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ANALIZA DYNAMICZNA WIRNIKÓW ŚRUBOWYCH OPARTA NA METODZIE ELEMENTÓW SKOŃCZONYCH

Streszczenie. W pracy przedstawiono metodę matematycznego modelowania drgań wirników śrubowych posiadających niesymetryczny przekrój poprzeczny, opartą na metodzie elementów skończonych. Wzięto pod uwagę bezwładność rotacyjną, momenty giroskopowe oraz tłumienie.

FEM DYNAMIC ANALYSIS OF THE HELICAL ROTORS

Summary. The FEM based mathematical modelling of vibration of helical rotors with non-symmetrical cross-section including the effects of rotational inertia, gyroscopic moments and damping is presented.

ДИНАМИЧЕСКИЙ АНАЛИЗ ВИНТОВЫХ РОТОРОВ С ПОМОЩЬЮ МКЭ

Резюме. В статье рассматривается применение метода конечных элементов для математического моделирования вибраций винтовых компрессоров с нессимметрическим сечением ротора с учетом моментов инерции, гироскопических моментов и затухания.

1. INTRODUCTION

1D-FEM model of the helical compressor rotor can be accepted [1] as a basis for computational simulation of vibration excited by pressure field of flowing medium and by dynamic unbalance. Taking into account high rotor speed (up to 20000 1/min), it is necessary to consider gyroscopic effects and rotational inertia. Discretization of rotors by FEM respecting presented effects is performed in available literature (e.g. [2, 3, 4]) only for the circular cross-section and it is not possible to apply it to the rotors of helical compressors having non-symmetrical cross-section.

2. MATRICES OF A ROTOR ELEMENT FOR THE GENERAL CASE

In order to investigate the displacement of a rotor element having length l, let us apply a coordinate system ξ, η, ζ which rotates about ξ axis at a constant frequency ω_0. These displacement can be then expressed by translations $\bar{u}(x)$, $\bar{v}(x)$, $\bar{w}(x)$ of neutral axis points and by small angles of rotation about the coordinate axes.

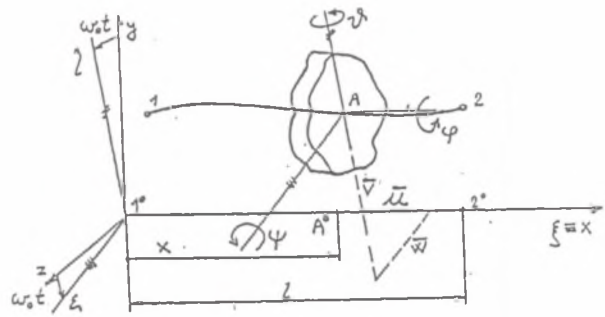

Fig. 1. Scheme of the rotor in the rotating system
Rys. 1. Schemat wirnika w wirującym układzie odniesienia

Displacements $\bar{v}(x)$, $\bar{w}(x)$, $\vartheta(x)$, $\psi(x)$ due to the bending phenomenon are approximated by cubic polynomials, longitudinal $\bar{u}(x)$ and torsional $\varphi(x)$ displacements by linear polynomials. Let us introduce generalized coordinate vectors of nodes 1 ($x=0$) and 2 ($x=l$),

$$\mathbf{q}_1=\begin{bmatrix}\bar{v}(0)\\ \psi(0)\\ \bar{v}(l)\\ \psi(l)\end{bmatrix},\quad \mathbf{q}_2=\begin{bmatrix}\bar{w}(0)\\ \vartheta(0)\\ \bar{w}(l)\\ \vartheta(l)\end{bmatrix},\quad \mathbf{q}_3=\begin{bmatrix}\bar{u}(0)\\ \bar{u}(l)\end{bmatrix},\quad \mathbf{q}_4=\begin{bmatrix}\varphi(0)\\ \varphi(l)\end{bmatrix} \tag{1}$$

Thus the generalized displacements of a rotor element internal points are given by expressions:

$$\bar{v}(x)=\boldsymbol{\Phi}(x)\mathbf{S}_1^{-1}\mathbf{q}_1,\qquad \psi(x)=\frac{\partial v(x)}{\partial x}=\boldsymbol{\Phi}'(x)\mathbf{S}_1^{-1}\mathbf{q}_1 \tag{2a}$$

$$\bar{w}(x)=\boldsymbol{\Phi}(x)\mathbf{S}_2^{-1}\mathbf{q}_2,\qquad \vartheta(x)=\frac{-\partial w(x)}{\partial x}=-\boldsymbol{\Phi}'(x)\mathbf{S}_2^{-1}\mathbf{q}_2 \tag{2b}$$

