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STABILNOŚĆ DYNAMICZNA PRĘTA CIENKOŚCIENNEGO OBCIĄŻONEGO MIMOŚRODOWO

Streszczenie. W referacie przedstawiono algorytm określania mapy obszarów stabilności prętów, oparty na metodzie różnic skończonych. Pokazano przykładowy wynik dla pręta cienkościennego obciążonego mimośrodowo.

THE DYNAMIC STABILITY OF A THIN-WALLED BAR SUBJECTED TO AN ECCENTRIC LOAD

Summary. A finite-difference based algorithm to determine a stability chart for bars is presented in the paper. An example result for a thin-walled bar subjected to an eccentric load is shown.

ДИНАМИЧЕСКАЯ УСТОЙЧИВОСТЬ ТОНКОСТЕННОГО СТЕРЖНЯ ПРИ ВНЕЦЕНТРЕННОМ ДЕЙСТВИИ НАГРУЗКИ

Резюме. В работе представлен алгоритм определения карты устойчивости стержней, основан на методе конечных разностей. Показан примерный результат для тонкостенного стержня нагруженного внецентренно.

1. INTRODUCTION

The so called technical theory of thin-walled bars of open cross-section has been proposed in the work [4]. This theory describes dynamics of the bar subjected to the load shown in Fig. 1 by four partial differential equations with variable coefficients. Three of these equations are coupled (cf. [4, 2, 1]). The uncoupled one defines longitudinal vibrations and is identical with the vibration equation of a prismatic bar. This equation is not to be analyzed here.

A general, analytical solution of the above mentioned system of equations is not known, however, for particular cases of a bar's geometry, arrangement of the load and boundary conditions, the system could be substantially simplified. In the extreme case, the stability investigation leads to an analysis of a system of Mathieu equations (cf. [1, 3]). If the conditions of the problem don't allow to uncouple the system and the exciting force is given by the formula

$$P = P_m(1 + \alpha \sin\omega t) \tag{1}$$

it is possible to apply an algorithm of finding stability regions of the thin-walled bar, taking advantage of properties of ordinary differential equations with periodic coefficients. A disadvantage of such an approach is a call for a decomposition of the exciting force into harmonics. The transition from the system of PDEs to the system of ODEs could be done by the Galerkin method (cf. [2, 3]) or applying the finite-difference method (cf. [9])[1]. It should be emphasized, that practical examples involved in [2, 3, 9] concern only the principal region of instability.

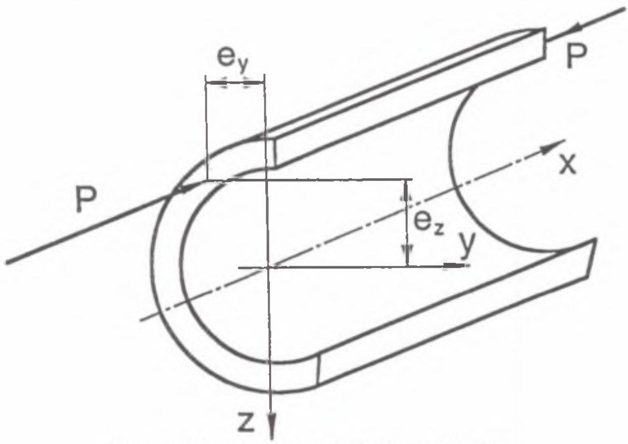

Fig. 1. An axially loaded thin-walled bar
Rys. 1. Osiowo obciążony pręt cienkościenny

A different approach is propounded in the present paper. The direct stability analysis gives a way to solving equations of dynamics for varying parameters of the load and judging (according to obtained responses of chosen variables) whether a given point of the input parameters state, e.g. (P_m, ω, α), is an element of the stability region or is not. Repetition of this operation for changed parameters comes up with a stability chart with respect to these parameters.

There are a few ways to find approximate solutions of the equations of the thin-walled bar's dynamics. A method based on power series is presented in papers [5, 6]. Assumption about displacement functions to be in form of power series though, is acceptable only for a harmonic load. Such a limitation doesn't exist in the finite-difference method. This method was employed in papers [7, 8] to solve an eigenproblem of a thin-walled bar of variable cross-section. On the other hand, the

[1] The paper [9] is not devoted to thin-walled bars, however, the problem touched there is also described by a system of coupled partial differential equations.

authors of [9] show an example of a numerically obtained displacement function for the midpoint of the shaft. Derivatives with respect to time have been approximated by the predictor–corrector method.

