ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1992
Seria: MECHANIKA z. 108 Nr kol. 1161

International Conference on
COMPUTER INTEGRATED MANUFACTURING
Internationale Konferenz über
RECHNERINTEGRIERTE FERTIGUNGSSYSTEME
Zakopane, March 24-27 1992

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DIRECT KINEMATIC SOLUTION OF THE GENERAL PLANAR STEWARD PLATFORM

Summary: The Subject of the present paper is the General Planar Steward Platform, a parallel (non-serial) type of manipulator with three degrees of freedom. This manipulator consists of a mobile rigid body, the platform, which is connected by three legs with a fixed base . Each of the legs is equipped with rotary joints on both of its ends and the distance of the two parallel joint axes is controlled either by a linear actuator (a driven P-joint) or a rotary actuator (a driven R-joint). An algebraic input – output equation of degree 6 is derived which allows to find the six theoretically possible positions of the platform corresponding to a given set of input variables. Furthermore an analysis of direct rate kinematics, force decomposition and direct acceleration for this three - degrees - of - freedom linkage is presented.

Keywords: Direct Kinematics,Parallel Manipulator,General Plane Steward Platform 3(RPR) ,3(RRR)

Introduction

Robot- arms in commonly used manipulators are anthropomorphic open- chain mechanisms with actuators connected in series along the arm. These serial, cantilever–like manipulators can be constructed with far-reaching end–effectors together with a sufficiently large workspace for easy maneouvering. But this type of manipulator also has some serious disadvantages. The summing- up of the backlashes at the end - effector and especially the low structural rigidity inherent to any serial manipulator limits their load capacity and make them unfit for tasks where accurate positioning capacity of the end-effector is indispensable.As an alternative to the serial type manipulator the concept of the parallel manipulator was proposed by K.H. Hunt in 1967 [1]. In his book on the "Kinematic Geometry of Mechanisms" Hunt referred to the "Steward Platform" which was developed for a flight simulator by D. Steward [2] in 1965 and suggested to construct parallelly activated manipulators which would avoid the shortcomings of the serial manipulators. In doing so Hunt opened a rather new field of scientific and practical activities. Some effort has since been directed to the investigation of parallelly activated manipulators. As their design is based on the use of closed kinematic chains, their forward (direct) kinematics becomes more difficult in comparison with the direct kinematics of the serial manipulator with its open chain structure. The inverse kinematics of the General Steward Platform is straightforward and was solved first by D.C.H. Yang and T.W.

Lee [3] in 1984. However no solution of the direct kinematics seems to have been presented so far. Recently, X. Shi and R.G: Fenton [4] developed a method to establish the six non linear equations determining the position of the platform and H. Rong and C.G. Liang [5] were able to derive the input-output equations for the 6(RPR) Triangle Steward Platform. A different result had been obtained by P. Nanua and K.J. Waldron in an earlier paper [6]. In the following we present the solution of the forward kinematics of the General Planar Steward Platforms 3(RPR) and 3(RRR). These platforms can be seen as special "triple arm mechanisms" [6] but it seems that they have never been investigated on its own.

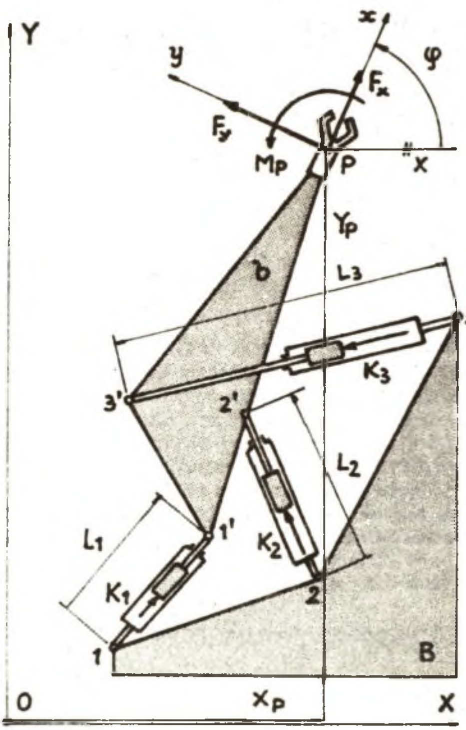

Fig. 1 Planar Parallel 3(RPR) Manipulator

Two types of planar parallel manipulators

There are two types of the planar parallel manipulator, the 3(RPR) manipulator shown in figure 1 and the 3(RRR) manipulator of figure 2. Generally speaking, the first manipulator has greater load capacity, the second a greater workspace. As only the distances between the axes of the rotary joints at the leg ends feature in the equations, both types of mechanisms can be treated mathematically in the same way.