$$\bar{u}(x)=\boldsymbol{\Psi}(x)\mathbf{S}_3^{-1}\mathbf{q}_3,\qquad \varphi(x)=\boldsymbol{\Psi}(x)\mathbf{S}_3^{-1}\mathbf{q}_4 \tag{2c}$$

where

$$S_1^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l} \\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2} \end{bmatrix},\quad S_2^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -\frac{3}{l^2} & \frac{2}{l} & \frac{3}{l^2} & \frac{1}{l} \\ \frac{2}{l^3} & -\frac{1}{l^2} & -\frac{2}{l^3} & -\frac{1}{l^2} \end{bmatrix},\quad S_3^{-1}=\begin{bmatrix} 1 & 0 \\ -\frac{1}{l} & \frac{1}{l} \end{bmatrix}$$

and

$$\underline{\Phi}(x)=[1,\ x,\ x^2,\ x^3],\qquad \underline{\Psi}(x)=[1,\ x]$$

For the creation of a mathematical model of the element let us cut a volume element in the distance x, the length of which is dx. This element performs a general spatial motion (Fig. 1). This motion can be decomposed into an entraining sliding motion given by the reference point A motion, the radius vector of which is

$$r(x)=\ [\bar{u}(x),\ \bar{v}(x),\ \bar{w}(x)]^T$$

and into a relative spherical motion at current angle speed $\underline{\omega}$ expressed in rotating space $\xi\eta\zeta$ has the approximate form

$$\underline{\omega}(x)=\ [\dot{\varphi}(x)+\omega_0,\ \dot{\vartheta}(x)-\omega_0\psi(x),\ \dot{\psi}(x)+\omega_0\vartheta(x)]^T \tag{3}$$

Kinetic energy of rotor element "e" is given by expression

$$E_k^e=\frac{1}{2}\int_0^l[A(x)\mathbf{v}^{\mathbf{T}}(x)\mathbf{v}(x)+\underline{\omega}^T(x)\mathbf{J}(x)\underline{\omega}(x)]\rho(x)dx \tag{4}$$

where $A(x)$ and $\rho(x)$ are the cross sectional area and density, respectively. The velocity of the reference point

$$\mathbf{v}(x)=\ [\dot{\bar{u}}(x),\ \dot{\bar{v}}(x)-\omega_0\bar{w}(x),\ \dot{\bar{w}}(x)+\omega_0\bar{v}(x)]^T \tag{5}$$

and the matrix of moments of inertia $\mathbf{J}(x)$

$$J(x) = \begin{bmatrix} J_p(x) & 0 & 0 \\ 0 & J_\eta(x) & -D_{\eta\zeta}(x) \\ 0 & -D_{\eta\zeta}(x) & J_\zeta(x) \end{bmatrix}, \quad J_p(x) = J_\eta(x) + J_\zeta(x) \tag{6}$$

were introduced in the expression (4). Potential energy of the rotor element is given by the known expression

$$E_p^e = \frac{1}{2}\int_0^l \underline{\varepsilon}^T(x)\,\mathbf{D}\,\underline{\varepsilon}(x)A(x)\,dx \tag{7}$$

When the shear displacements are neglected, the deformation vector has the form

$$\underline{\varepsilon}(x) = \begin{bmatrix} \varepsilon_x \\ \gamma \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u(x)}{\partial x} \\ \dfrac{\partial \varphi(x)}{\partial x}\sqrt{\eta^2+\zeta^2} \end{bmatrix} \tag{8}$$

where longitudinal displacement of an arbitrary point of the element, which has coordinates x, η, ζ is

$$u(x) = \bar{u}(x) - \eta\,\psi(x) + \zeta\,\vartheta(x) \tag{9}$$

The elasticity matrix has the form

$$\mathbf{D} = \begin{bmatrix} E & 0 \\ 0 & G \end{bmatrix} \tag{10}$$

where E and G are Young's modulus and shear modulus, respectively. A discrete mathematical model of spatial internally undamped vibration of a rotor element we derive from Lagrangian equations in matrix form

$$\frac{d}{dt}\left(\frac{\partial E_k^e}{\partial \dot{\mathbf{q}}^e}\right) - \frac{\partial E_k^e}{\partial \mathbf{q}^e} + \frac{\partial E_p^e}{\partial \mathbf{q}^e} = 0 \tag{11}$$

where the generalized coordinate (nodal displacement) vector of the dimension 12 is defined by

$$\mathbf{q}^e = [\mathbf{q}_1^T, \mathbf{q}_2^T, \mathbf{q}_3^T, \mathbf{q}_4^T]^T \tag{12}$$