Numerous exemplifications on the subject for statics and dynamics, but first of all the simple idea of the finite–difference method have persuaded the author to be based on the method in the present paper as well as in the dissertation [1]. The most complicated case considered in [1] concerns an I–beam subjected to an eccentric load, but it seems to be admissible to put a hypothesis about effectiveness of the method in the general case. Much attention was paid to the comparison of results obtained for a bar subjected to a central load with corresponding analytical ones.

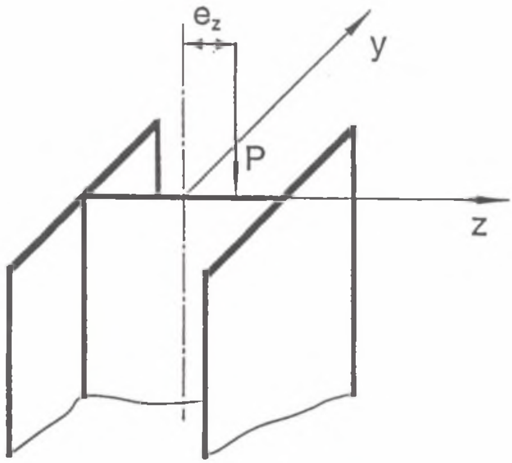

Fig. 2. An I–beam subjected to an eccentric load
Rys. 2. Dwuteownik obciążony mimośrodowo

2. A BISYMMETRIC THIN–WALLED BAR SUBJECTED TO AN ECCENTRIC LOAD

The dynamic stability of a thin–walled bar which cross–section has two axes of symmetry, loaded as shown in Fig. 2 is described by two[2] equations (cf. [2, 4, 1])

$$\begin{aligned} &EI_z\frac{\partial^4\eta}{\partial x^4} + \rho F\frac{\partial^2\eta}{\partial t^2} + b\frac{\partial\eta}{\partial t} + P\frac{\partial^2\eta}{\partial x^2} - e_zP\frac{\partial^2\varphi}{\partial x^2} = 0 \\ &EI_\omega\frac{\partial^4\varphi}{\partial x^4} + \rho F r^2\frac{\partial^2\varphi}{\partial t^2} + b_T\frac{\partial\varphi}{\partial t} + (r^2P - GI_o)\frac{\partial^2\varphi}{\partial x^2} - e_zP\frac{\partial^2\eta}{\partial x^2} = 0 \end{aligned} \tag{2}$$

[2] Disregarding longitudinal vibrations as well as vibrations in direction of z-axis ($I_z>I_y$).

where

η – displacement of the shear centre in direction of y–axis,
φ – angle of torsion about x–axis,
F – cross–sectional area,
$I_z = \int y^2\, dF$, $I_\omega = \int \omega^2\, dF$,
$I_o = \alpha/3\ \Sigma\ h_i \delta_i^3$ (cf. [4]),
ω – sectorial coordinate,
E – Young's modulus,
G – shear modulus,
ρ – density of material,
b – damping coefficient for longitudinal vibration,
b_T – damping coefficient for torsional vibration,
$r^2 = (I_y + I_z)/F + y_D^2 + z_D^2$,
y_D, z_D – coordinates of the shear centre.

3. THE FINITE–DIFFERENCE METHOD

Let us decompose the set of equations (2) into a system of first–order equations

$$
\begin{aligned}
v &= \frac{\partial \eta}{\partial t} \\
\vartheta &= \frac{\partial \eta}{\partial x} \\
\bar{M} &= \frac{\partial \vartheta}{\partial x} \\
\bar{T} &= \frac{\partial \bar{M}}{\partial x} \\
EI_z \frac{\partial \bar{T}}{\partial x} &+ \rho F \frac{\partial v}{\partial t} + bv + P\bar{M} - e_z P \bar{B} = 0 \\
\Omega &= \frac{\partial \varphi}{\partial t} \\
\Theta &= \frac{\partial \varphi}{\partial x} \\
\bar{B} &= \frac{\partial \Theta}{\partial x} \\
\bar{H} &= \frac{\partial \bar{B}}{\partial x} \\
EI_\omega \frac{\partial \bar{H}}{\partial x} &+ \rho F r^2 \frac{\partial \Omega}{\partial t} + b_T \Omega + (Pr^2 - GI_o)\bar{B} - e_z P \bar{M} = 0
\end{aligned}
\tag{3}
$$

As far as derivatives with respect to x are concerned, let us approximate them by central differences.