The planar parallel manipulator essentially consists of two bodies, one fixed (B) and one mobile (b), interconnected by three legs. There are two basic loops in the mechanisms and accordingly their order is : $\Omega = 1$ .The structure-formula of Grübler-Kutzbach confirms that the mechanism has three global degrees of freedom :

$$F = \sum_1^9 f_i - 3(\Omega+1) = 9 - 3.2 = 3.$$

The motion (translation and rotation) of body b (the platform) is determined by the time - dependent lengths $L_\alpha(t)$; $\alpha = 1 \div 3$ of the platform legs in the case of the 3(RPR) mechanism and the time - dependent link angles $\varphi_\alpha(t)$; $\alpha = 1 \div 3$ in the case of the 3(RRR) mechanism.

We take as given the coordinates X_α, Y_α ; $\alpha = 1 \div 3$ of the three rotary joint axes 1,2,3 in body B, in relation to the coordinate system $(O; X, Y)$ fixed in B, and the coordinates x_α, y_α ; $\alpha = 1 \div 3$ of the rotary joint axes 1'2'3' in the mobile body b, in relation to the coordinate system $(P; x, y)$ fixed in b. If the straight lines through the points 1 and 1' , 2 and 2', and 3 and 3' [see Fig. 2] intersect in one point, then the mechanism becomes shaky, because it is infinitesimally moveable without changement of the leg - lengths L_α . In this singular position any force and/or moment applied to platform b would cause infinite forces (or moments) in the actuators. In practice, therefore this critical positions has to be carefully avoided.

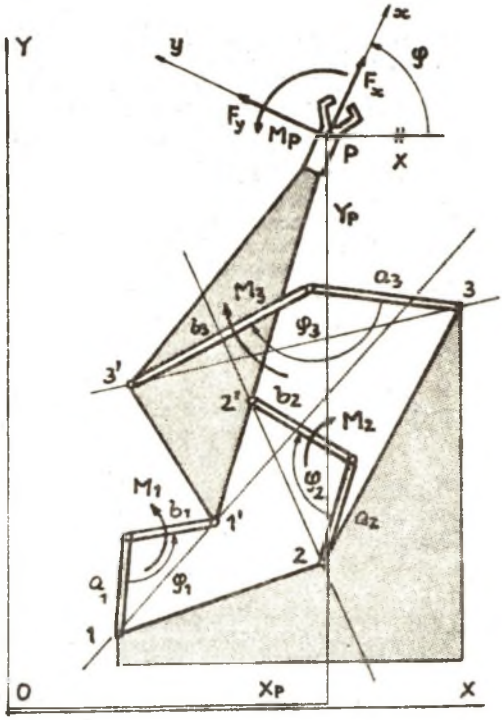

Fig.2 Planar Parallel 3(RRR) Manipulator

The input-output equation

The position of the mobile body b relative to body B can be given by the coordinates X_P, Y_P of the point P and the rotation angle φ as shown in Figs.1 and 2 . The values X_P, Y_P and φ can therefore serve as output variables which are to be determined for a given set of input variables L_α .To any set of input variables correspond a maximum of six sets of output variables. This can easily be shown in the following way. If the connection of one of the legs with the mobile body b is released; e.g. point 3´ in Fig. 3, the path of point 3' is a coupler curve, and a circle with the radius $r = L_3$ and with the centerpoint in 3 intersects this coupler curve in either two or four or six points. Since the coupler curve is a tricircular algebraic curve of order six, there cannot be more than six intersecting points with the circle. In general a six order algebraic curve is intersected by a circle in 12 points. But as the coupler curve contains the imaginary circle three times, only a maximum of six real intersecting points with a given circle are possible. In principle the input - output equation must, therefore provide six output solutions for any given set of input variables .The input variables and the output variables are interconnected by the following conditions:

$$[(X_\alpha - X_P) - (x_\alpha \cos\varphi - y_\alpha \sin\varphi)]^2 + [(Y_\alpha - Y_P) - (x_\alpha \sin\varphi + y_\alpha \cos\varphi)]^2 - L_\alpha^2 = G_\alpha = 0 \qquad (1)$$