Substituting (4) and (7) into (11) and considering all above presented relations, it is possible to derive equation of motion of rotor element

$$\mathbf{M}^e\ddot{\mathbf{q}}^e(t) + \omega_0\mathbf{G}^e\dot{\mathbf{q}}^e(t) + (\mathbf{K}_S^e - \omega_0^2\mathbf{K}_D^e)\mathbf{q}^e(t) = 0 \tag{13}$$

where the coefficient matrices have the form:

— symmetrical mass matrix

$$\mathbf{M}^e = \begin{bmatrix} S_1^{-T}(I_1+I_4)S_1^{-1} & S_1^{-T}I_3S_2^{-1} & 0 & 0 \\ S_2^{-T}I_3S_1^{-1} & S_2^{-T}(I_2+I_4)S_2^{-1} & 0 & 0 \\ 0 & 0 & S_3^{-T}I_5S_3^{-1} & 0 \\ 0 & 0 & 0 & S_3^{-T}I_6S_3^{-1} \end{bmatrix} \tag{14a}$$

— antisymmetrical matrix of gyroscopic effects

$$\omega_0\mathbf{G}^e = \omega_0 \begin{bmatrix} 0 & -S_1^{-T}(I_1+I_2+2I_4)S_2^{-1} & 0 & 0 \\ S_2^{-T}(I_1+I_2+2I_4)S_1^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{14b}$$

— symmetrical static stiffness matrix

$$\mathbf{K}_S^e = \begin{bmatrix} S_1^{-T}I_7S_1^{-1} & S_1^{-T}I_9S_2^{-1} & 0 & 0 \\ S_2^{-T}I_9S_1^{-1} & S_2^{-T}I_8S_2^{-1} & 0 & 0 \\ 0 & 0 & S_3^{-T}I_{10}S_3^{-1} & 0 \\ 0 & 0 & 0 & S_3^{-T}I_{11}S_3^{-1} \end{bmatrix} \tag{14c}$$

and symmetrical dynamic stiffness matrix

$$-\omega_0^2\mathbf{K}_D^e = -\omega_0^2 \begin{bmatrix} S_1^{-T}(I_1+I_4)S_1^{-1} & S_1^{-T}I_3S_2^{-1} & 0 & 0 \\ S_2^{-T}I_3S_1^{-1} & S_2^{-T}(I_2+I_4)S_2^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{14d}$$

In presented matrices we introduced auxiliary matrices

$$
\begin{aligned}
&\mathbf{I}_1 = \int_0^l J_\zeta(x)\,\underline{\Phi}'^T(x)\,\underline{\Phi}'(x)\,\rho(x)\,dx, && \mathbf{I}_7 = \int_0^l EJ_\zeta(x)\,\underline{\Phi}''^T(x)\,\underline{\Phi}''(x)\,\rho(x)\,dx,\\
&\mathbf{I}_2 = \int_0^l J_\eta(x)\,\underline{\Phi}'^T(x)\,\underline{\Phi}'(x)\,\rho(x)\,dx, && \mathbf{I}_8 = \int_0^l EJ_\eta(x)\,\underline{\Phi}''^T(x)\,\underline{\Phi}''(x)\,\rho(x)\,dx,\\
&\mathbf{I}_3 = \int_0^l D_{\eta\zeta}(x)\,\underline{\Phi}'^T(x)\,\underline{\Phi}'(x)\,\rho(x)\,dx, && \mathbf{I}_9 = \int_0^l ED_{\eta\zeta}(x)\,\underline{\Phi}''^T(x)\,\underline{\Phi}''(x)\,\rho(x)\,dx,\\
&\mathbf{I}_4 = \int_0^l A\,\underline{\Phi}^T(x)\,\Phi(x)\,\rho(x)\,dx, && \mathbf{I}_{10} = \int_0^l EA\,\underline{\Psi}'^T(x)\,\Psi'(x)\,dx,\\
&\mathbf{I}_5 = \int_0^l A\,\underline{\Psi}^T(x)\,\underline{\Psi}(x)\,\rho(x)\,dx, && \mathbf{I}_{11} = \int_0^l GJ_P\,\underline{\Psi}'^T(x)\,\underline{\Psi}'(x)\,dx\\
&\mathbf{I}_6 = \int_0^l J_P\,\underline{\Psi}^T(x)\,\underline{\Psi}(x)\,\rho(x)\,dx
\end{aligned}
\tag{15}
$$

