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MODELLING OF VIBRATION OF GEAR TRANSMISSIONS

Streszczenie. W artykule przedstawiono metodę matematycznego modelowania przestrzennych układów drgających z walcowymi kołami zębatymi. Metoda umożliwia znaczną redukcję liczby stopni swobody układu przy stosunkowo dużej dokładności obliczania amplitud drgań.

Резюме. В статье приводится метод математического моделирования пространственных колебаний систем с цилиндрическими зубчатыми колесами. Метод дает возможность значительной редукции числа степени свободу системы при относительно большой точности вычисления амплитуды колебания.

Summary. The method for mathematical modelling of spatial vibrations of the spur gear transmissions is presented. This method enables a substantial reduction of the number of degrees of freedom with relatively high accuracy in calculating vibration amplitudes.

1. INTRODUCTION

We consider linear shaft systems with gears composed of N shafts (Fig. 1). The motion of each dismembered shaft "k" is, after discretization by finite element method in a coordinate system x, y, z, described by an equation of motion in a matrix form [1]

$$\mathbf{M}_k\ddot{\mathbf{q}}_k(t) + (\mathbf{B}_k + \mathbf{G}_k)\dot{\mathbf{q}}_k(t) + \mathbf{K}_k\mathbf{q}_k(t) = \mathbf{f}_k^I(t)\ ,\ k = 1,\ 2,\ \dots,\ N. \qquad (1)$$

Matrices $\mathbf{M}_k$, $\mathbf{B}_k$, $\mathbf{K}_k$ are real, time - independent square mass, damping and stiffness matrices of order n_k of the isolated shaft "k". The shaft is supported on viscous - elastic bearings in nodal points "i". Matrices $\mathbf{G}_k$ express gyroscopic effect of the rigid discs connected with shaft in nodal points "j". The generalized coordinate vectors $\mathbf{q}_k(t)$ express displacements u_i, v_i, ψ_i, w_i, ϑ_i, φ_i of the all nodal points $i=1,2,..,N_k$

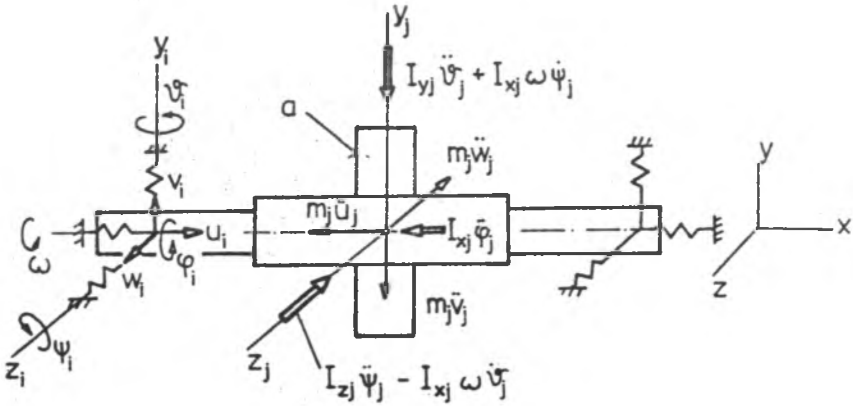

Fig. 1. Shaft model

Rys.1 Model wału

of the shaft. Vectors $f_k^I(t)$ describe the force effect of other shafts in gear meshings. Structure of matrices is shown in 1 .

2. MATHEMATICAL MODEL OF THE SHAFT SYSTEM WITH SPUR GEARS

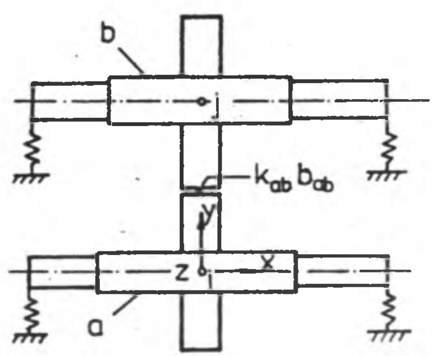

Fig. 2. The single - stage spur gear transmission

Rys.2. Jedostopniowa walcowa przekładnia zębata

We consider the system composed of two shafts (Fig. 2) joined by elastic - viscous discrete coupling in gear mesch between gear "i" on the shaft (a) and gear "j" on the shaft (b) . The internal coupling force in spur gearing with spiral teeth in central mesh point can be expressed in first approximation in form