With the abbreviations $-(x_\alpha X_\alpha + y_\alpha Y_\alpha) = a_\alpha$, $-(x_\alpha Y_\alpha - y_\alpha X_\alpha) = b_\alpha$

$$(x_\alpha^2 + y_\alpha^2 + X_\alpha^2 + Y_\alpha^2 - L_\alpha^2)/2 = c_\alpha \qquad (2)$$

we obtain

$$(x_\alpha \cos\varphi - y_\alpha \sin\varphi - X_\alpha) X_P + (y_\alpha \cos\varphi + x_\alpha \sin\varphi - Y_\alpha) Y_P + (X_\alpha^2 + Y_\alpha^2)/2 + a_\alpha \cos\varphi + b_\alpha \sin\varphi + c_\alpha = 0 \qquad (3)$$

From this set of equations two equations that are linear in the output variables X_P, Y_P and independent from each other can be derived by substracting the second ($\alpha = 2$) and the third ($\alpha = 3$) equation from the first ($\alpha = 1$).
With

$$x_\alpha - x_\beta = x_{\alpha\beta} , \quad y_\alpha - y_\beta = y_{\alpha\beta} , \quad X_\alpha - X_\beta = X_{\alpha\beta} , \quad Y_\alpha - Y_\beta = Y_{\alpha\beta}$$
$$a_\alpha - a_\beta = a_{\alpha\beta} , \quad b_\alpha - b_\beta = b_{\alpha\beta} , \quad c_\alpha - c_\beta = c_{\alpha\beta} \qquad (4)$$

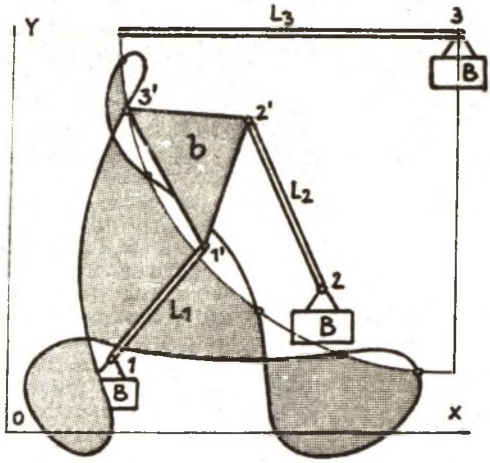

Fig.3 Circle intersecting a coupler courve

and the functions

$$x_{12}\cos\varphi - y_{12}\sin\varphi - X_{12} = K_{11}$$
$$y_{12}\cos\varphi + x_{12}\sin\varphi - Y_{12} = K_{12}$$
$$a_{12}\cos\varphi + b_{12}\sin\varphi + c_{12} = K_{13}$$
$$x_{23}\cos\varphi - y_{23}\sin\varphi - X_{23} = K_{21}$$
$$y_{23}\cos\varphi + x_{23}\sin\varphi - Y_{23} = K_{22}$$
$$a_{23}\cos\varphi + b_{23}\sin\varphi + c_{23} = K_{23} \qquad (5)$$

we find for the translation parameters X_P and Y_P

$$K_{11}X_P + K_{12}Y_P = -K_{13}$$
$$K_{21}X_P + K_{22}Y_P = -K_{23} \qquad (6)$$

and write with

$$\det\begin{Vmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{Vmatrix} = N(\varphi)$$

$$\det\begin{Vmatrix} -K_{13} & K_{12} \\ -K_{23} & K_{22} \end{Vmatrix} = Z_1(\varphi)\,,\quad \det\begin{Vmatrix} K_{11} & -K_{13} \\ K_{21} & -K_{23} \end{Vmatrix} = Z_2(\varphi) \qquad (7)$$

for the solution of (6):

$$X_P(\varphi) = Z_1(\varphi)/N(\varphi)\,,\quad Y_P(\varphi) = Z_2(\varphi)/N(\varphi). \qquad (8)$$

