ZESZYTY NAUKOWE POLITECHNIKI SLASKIEJ 1992

Seria: MECHANIKA z. 107 Nr kol. 1154

Natalija Pankratova
Institute of Mechanics of the Academy of Sciences
of the Ukrainian

THE CALCULATION OF ANISOTROPIC CONSTRUCTION ELEMENTS IN SPACE FORMULATION

Streszczenie. Przedstawiono model wyznaczania stanu naprężenia i odkształcenia anizotropowych warstwowych drążonych sprężystych ciał z jedną płaszczyzną sprężystej symetrii w zastosowaniu do elementów konstrukcyjnych. Równania konstytutywne przyjęto jak w trójwymiarowym zagadnieniu teorii sprężystości. W wyniku analitycznych przekształceń zagadnienie trójwymiarowe sprowadza się do zadania jednowymiarowego, odpowiadającego określonym postaciom wybranych funkcji rozwiązujących. Zagadnienie jednowymiarowe rozwiązuje się wykorzystując stabilne metody numeryczne. Taki sposób modelowania można zastosować do obliczeń wytrzymałościowych drążonych cylindrów i stożków otrzymywanych przez nawijanie.

Резюме. Предлагается модель расчета напряьенно-деформированного состояния анизотропных слоистых полых упругих тел с одной плоскостью упругой симметрии применительно к конструктивным элементам. В качестве исходных здесь выбираются уравнения трехмерной задачи теории упругости. Посредством различных аналитических преобразований трехмерная задача точно сводится к одномерной задаче относительно определенным образом выбранных разрешающих функций. Одномерная задача решается численным устойчивым методом. Этот подход может быть использован для расчета полых цилиндров или конусов, образованных армированием.

Summary. Model is presented for calculation stressed-deformed state of anisotropic laminated hollow elastic bodies with one plane of elastic symmetry applied by constructions elements. Constitutive relations in this model are equations of three-dimensional problem of elasticity theory. By means of various analytical trasformations three-dimensional problem is accurately reduced to one-dimensional problem relative to some appropriately chosen resolving functions. The one-dimensional problem is solved by stable numerical method. This approach may be used for design and calculation of hollow cylinders or cones fabricated by winding.

1. INTRODUCTION

The problems of calculation of the stressed-deformed state of elastic bodies from modern materials demand to take into account of anisotropy and non-homogeneity of elastic properties of materials. Such problems were obtained in more common form for some simple cases - for elastic bodies isotropy homogeneity materials, varios approximate analitical and numerical methods were used. The solution is very diffucult for problems of the elastic bodies with one plane of elastic symmetry paralleled of cylindrical space or perpendiculared to axis of revolution. It may explained, on the one hand, complication of resolving equtions. On the other hand, there are effects in stressed state, which absent in orthotropy's bodies. There is similar situation for designation and calculation of hollow elastic bodies fabricated from orthotropic materials in case, when main elasticity directions do not coincide with coordinates. These problems have a practical interest for determination of stressed state of the hollow elastic elements fabricated by winding.More fully and exact calculation of elastic bodies with anisotropy and non- homogenity of elastic properties of materials may be carried out when constitutive relations are equations of three-dimensional problem of elastisity theory.

2. THE STRESSED STATE OF ELASTIC BODIES

2.1. The calculation of model

This model was used for investigation of the stressed-deformed state of anisotropy laminated elastic bodies in the form of cylinders or cones.The packet of body's layers consists of arbitrary number of isotropic,orthotropic or anisotropic layers with the constant thickness.These layers have different structure and mechanical and thermophysical characteristics and elastic properties of materials. All layers of the packet are supposed to deform without sliding and separation and with sliding too.The body's materials obeys the generalised Hooke's law with taking into account the Dugamel-Neumann hypothesis.The model is presented based on the combination of elastisity theory equations and the method of numerical analysis.

The relations of generalized Hooke law is reffered to the curvilinear system of coordinates α ,β ,γ .These equations may be written for i-layer