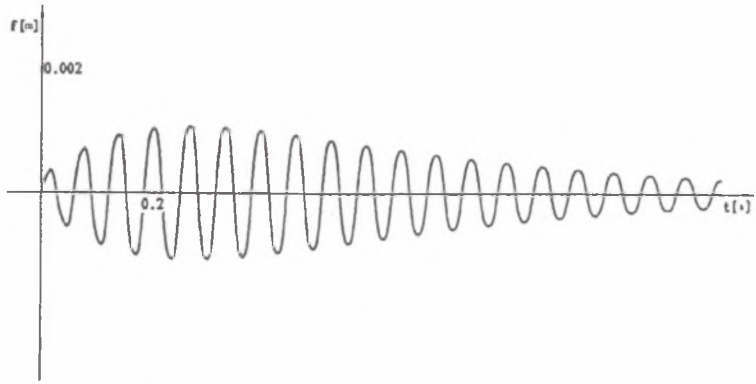

Fig. 3. An example of the displacement function
Rys. 3. Przykładowy przebieg przemieszczenia

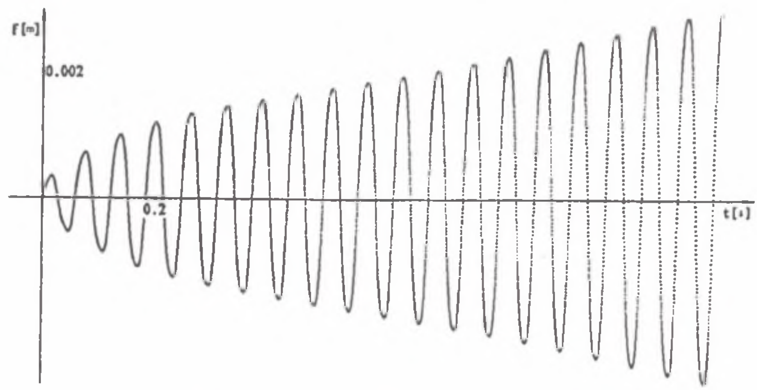

Fig. 4. An example of the displacement function (loss of stability)
Rys. 4. Przykładowy przebieg przemieszczenia (utrata stabilności)

For example, the angle of deflection could be approximated as follows[3]

$$\vartheta_{i+1}^{k} = \frac{1}{2h}(\eta_{i+1}^{k+1} - \eta_{i+1}^{k-1}) \tag{4}$$

where h is the length of the segment $(k, k+1)$.

As for derivatives with respect to time we'll use *the implicit method* (the Crank-Nicholson method, cf. [10]).

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[3] The superscript depicts a point along the bar, the subscript — a moment of time.

Thus for the first of equations (3) we have

$$\frac{\eta_{i+1}^{k} - \eta_{i}^{k}}{\tau} = \frac{1}{2}(v_i^k + v_{i+1}^k) \tag{5}$$

where τ depicts the time step length.