The functions $N(\varphi)$, $Z_1(\varphi)$ and $Z_2(\varphi)$ can be written as

$$N(\varphi) = N_1\cos\varphi + N_2\sin\varphi + N_3$$
$$Z_1(\varphi) = Z_{11}\cos^2\varphi + Z_{12}\cos\varphi\sin\varphi + Z_{13}\cos\varphi + Z_{14}\sin\varphi + Z_{15}$$
$$Z_2(\varphi) = Z_{21}\cos^2\varphi + Z_{22}\cos\varphi\sin\varphi + Z_{23}\cos\varphi + Z_{24}\sin\varphi + Z_{25} \qquad (9)$$

with

$$N_1 = -Y_{23}x_{12} + Y_{12}x_{23} + X_{23}y_{12} - X_{12}y_{23}$$
$$N_2 = X_{23}x_{12} - X_{12}x_{23} + Y_{23}y_{12} - Y_{12}y_{23}$$
$$N_3 = -X_{23}Y_{12} + X_{12}Y_{23} - x_{23}y_{12} + x_{12}y_{23}$$
$$Z_{11} = -b_{23}x_{12} + b_{12}x_{23} + a_{23}y_{12} - a_{12}y_{23}$$
$$Z_{12} = a_{23}x_{12} - a_{12}x_{23} + b_{23}y_{12} - b_{12}y_{23}$$
$$Z_{13} = Y_{22}a_{12} - Y_{12}a_{23} + c_{23}y_{12} - c_{12}y_{23}$$
$$Z_{14} = Y_{23}b_{12} - Y_{12}b_{23} + c_{23}x_{12} - c_{12}x_{23}$$
$$Z_{15} = Y_{23}c_{12} - Y_{12}c_{23} + b_{23}x_{12} - b_{12}x_{23}$$
$$Z_{21} = -Z_{12}\,,\quad Z_{22} = Z_{11}$$
$$Z_{23} = -X_{23}a_{12} + X_{12}a_{23} - c_{23}x_{12} + c_{12}x_{23}$$
$$Z_{24} = -X_{23}b_{12} + X_{12}b_{23} + c_{23}y_{12} - c_{12}y_{23}$$
$$Z_{25} = -X_{23}c_{12} + X_{12}c_{23} + b_{23}y_{12} - b_{12}y_{23} \qquad (10)$$

Substitution of $X_P = Z_1/N$ and $Y_P = Z_2/N$ into the first equation of (3) leads to the final equation which allows to determine the rotation angle φ . This equation has the structure:

$$(f_1 \cos^3\varphi + f_2 \cos^2\varphi + f_3 \cos\varphi + f_4) + \sin\varphi\,(f_5 \cos^2\varphi + f_6 \cos\varphi + f_7) = f(\varphi) = 0. \qquad (11)$$

The coefficients f_i are functions of the coordinates $x_\alpha, y_\alpha, X_\alpha, Y_\alpha$. The products $(x_1\cos\varphi - y_1\sin\varphi - X_1)\,Z_1(\varphi)\,N(\varphi)$ and $(y_1\cos\varphi + x_1\sin\varphi - Y_1)\,Z_2(\varphi)\,N(\varphi)$ let expect that in the final equation terms with $(\cos\varphi)^4$ and $(\cos\varphi)^3\sin\varphi$ would also occur.But they disappear because their coefficients are:

$$x_1(Z_{11} - Z_{22}) + y_1(Z_{12} + Z_{21}) \text{ and } x_1(Z_{12} + Z_{21}) + y_1(Z_{11} + Z_{22})$$

and these coefficients are, according to (10), equal to zero .

