+621, 1990.
[5] Mianowski,K.: Analiza i synteza oraz badania doswiadczalne wybranych napẹdów rownoleglych raraion manipulatorow. Praca doktorska. Politechnika Warszawska Wydzial MEiL, Warszawa.
[6] Schielen, W.: Multibody Systems and Robot Dynamics. Preprints 8th Symposium on Theory and Practice of Robots and Manipulators. ROMANSY'90, July 1990, Cracow Poland.
[7] Thornton, G.S.: The GEC Tetrabot - A New Serial-Parallel Assembly Robot. Proc. IEEE Ir.t Conf. Rob. and Autom. Philadelphia, Pa, Apr. 24-29, 1988. Vol. 1 Washington (D.C.) 1988, 437+439.
[8] Walker. M.W. Orin D.E.: Efficlent Dynamic Computer Simulation of Robotic Mechanisms, JACC, Charlotesville, VA. June 1981, pp. 1+9.

DYNAMISCHE ANALYSE IN DER PROJEKTIERUNG VON MANIPULATORANTRIEBEN MIT REICHEN/

## PARALIELSTRUCTUR

## Zusammenfassung

In der Arbeit wird ein Algorithmus zur dynamischen Analyse eines neven Manipulatortyps mit Reihen/Parallelstructur vorgestellt. Auferdem wird ein Computerprogramm beschrieben, das in der Lage ist, diese Analyse durchzufüren und die unproblematische Anwendung ihrer Ergebnisse wahrend der Projektierung des Antriebssystems garantiert.

ANALIZA DYNAMICZNA W PROJEKTDWANIU NAPEDU MANIPULATOAA O STRUKTURZE SZEREGOWO-
-RÓWNOLEG\&EJ
Streszczenie
W pracy przedstawiono algorytm ainalizy dynamicznej nowego typu manipulatora $z$ ramieniem o strukturze szeregoworównoległej. Opisano program komputerowy dokonujacy takiej analizy i zapewniajacy latwe wykorzystanie jej wynikow podczas projektowania układu napẹdowegn.

Wpłyneło do redakcji w styczniu 1992 r.
Recenzent: Tadeusz Burczyński