$$N_{ab} = k_{ab}({}^a\delta_i^T\, {}^a q_i - {}^b\delta_j^T\, q_j + \,_{ab}(t)) + b_{ab}({}^a\delta_i^T\, {}^a\dot{q}_i - {}^b\delta_j^T\, \dot{q}_j + \dot{\Delta}_{ab}(t)) \qquad (2)$$

where:

k_{ab}, b_{ab} - central stiffness, coefficient of viscous damping of gearing in normal direction,

${}^a q_i$, ${}^b q_j$ - displacement vector of gear mass centre in point "i" on the shaft (a) and in point "j" on the shaft (b),

${}^a\delta_i$, ${}^b\delta_j$ - vector of geometrical parameters of gear "i" on the shaft (a) and gear "j" on the shaft (b) in form (3.5) in 1,

$\Delta_{ab}(t)$ - kinematic transmissions error on the line of gear mesh.

Mathematical model of the single - stage spur gear transmission (Fig.2) is described by two equations (1) for k = a,b. The internal coupling force vector of the whole system can be written as

$$\mathbf{f}_I(t) = \begin{bmatrix} \mathbf{f}_a^I(t) \\ \mathbf{f}_b^I(t) \end{bmatrix} = -\mathbf{K}_v\, \mathbf{q}(t) - \mathbf{B}_v\, \dot{\mathbf{q}}(t) + \mathbf{f}(t) \qquad (3)$$

where $\mathbf{q}(t) = \mathbf{q}_k(t)$ is generalized coordinate vector of dimension $n = \sum_{k=a,b} n_k$ and $\mathbf{K}_v$ ($\mathbf{B}_v$) is stiffness (damping) matrix of gear mesh coupling between shafts. Effect of kinemetic error in gear mesh is expressed by internal excitation force vector $\mathbf{f}(t)$. These matrices and vectors satisfy the condition

$$\frac{\partial E_p^{(c)}}{\partial \mathbf{q}} + \frac{\partial R^{(c)}}{\partial \dot{\mathbf{q}}} = \mathbf{K}_v \mathbf{q}(t) + \mathbf{B}_v \dot{\mathbf{q}}(t) - \mathbf{f}(t) \qquad (4)$$

where

$$\begin{aligned} E_p^{(c)} &= \frac{1}{2}\, k_{ab}({}^a\delta_i^T\, {}^a\mathbf{q}_i - {}^b\delta_j^T\, {}^b\mathbf{q}_j + \Delta_{ab}(t))^2, \\ R^{(c)} &= \frac{1}{2}\, b_{ab}({}^a\delta_i^T\, {}^a\dot{\mathbf{q}}_i - {}^b\delta_j^T\, {}^b\dot{\mathbf{q}}_j + \Delta_{ab}(t))^2 \end{aligned} \qquad (5)$$

are potential and dissipative energy of the coupling. From (4) and (5) we can obtain coupling matrices in form

$$X = x \begin{bmatrix} & \vdots & & \vdots & \\ \cdots & {}^a\delta_i\,{}^a\delta_i^T & \cdots & -{}^a\delta_i\,{}^b\delta_j^T & \cdots \\ \cdots & -{}^b\delta_j\,{}^a\delta_i^T & \cdots & {}^b\delta_j\,{}^b\delta_j^T & \cdots \\ & \vdots & & \vdots & \end{bmatrix}, \quad \begin{matrix} X = K_v\,,\ B_v \\ x = k_{ab},\ b_{ab} \end{matrix} \tag{6}$$

and internal excitation force vector

$$f(t) = \begin{bmatrix} \vdots \\ -\left(k_{ab}\Delta_{ab}(t) + b_{ab}\dot{\Delta}_{ab}(t)\right)\,{}^a\delta_i \\ \vdots \\ \left(k_{ab}\Delta_{ab}(t) + b_{ab}\dot{\Delta}_{ab}(t)\right)\,{}^b\delta_j \\ \vdots \end{bmatrix}. \tag{7}$$