$\{\gamma_{i+1} < \gamma < \gamma_i, i=1,2,\dots,N\}$ for anisotropic materials with one plane of elastic symmetry γ =const in following form [1]

$$
\begin{aligned}
e^i_\alpha &= a^i_{11}\sigma^i_\alpha + a^i_{12}\sigma^i_\beta + a^i_{13}\sigma^i_\gamma + a^i_{16}\tau^i_{\alpha\beta} + \alpha^i_1 T^i; \\
e^i_\beta &= a^i_{12}\sigma^i_\alpha + a^i_{22}\sigma^i_\beta + a^i_{23}\sigma^i_\gamma + a^i_{26}\tau^i_{\alpha\beta} + \alpha^i_2 T^i; \\
e^i_\gamma &= a^i_{13}\sigma^i_\alpha + a^i_{23}\sigma^i_\beta + a^i_{33}\sigma^i_\gamma + a^i_{36}\tau^i_{\alpha\beta} + \alpha^i_3 T^i; \\
e^i_{\beta\gamma} &= a^i_{44}\tau^i_{\beta\gamma} + a^i_{45}\tau^i_{\alpha\gamma} + \alpha^i_{23} T^i; \\
e^i_{\alpha\gamma} &= a^i_{45}\tau^i_{\beta\gamma} + a^i_{55}\tau^i_{\alpha\gamma} + \alpha^i_{13} T^i; \\
e^i_{\alpha\beta} &= a^i_{16}\sigma^i_\alpha + a^i_{26}\sigma^i_\beta + a^i_{36}\sigma^i_\gamma + a^i_{66}\tau^i_{\alpha\beta} + \alpha^i_{12} T^i,
\end{aligned}
\tag{1}
$$

where $e^i_\alpha, e^i_\beta, e^i_\gamma, e^i_{\alpha\beta}, e^i_{\beta\gamma}, e^i_{\alpha\beta}$ -components of tensor's deformations,

$\sigma^i_\alpha, \sigma^i_\beta, \sigma^i_\gamma, \tau^i_{\alpha\beta}, \tau^i_{\alpha\gamma}, \tau^i_{\beta\gamma}$ - components of tensor's stresses.

The elastic characteristics a^i_{mn}, the coefficients of linear temperature extension $\alpha^i_1, \alpha^i_2, \alpha^i_3$ in directions α, β, γ respectivly and coefficients of shear $\alpha^i_{12}, \alpha^i_{13}, \alpha^i_{23}$ are functions of γ coordinate. It should be pointed out, that last circumstanse taking accountthe arbitrarry of elastic properties of materials in the body's thickness.

For solution the problem of the theory of elasticity ought to satisfy not only the equations of equilibrium, the expressions for deformations by displacements, the Hooke law but and the boundary conditions on all spaces of body and conditions junction of layers in the united packet. There is rigid contact in more cases, when all layers of the packet are deformed without sliding and separation. Then condition of unbrokenly of components vector's displacement and components of tensor's stresses must be carried out respectively

$$
\begin{aligned}
&\sigma^i_\gamma = \sigma^{i+1}_\gamma; \quad \tau^i_{\alpha\gamma} = \tau^{i+1}_{\alpha\gamma}; \quad \tau^i_{\beta\gamma} = \tau^{i+1}_{\beta\gamma}; \\
&u^i_\gamma = u^{i+1}_\gamma; \quad u^i_\alpha = u^{i+1}_\alpha; \quad u^i_\beta = u^{i+1}_\beta.
\end{aligned}
\tag{2}
$$

These conditions (2) may be to break in many cases and the regions of layers separtion are formed. When the friction's forceies are very small and it may be don't take into account the model of ideal sliding of layers may be put for these regions in form

$$\sigma_{\gamma}^{i} = \sigma_{\gamma}^{i+1};\ \tau_{\alpha\gamma}^{i} = \tau_{\alpha\gamma}^{i+1} = \tau_{\beta\gamma}^{i} = \tau_{\beta\gamma}^{i+1} = 0;$$

$$u_{\gamma}^{i} = u_{\gamma}^{i+1};\ u_{\alpha}^{i} \neq u_{\alpha}^{i+1};\ u_{\beta}^{i} \neq u_{\beta}^{i+1}. \tag{3}$$

These conditions formulate in functions, which are chosen as resolving functions for solution problems of this class. The resolving system of differential equations may be represented in cylindrical system of coordinates z, θ, r for axissymmetrical loading cylinder in the form [2]

$$\frac{\partial\bar{\sigma}^{i}}{\partial r} = B_{0}^{i}\,\bar{\sigma}^{i} + B_{1}^{i}\frac{\partial\bar{\sigma}^{i}}{\partial z} + B_{2}^{i}\frac{\partial^{2}\bar{\sigma}^{i}}{\partial z^{2}} + \bar{f}^{i};\quad B_{m}^{i} = |\ b_{m}^{pj}(r)|\ ,$$

$$\bar{\sigma}^{i} = \{\sigma_{r}^{i},\tau_{rz}^{i},\tau_{r\theta}^{i},u_{r}^{i},u_{z}^{i},u_{\theta}^{i}\};\ \bar{f}^{i} = \{f_{1}^{i},f_{2}^{i},\dots,f_{6}^{i}\}; \tag{4}$$

$$(r_{0} \leqslant r \leqslant r_{N};\ 0 \leqslant z \leqslant l;\ m = 0,1,2;\ p,j = 1,2,\dots,6)$$