3. MATRICES OF THE ROTOR ELEMENT FOR SPECIAL CASES

In the simplest case of a cylindrical rotor element is $D_{\eta\zeta}(x) = 0$, $J_\eta(x) = J_\zeta(x) = J$, $J_P(x) = J_P = 2J$ and obviously $\mathbf{I}_1 = \mathbf{I}_2$, $\mathbf{I}_7 = \mathbf{I}_8$ and $\mathbf{I}_3 = \mathbf{I}_9 = 0$. Consequently $\mathbf{M}^e$, $\mathbf{K}_S^e$, $\mathbf{K}_D^e$ are the block diagonal matrices. A linkage between $\xi\eta$ plane $\xi\zeta$ plane bending vibration exist only of gyroscopic effects.

I the case of helical compressor rotors, the helical part of rotor (macroelement) of length l is subdivided into N rotor elements. Each element is defined by cross sectional product of inertia and moments of inertia.

$$
\begin{aligned}
D_{\eta\zeta}(x) &= \frac{J_\zeta^i - J_\eta^i}{2}\sin\frac{4\pi x}{s} + D_{\eta\zeta}^i\cos\frac{4\pi x}{s}\\
J_\eta(x) &= J_\zeta^i\sin^2\frac{2\pi x}{s} + D_{\eta\zeta}^i\sin\frac{4\pi x}{s} + J_\eta^i\cos^2\frac{2\pi x}{s}\\
J_\zeta(x) &= J_\zeta^i\cos^2\frac{2\pi x}{s} - D_{\eta\zeta}^i\sin\frac{4\pi x}{s} + J_\eta^i\sin^2\frac{2\pi x}{s}
\end{aligned}
$$

Here variables with the subscript of node "i" correspond to cross-section on the left edge and s is the lead of the helix.

4. MATHEMATICAL MODEL OF THE ROTOR

Introducing a generalized displacement vector of rotor nodes

$$\mathbf{q} = [\ldots, \bar{u}_i, \bar{v}_i, \psi_i, \bar{w}_i, \vartheta_i, \varphi_i, \ldots]^T \tag{16}$$

a mathematical model of free vibrating rotor supported by discrete isotropic supports has in rotating space the form

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{B}_B + \omega_0\mathbf{G})\dot{\mathbf{q}}(t) + (\mathbf{K}_S - \omega_0^2\mathbf{K}_D + \mathbf{K}_B)\mathbf{q}(t) = 0 \tag{17}$$

where matrices of isolated shaft $\mathbf{M}$, $\mathbf{G}$, $\mathbf{K_S}$, $\mathbf{K_D}$ have the band structure. The matrices represent transformed matrices of rotor elements $\mathbf{T}^T\mathbf{X}\mathbf{T}$, where $\mathbf{X} \in \{\mathbf{M}^e, \mathbf{G}^e, \mathbf{K}_S^e, \mathbf{K}_D^e\}$ and the transformational matrix $\mathbf{T}$ exchanges only rows and columns of the original element matrices. $\mathbf{B_B}$ and $\mathbf{K_B}$ are damping and stiffness matrices of supports.

5. CONCLUSION

Mathematical 1D FEM model (17) of the non-symmetrical cross-section rotor includes an influence of gyroscopic effects and inertia. As a result of the non-symmetrical cross-section, the equation of the motion in the matrix form is derived in rotating space. The advantage of modelling in rotating space is that the coefficient matrices depend on the operating speed only and are time independent.

1D FEM model can be suitably used for a modal analysis of coupled rotors of the helical compressors and simulation of vibration excited by pressure field and dynamic unbalance.

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Streszczenie

Praca poświęcona jest modelowaniu drgań wirnika sprężarki śrubowej, charakteryzującego się niesymetrycznym przekrojem poprzecznym. Duża prędkość obrotowa wirnika sprawia, iż w modelowaniu należy wziąć pod uwagę efekty giroskopowe, jak również bezwładność związaną z ruchem obrotowym. Do dyskretyzacji zastosowano metodę elementów skończonych. Należy zaznaczyć, że przykłady literaturowe ([2, 3, 4]) dotyczą jedynie przekrojów symetrycznych.

W pracy wyprowadzono równanie (w postaci macierzowej) drgań swobodnych wirnika w przestrzeni współrzędnych układu wirującego wraz z wirnikiem. W efekcie przyjęcia takiego układu współrzędnych macierze-współczynniki zależą jedynie od prędkości wirowania i są niezależne od czasu.

Przedstawiony model nadaje się do analizy modalnej, jak również do symulacji drgań wymuszonych sprężarki.