The mathematical model of the gear transmission (Fig.2) can be written as

$$M\ddot{q}(t) + (B + B_v + G)\dot{q}(t) + (K + K_v)\,q(t) = f\,(t) \tag{8}$$

where $M = \mathrm{diag}(M_k)$, $B = \mathrm{diag}(B_k)$, $K = \mathrm{diag}(K_k)$, $G = \mathrm{diag}(G_k)$, are block - diagonal matrices of order $n = \sum_{k=1}^{N} n_k$ and $q\,(t) = [q_k(t)]$ is vector of all generalized coordinates of dimension n . In our case is $N = 2$, $n = n_a + n_b$.

The presented mathematical modelling method can be generalized for systems with arbitrary number of shafts and gear meshings [1], [2].

3. CALCULATION OF DYNAMICS RESPONSE USING REDUCED MODEL

The steady dynamic response of the gear transmission excited by the harmonic transmissions error $\Delta_{ab}(t) = \Delta_z e^{i\omega_z t}$ is characterized by dimensionless complex amplitudes of the generalized coordinates

$$\bar{q}_i(\omega_z) = \frac{q_i(\omega_z)}{\Delta} = (k_{ab} + i\omega_z b_{ab})\, e_i^T q(\omega_z)\ ,\ i = 1,\ 2,\ \ldots,\ n \tag{9}$$

where e_i are unit vectors. Under the conditions that the gyroscopic effect of the discs is neglected and the damping matrices B_k, B_V satisfy the proportionality conditions

$$B_k = c_0 M_k + c_1 K_k,\quad B_V = c_1 K_V \tag{10}$$

vector $q(\omega_z)$ in (9) can be written as

$$q(\omega_z) = \sum_{\nu=1}^{n} v_\nu \frac{v_\nu^T \gamma_Z}{\Omega_\nu^2 - \omega_z^2 + i\,2\,D_\nu \Omega_\nu \omega_z} \tag{11}$$

In (11) $\gamma_Z = [\,..{}^a\delta_i^T..{}^b\delta_j^T..\,]^T$ is vector of geometric parameters of gears in mesh [2]. Parameters Ω_ν, v_ν and D_ν are natural frequencies, eigenvectors of the conservative system corresponding to (8) and relative damping factors satisfy the condition $v_\nu^T\,(B + B_V)v_\nu = 2\,D_\nu\Omega_\nu$.

For calculation of the natural frequencies and eigenvectors in (11) we can used a conservative reduced model [2]. Let modal matrices V_k and diagonal spectral matrices Λ_k of isolated undamped shafts

$$M_k \ddot{q}_k(t) + K_k q(t) = 0\ ,\ k = 1,\ 2,\ \ldots,\ N \tag{12}$$

are divided into submatrices describing m_k master mode shapes (index m) and s_k slave mode shapes (index s) resp.

$$V_k = [\,{}^mV_k, {}^sV_k\,]\ ,\ \Lambda_k = [\,{}^m\Lambda_k,\ {}^s\Lambda_k\,] \tag{13}$$

The generalized coordinate vectors $q_k(t)$ of each shaft can be transformed by appropriate modal matrix of the isolated subsystem into the form

$$q_k(t) = {}^mV_k\,{}^mx_k(t) + {}^sV_k\,{}^sx_k(t),\ k = 1,\ 2,\ \ldots,\ N. \tag{14}$$

Slave modal coordinates can be approximated [3]

$${}^sx_k(t) \approx {}^s\Lambda_k^{-1}\,{}^sV_k^T\,f_k^I(t)\ . \tag{15}$$

Eliminating slave modal coordinates in (14) by means of (15) we get

$$q_k(t) = {}^mV_k\,{}^mx_k(t) + {}^sH_k\,f_k^I(t),\ k = 1, 2, \ldots, N \tag{16}$$

where

$${}^sH_k = {}^sV_k\,{}^s\Lambda_k^{-1}\,{}^sV_k^T,\ k = 1, 2, \ldots, N \tag{17}$$

is so called residual flexibility matrix 4 .