The matrix's elements B_{m}^{i} are function of elastic properties of layers materials, components of vector $\bar{f}^{i}$ determinate of volume's forces and temperature field. Joing to the system of equation (4) the boundary conditions on body contours z=0, z=l

$$\tau_{rz} = u_{z}^{i} = u_{\theta}^{i} = 0, \tag{5}$$

we formulate the corresponding boundary value problem.

For solution of this problem we apply of double trigonometrical rows. It is allows to satisfy strictly of boundary conditions and to divide the variables in system (4). By means of analytical transformations three-dimensional problem is accurately reduced to one-dimensional problem with variable coefficients.

For body's cone the resolving system may be represented in spherical system of coordinates φ, θ, r in the following form [3]

$$\frac{\partial\bar{\sigma}^{i}}{\partial\varphi} = C_{0}^{i}\,\bar{\sigma}^{i} + C_{1}^{i}\,\frac{\partial\bar{\sigma}^{i}}{\partial r} + C_{2}^{i}\,\frac{\partial^{2}\bar{\sigma}^{i}}{\partial r^{2}} + \bar{g}^{i};\ C_{m}^{i} = |\ c_{m}^{pj}(\varphi)\ |;$$

$$\bar{\sigma}^{i} = \{\sigma_{\varphi}^{i},\tau_{r\varphi}^{i},\tau_{\varphi\theta}^{i},u_{\varphi}^{i},u_{r}^{i},u_{\theta}^{i}\};\ \bar{g}^{i} = \{g_{1}^{i},g_{2}^{i},\dots,g_{6}^{i}\} \tag{6}$$

$$(\varphi_{0} \leqslant \varphi \leqslant \varphi_{N};\ 0 \leqslant r \leqslant \infty;\ 0 \leqslant \theta \leqslant 2\pi;\ m = 0,1,2;$$

$$p,j = 1,2,\dots,6\ )$$

where the matrix's elements C_m^i are functions of elastic properties of layers materials too, components of vector $\bar{g}^i$ are determinated of force's loading. It should be pointed out that the coordinate surface in the circumferential direction is closed and the load is changed by polinominal law. With these conditions the solution of differentional system (6) is accurately reduced to one-dimensional problem. The one-dimensional system of equations was represented in the normal form of Cauchy, so that this system was resolved relative to the first derivatives of the sought function in γ coordinate. The succession of boundary value problem is solved by the discrete orthogonalization method. In this method by means of the orthogonalization of the vectors-solutions of Cauchy problems in certain points of integration the increase of error is eliminated resulting in stable computational process.

The resolving system (4),(6) may be used too for designation of the stressed-deformed state of elastic bodies fabricated from orthotropic materials in case, when main elasticity direction do not coincide with coordinates.

The solution obtained may be used for evalution of acceptability of assumptions in various applied models of shells, for choice of regularities of distributions of stresses and displacements used in these models - taking into account nonhomhgenity and anisotropy of elastic properties of materials.

2.2. The calculation of construction's element

The stressed-deformed is considered on the construction's element in form of the two-layered cylinders. The layers are fabricated by winding of the fibers, which have equallity and opposite of the angles by longitudinal axis relativly. That is main directions of elasticity are turned on angle φ of the internal layers and $-\varphi$ of external layers to axies z and θ accordingly. The material is orthotropic with following elastic characteristics

α_{11} = E/5.7; α_{22} =α_{33} =E/1.4; α_{12} = α_{13} =-0.068E/1.4; α_{23} =-0.4E/1.4;

α_{44} = E/0.6; α_{55} = α_{66} = E/0.575.

The cylinders are under applied by external pressure

$$\sigma_r(r_N) = - q_0 \sin^P \pi z/l.$$

The internal of radius is r_0, external - $r_N = 1,1r_0$. The layers thickness are equall. In the solution of this problem parameters have following values $r_0 = 1$; $l = 2r_0$; $p = 8$.

The some results of this problem solution are shown in figures.