The substitutions of $\cos\varphi = (1 - \tan^2\frac{\varphi}{2})/(1 + \tan^2\frac{\varphi}{2}),\ \sin\varphi = 2\tan\frac{\varphi}{2}/(1 + \tan^2\frac{\varphi}{2})$

transform equation (11) into an algebraic equation of order six in $\tan\varphi/2$. Equation (11), therefore allows to determine numerically a maximum of six values for the rotation angle φ . With the equations (8) then the corresponding translation variables X_P and Y_P can be found .

<u>Numerical Examples</u>

For the system parameter set (corresponding to (Fig.1)

$X_1 = 8.3\,dm,\ Y_1 = 5.6\,dm,\ X_2 = 24.3\,dm,\ Y_2 = 10.8\,dm,\ X_3 = 35\,dm,\ Y_3 = 31\,dm$

$x_1 = -31\,dm,\ y_1 = -4.3\,dm,\ x_2 = -20.9\,dm,\ y_2 = -2.9\,dm,\ x_3 = -23.8\,dm,\ y_3 = 6\,dm$

and the input variables $L_1 = 11.204\,dm,\ L_2 = 14.235\,dm,\ L_3 = 26.445\,dm$

we obtain the output equation (11) in the form

$$\begin{aligned} &1.1524 \cos^3\varphi - 2.9832 \cos^2\varphi + 2.3457 \cos\varphi - 0.52421 \\ &\quad + \sin\varphi(-1.2497 \cos^2\varphi + 1.7391 \cos\varphi - 0.52742) = f(\varphi) = 0. \end{aligned}$$

This equation has six real solutions:

$$\varphi = \{65.426°,\ 18{,}618°,\ -23.627°,\ -90.298°,\ -50.031°,\ -14.729°\}$$

The corresponding translation variables then are

$$X_P = \{24.624\,dm,\ 33.752\,dm,\ 30.376\,dm,\ 13.475\,dm,\ 42.540\,dm,\ 47.580\,dm\}$$

$$Y_P = \{44.043\,dm,\ 30.323\,dm,\ 4.9105\,dm,\ -14.266\,dm,\ -17.351\,dm,\ -5.7520\,dm\}.$$

Fig. 4 shows the six position of the "platform" body b which belong to the given set of input variables. They correspond to different "closure modes" of the mechanism. But because the leg lengths can be adapted, a change from one "mode" to an other is possible without dismantling the mechanism.

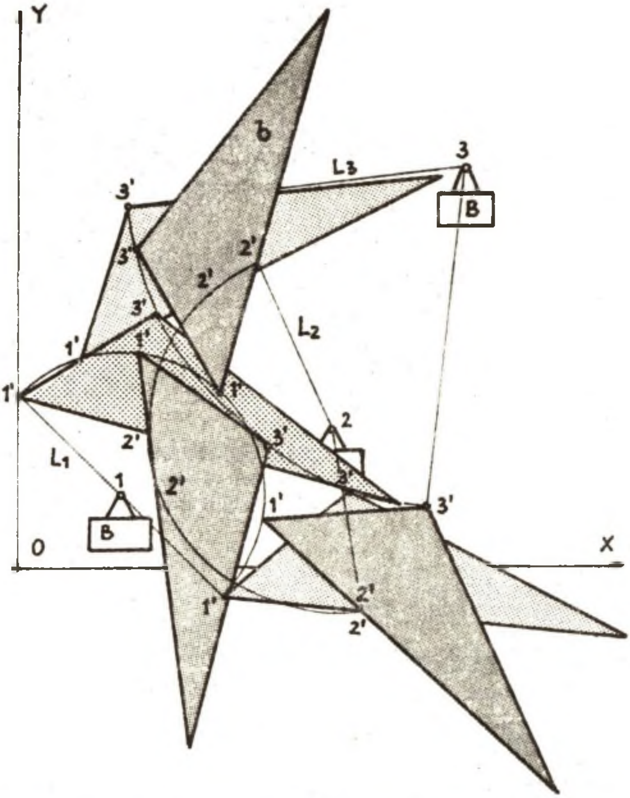

Fig. 4 Six positions of the platform body corresponding to a given set of leg lenghts.