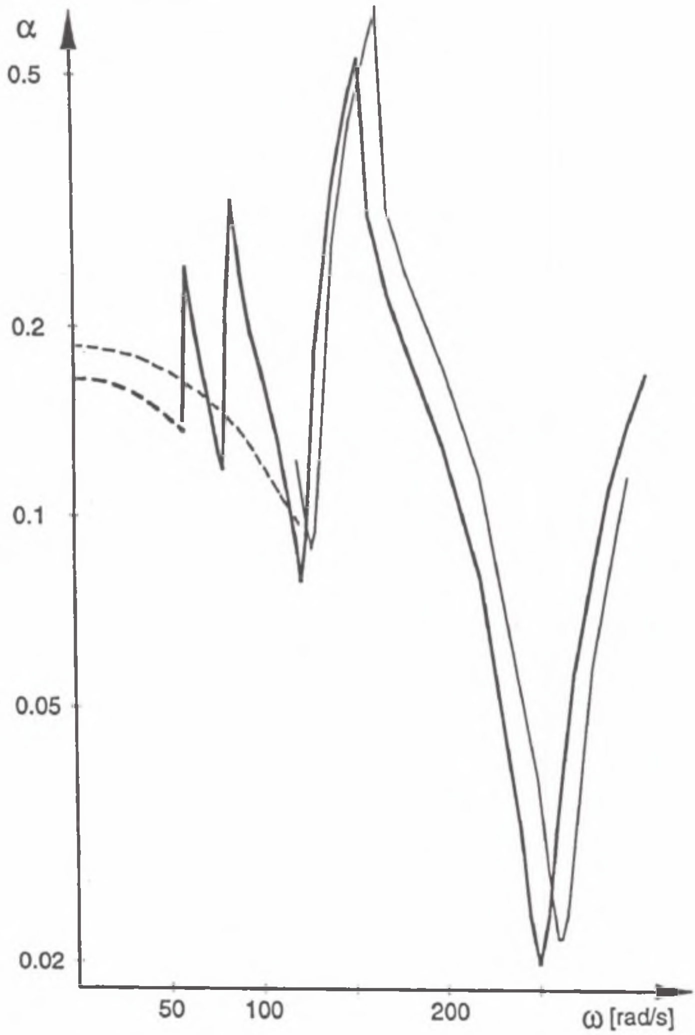

Fig. 5. A stability chart with respect to ω and α
Rys. 5 Mapa stabilności ze względu na ω i α

As the effect of the above described approximation, instead of the system (3) we get a system of linear, algebraic equations. After having presumed boundary conditions in the form

$$\begin{aligned} &\underset{i}{\forall}\ \eta_i^1 = \eta_i^N = 0 \qquad (N = \max(k)) \\ &\underset{i}{\forall}\ \varphi_i^1 = \varphi_i^N = 0 \\ &\underset{i}{\forall}\ \bar{M}_i^1 = \bar{M}_i^N = 0 \\ &\underset{i}{\forall}\ \bar{B}_i^1 = \bar{B}_i^N = 0 \end{aligned} \tag{6}$$

as well as non-trivial initial conditions ($v \neq 0$, $\Omega \neq 0$) and after having substituted numerical values of geometric and material constants, solving the system step by step, we come into possession of output functions of desirable variables.

Examples of these functions are shown in Fig. 3 and Fig. 4. Being based on such output graphs[4] we decide about stability (or about lack of stability).

Changing some parameters and repeating the reported procedure, it is possible to generate a stability chart with respect to these parameters.

A stability chart with respect to ω and α, in the case when the exciting force is given by the formula (1) is shown in Fig. 5. The thick line depicts the border between the stability region and the instability region for a bar subjected to an eccentric load. The thin line is the border in the case of a central load. The influence of the shift of the force application point on stability is clearly visible.

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Streszczenie

Dynamika pręta cienkościennego o profilu otwartym w ogólnym przypadku opisana jest przez układ czterech równań różniczkowych cząstkowych, z których trzy są ze sobą sprzężone. W przypadku gdy przekrój poprzeczny rozpatrywanego pręta cienkościennego posiada dwie osie symetrii, a siła osiowa nie jest przyłożona centralnie, to drgania giętno-skrętne opisane są układem dwu równań sprzężonych (2). W odniesieniu do pręta o wymiarach takich jak założono w niniejszej pracy, równania te określają możliwość wystąpienia wyboczenia. W pracach [2, 3, 9] podany jest algorytm określania mapy obszarów stabilności, wykorzystujący własności równań różniczkowych o okresowych współczynnikach. W niniejszej pracy przedstawiona jest odmienna metoda badania stabilności. Opiera się ona na rozwiązywaniu układu równań (2) metodą różnic skończonych. Otrzymuje się wtedy wykresy przebiegów wybranych zmiennych, na przykład przemieszczenia środka ścinania przekroju leżącego w połowie długości pręta. Przykładowe przebiegi pokazane są na rys. 3 oraz 4. Powtarzając obliczenia dla różnych wielkości parametrów, można utworzyć mapę obszarów stabilności ze względu na te parametry. Na rys. 5 pokazana jest mapa stabilności ze względu na parametry siły wymuszającej (porównaj równanie (1)) ω i α. Na rysunku tym można zauważyć wpływ przesunięcia punktu przyłożenia siły na stabilność.