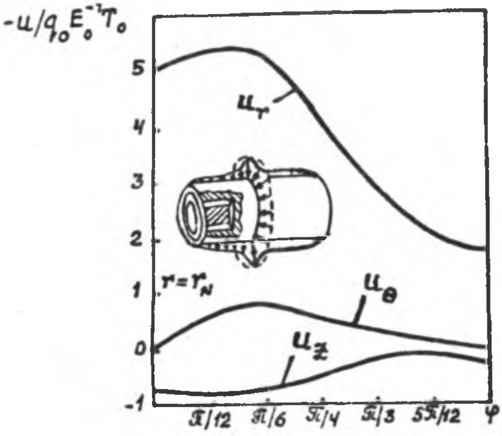

Fig. 1 Distribution of displacements

Rys. 1 Rozkład przemieszczeń

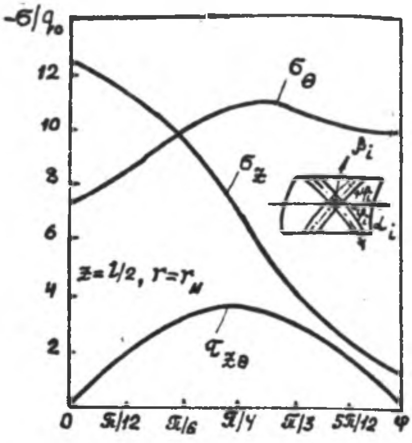

Fig. 2 Distribution of stresses

Rys. 2 Rozkład naprężeń

In fig. 1 the distribution of displacements is shown for u_r in the section z = 0.5l and u_z, u_θ in the section z = 0.475l. In Figure 2 the distribution of stresses $\sigma_z, \sigma_\theta, \tau_{z\theta}$ is illustrated on external space with z = 0.5l for varios values of parametrs φ ,which characterizes the variability of winding. These results allow the conclusion, that the stresses $\tau_{r\theta}, \tau_{z\theta}$ and displacements u_θ is appeared. But these stresses and displacement are absent when main elasticity of directions coincide with coordinates.

The computations carried out in prescribed rangers of changes of the geometric, mechanical and thermophysical characteristics of the elements allowed to find effects resulting from noncoincidence of main directions of elasticity with directions of coordinate lines.

REFERENCES

[1] Lehnitskij S.G. Theory of elasticity of anisotropy body. Nauka, Moskwa 1977.

[2] Pankratova N.D. The investigation of stressed-deformed state of cylindrical shell is based on theory of elasticity equations. Prikladnaja Mechanika 1983, 19.N12.

[3] Pankratova N.D. The solution of problems of the stressed state of anisotropic laminated cone. Doklady AN USSR. 1988, N10.

OBLICZENIA ANIZOTROPOWYCH ELEMENTÓW KONSTRUKCYJNYCH W ZAGADNIENIACH PRZESTRZENNYCH

W pracy przedstawiono model obliczeń stanu naprężeń i odkształceń niejednorodnych wydrążonych cylindrów i stożków w zastosowaniu do elementów konstrukcyjnych wykonywanych przez nawijanie. Przedstawiony sposób przygotowania powoduje, że główne kierunki sprężystości ortotropowego materiału nie pokrywają się z kierunkami układu współrzędnych. Zależności uogólnionego prawa Hooke'a zapisuje się w tym przypadku w postaci jak dla jednej płaszczyzny sprężystej symetrii. Rozwiązanie tego typu zadań związane jest z koniecznością uwzględnienia anizotropii i niejednorodności materiału. Uwarunkowane to jest istnieniem kolejno następujących warstw z różnym kątem ich układania, co prowadzi do bardziej złożonego opisu zagadnienia. Pozwala to jednak na dokładniejsze wyznaczenie stanu naprężenia i przemieszczenia w porównaniu z elementami ortotropowymi.

W przedstawionym modelu wychodzi się z równań teorii sprężystości niejednorodnego anizotropowego ciała [1]. Przyjmuje się, że ciało sprężyste może składać się z dowolnej liczby warstw, które mają określoną charakterystykę anizotropii i niejednorodności. Otrzymane rozwiązanie w postaci podwójnego szeregu trygonometrycznego pozwala wystarczająco dobrze spełnić warunki graniczne, dokonać rozdziału zmiennych równań (4), (6) i sprowadzić rozwiązanie zadania brzegowego do rozwiązania układu zwyczajnych równań różniczkowych o zmiennych współczynnikach.
Przedstawione rozwiązanie elementu konstrukcyjnego w postaci dwuwarstwowego cylindra wskazuje, że kierunki główne sprężystości nie pokrywają się z przyjętymi kierunkami współrzędnych.