Velocities, forces moments and accelerations

If the position of the mobile body b is known and the leg lengths are given as time functions then the velocity of point P and the angular velocity can be found by solving a set of three linear equations.

Differentiation of the equations (1) with respect to time yields with

$p = (x_P, y_P, \varphi)^T$:

$$\sum \frac{\partial G_\alpha}{\partial p_i} \dot{p}_i = 2 L_\alpha \dot{L}_\alpha , \qquad \alpha = 1 \div 3 \qquad (12)$$

Introducting the diagonal matrix B:

$$B = \begin{Vmatrix} 2L_1 & 0 & 0 \\ 0 & 2L_2 & 0 \\ 0 & 0 & 2L_3 \end{Vmatrix}$$

the matrix A:

$$A = \left\Vert \frac{\partial G_\alpha}{\partial p_i} \right\Vert$$

and the column matrix $L = (L_1, L_2, L_3)^T$ we can write eqs. (12) as a matrix equation $A\dot{p} = B\dot{L}$ and obtain:

$$\dot{p} = (\dot{x}_P, \dot{y}_P, \dot{\varphi})^T = (A^{-1}B)\dot{L} = J.(\dot{L}_1, \dot{L}_2, \dot{L}_3)^T . \qquad (13)$$

For any position of the mechanism the Jakobian matrix $J = A^{-1}B$ can be calculated. With equation (13) we find the velocities $\dot{x}_P$ and $\dot{y}_P$ and the angular velocity $\dot{\varphi}$ for any given set of relative velocities of the linear activators $\dot{L}_\alpha$ $(\alpha = 1 \div 3)$. This result can immediately be used for the determination of the actuator forces of the 3(RPR) manipulator and the actuator moments of the 3(RRR) manipulator.

The actuator forces in the active P-joints of the 3(RPR) manipulator collected in the matrix K:

$$K = (K_1, K_2, K_3)^T$$

and the forces and the Moment acting on the body b (and reduced to point P) in the matrix F:

$$F = (F_x, F_y, M_P)^T$$

are connected by

$$K = -J^T F \tag{14}$$

This follows immediately from the principle of virtual displacements

$$\delta W = 0 = \delta p^T F + \delta L^T K$$

together with

$$\delta p = J \, \delta L \, .$$

The moments in the rotary actuators of the 3(RRR) manipulator collected in the matrix

$$M = (M_1, M_2, M_3)^T$$

are related to the elements of the matrix $F = (F_x, F_y, M_P)^T$ in the following way.

From

$$L_\alpha^2 = a_\alpha^2 + b_\alpha^2 - 2 a_\alpha b_\alpha \cos\varphi_\alpha$$

we get

$$\delta L_\alpha = (a_\alpha b_\alpha \sin\varphi_\alpha / L_\alpha)\, \delta\varphi_\alpha$$

and with the matrices

$$D = diag \, \| a_\alpha b_\alpha \sin\varphi_\alpha / L_\alpha \|$$

$$\varphi = (\varphi_1, \varphi_2, \varphi_3)^T \quad , \quad \delta\varphi = (\delta\varphi_1, \delta\varphi_2, \delta\varphi_3)^T$$

we find

$$\delta p = J \delta L = J D \, \delta\varphi \, .$$

From the principle of virtual displacements $\delta W = 0 = \delta p^T F + \delta\varphi^T M$ we obtain

$$M = -D^T J^T F \tag{15}$$

In our numerical example [Fig. 1] we find with:

$$X_P = 24.624 \text{ dm} \; , \; Y_P = 44.043 \text{ dm} \; , \; \varphi = 65.426^\circ$$

$$L_1 = 11.204 \text{ dm} \; , \; L_2 = 14.234 \text{ dm} \; , \; L_3 = 26.445 \text{ dm}$$

for:

$$\dot{p} = \begin{bmatrix} \dot{x}_P \\ \dot{y}_P \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} -2.4248 \\ 1.0341 \\ 0.14056 \end{bmatrix} \dot{L}_1 + \begin{bmatrix} 1.2040 \\ 0.37563 \\ -0.08340 \end{bmatrix} \dot{L}_2 + \begin{bmatrix} -3.1591 \\ 0.54105 \\ 0.12918 \end{bmatrix} \dot{L}_3 \tag{16}$$