Let us introduce following vectors and matrices of the whole shaft system

$${}^mx(t) = {}^mx_k(t)\ ,\ f_I(t) = f_k^I(t)\ ,\ {}^m\Lambda(t) = \mathrm{diag}({}^m\Lambda_k)\ ,\ {}^mV = \mathrm{diag}({}^mV_k)\ ,\ {}^sH = ({}^sH_k).$$

All equations (16) can then be written in form

$$q\,(t) = {}^mV\,{}^mx\,(t) + {}^sH\,f_I(t). \tag{18}$$

Eliminating vector $f_I(t)$ from (18) according to (3) for $B_v=0$ and $f(t)=0$ (conservative system) we get

$$q\,(t) = (\ I + {}^sH\,K_v)^{-1}\ {}^mV\,{}^mx(t). \tag{19}$$

Master modal coordinates vector ${}^mx(t)$ follows from the reduced mathematic model of order $m = \sum_{k=1}^{m} m_k < n$

$${}^m\ddot{x}(t) + \left[{}^m\Lambda + {}^mV^T K_k (I + {}^sH\,K_v)^{-1}\,{}^mV\right]{}^mx(t) = 0. \tag{20}$$

By means of QL or Jacobi algorithm we can calculate natural frequencies Ω_ν and eigenvectors ${}^mx_\nu$, $\nu = 1, 2, \ldots, m$ of the reduced model (20). Frequencies Ω_ν represent m approximate natural frequencies of whole shaft system. Eigenvectors ${}^mx_\nu$ have to be transformed from the basis of modal coordinates of isolated shafts master modes into the basis of general coordinates by formula

$$v_\nu = (\ I + {}^sH\,K_v)^{-1}\,{}^mV\,{}^mx_\nu\ ,\ \nu = 1, 2, \ldots, m \tag{21}$$

corresponding to (19).

The method enables to gain substantial reduction of number of degrees of freedom of gear transmissions with relatively high accuracy of the dynamic response calculation.

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MODELOWANIE DRGAŃ PRZEKŁADNI ZĘBATYCH

Artykuł przedstawia oryginalną metodę tworzenia liniowo przestrzennych modeli matematycznych walcowych przekładni zębatych obciążonych dynamicznie o postaci funkcji okresowej uwzględniającej błędy przekładni. Modele uwzględniają podstawowe własności walcowej przekładni zębatej, w której przyjęto założenia o ciągłym rozkładzie mas, podatności i tłumienia w łożyskach oraz w zazębieniu. Wpływ własności lepkosprężystych zębów oraz błędów kinematycznych w zazębieniach kół zębatych określany jest w postaci macierzy sprzężenia pomiędzy wałami oraz wektorem wymuszenia kinematycznego. Przy tak przyjętych założeniach w poszczególnych elementch przekładni powstaje złożony stan naprężenia wynikający z drgań giętno-skrętnych. Dyskretyzacji wałów dokonuje się przy wykorzystaniu metody elementów skończonych. Przedstawioną metoda syntezy modalnej zastosowano w celu redukcji liczby stopni swobody, a tym samym skrócenia czasu obliczeń numerycznych. Metoda opiera się na obliczonych częstotliwościach drgań swobodnych i wektorach modalnych przedstawiających postacie modalne wałów niesprzężonych. Wpływ postaci modalnych o wyższych częstotliwościach oceniany jest za pomocą macierzy o podatności szczątkowej.

Metoda ta umożliwia znaczną redukcję liczby stopni swobody przy stosunkowo dużej dokładności obliczeń częstotliwości drgań swobodnych wektorow modalnych i odpowiedzi dynamicznej całej przekładni zębatej. Dokładność metody oceniano uwzględniajac liczbę wzorcowych postaci modalnych dla istniejącej przekładni zębatej.