Differentiation of equation $A \dot{p} = B \dot{L}$ with espect to time gives

$$\Sigma (\partial A / \partial p_i) \dot{p}_i \, \dot{p} + A \ddot{p} = \dot{B} \dot{L} + B \ddot{L}$$

an equation wich can be solved for $\ddot{p}$. With the matrix operator

$$\dot{p}_1\frac{\partial}{\partial p_1} + \dot{p}_2\frac{\partial}{\partial p_2} + \dot{p}_3\frac{\partial}{\partial p_3} = \dot{p}^T\frac{\partial}{\partial p}$$

we obtain

$$\ddot{p} = A^{-1}(\dot{B}\dot{L} - \dot{p}^T\frac{\partial A}{\partial p}\dot{p}) + A^{-1}B\ddot{L}. \qquad (17)$$

The first term on the right side of this equation is fully determined by the position matrix p and its time derivative $\dot{p}$.
In our example we get, using the result of (16), with

$$\dot{L} = (-1.2\ dm/sec\ , -0.9\ dm/sec\ , 1.4\ dm/sec\ )^T$$

from eq. (17) for the accelerations $\ddot{x}_P$, $\ddot{y}_P$ and the angular acceleration $\ddot{\varphi}$:

$$\ddot{p} = \begin{bmatrix} \ddot{x}_P \\ \ddot{y}_P \\ \ddot{\varphi} \end{bmatrix} = \begin{bmatrix} -0.019787 \\ -2.1739 \\ -0.055722 \end{bmatrix} + \begin{bmatrix} -2.4248 \\ 1.0341 \\ 0.14056 \end{bmatrix} \ddot{L}_1 + \begin{bmatrix} 1.2040 \\ 0.37563 \\ -0.08340 \end{bmatrix} \ddot{L}_2 + \begin{bmatrix} -3.1591 \\ 0.54105 \\ 0.12918 \end{bmatrix} \ddot{L}_3 .$$

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KINEMATISCHE ANALYSE DER STEWARD-PLATFORM MECHANISMUS

Zusammenfassung

Es wird der ebene Steward-Plattform Mechanismus mit drei Freiheitsgraden der Bewegung analysiert. Dieser zweischleifige Mechanismus stellt einen Sonderfall des sogenannten "Triple arm mechanism" [6] dar und besteht aus einem starren (Plattform-) Körper dessen ebene Bewegung relativ zum Basiskörper durch drei Beine veränderlicher Längen kontrolliert wird. Um die Lage der Plattform-Körpers aus den drei Längen zu bestimmen ist eine algebraische Gleichung sechster Ordnung numerisch zu lösen. Einem Längensatz entsprechen demnach maximal sechs reelle Lagen des Plattform-Körpers. Lineare Gleichungen erlauben die Bestimmung des Geschwindigkeits- und des Beschleunigungszustandes aus den Längenänderungs-Geschwindigkeiten bzw.-Beschleunigungen , sowie die Klärung der Kraftverhältnisse.

ROZWIĄZANIA KINEMATYKI DLA OGÓLNEJ PŁASKIEJ PLATFORMY STEWARDA

Streszczenie

Przedmiotem niniejszej pracy jest ogólna płaska platforma Stewarda, równoległy manipulator o trzech stopniach swobody. Manipulator ten składa się z cześci jezdnej, platformy,składającej się z trzech nóg połączonych z ustaloną podstawą. Każda noga jest wyposażona w połączenia obrotowe na obydwu końcach, a odległość dwóch osi równoległych połączeń jest sterowana przy pomocy serwomechanizmu liniowego lub obrotowego.. Algebraiczne równanie szóstego stopnia pozwala na znalezienie sześciu teoretycznie możliwych położeń platformy odpowiadających danemu zbiorowi zmiennych wejściowych. Ponadto przedstawiono analizę rozkładu sił i przyspieszeń dla tego mechanizmu o trzech stopniach swobody.

Wpłynęło do redakcji w styczniu 1992 r.   Recenzent: Jerzy Wróbel