ZESZYTY NAUKOWE POLITECHNIKI ŚLĄSKIEJ 1993

Seria: MECHANIKA z. 116 Nr kol. 1231

Natalija PANKRATOWA
Institute of Mechanics
Ukrainian Academy of Sciences

STAN NAPRĘŻEŃ GRUBOŚCIENNYCH ELEMENTÓW STOŻKOWYCH

Streszczenie. Rozpatruje się pewne podejście do rozwiązania zadań o stanie naprężeń w stożkowych elementach konstrukcji, formowanych na drodze zbrojenia przez nawinięcie. Metoda jest oparta na racjonalnym połączeniu równań teorii sprężystości niejednorodnego ciała anizotropowego, przekształcenia analitycznego i na metodach analizy numerycznej. Rozpatrywana jest postać anizotropii, gdy materiał ma jedną płaszczyznę symetrii sprężystej, prostopadłą do powierzchni stożka. Obliczenia pozwoliły ujawnić efekty, uwarunkowane niejednorodnością i anizotropią materiału,rodzajami zbrojenia i obciążeń.

STRESSED STATE OF THICK-WALLED LAMINATED CONIC ELEMENTS

Summary. An approach is presented to solution of problems of stressed state of anisotropic hollow conic elements with one plane of elastic symmetry fabricated by winding. Constitutive relations in this approach are equations of three-dimensional problem of elasticity theory. By means of various analytic al transformations three-dimensional problem is accurately reduced to one dimensional problem which is solved by stable numerical method. Computations allowed to find effects preconditioned of non-homogeneity and anisotropy of elastic properties of materials and types of applied loads.

НАПРЯЖЕННОЕ СОСТОНИЕ ТОЛСТОСТЕННЫХ СЛОИСТЫХ КОНИЧЕСКИХ ЭЛЕМЕНТОВ

Резюме. Приводится подход к решению задач о напряженном состоянии анизотропных полых коничэских элементов, образованных путем армирования. Подход к решению базируется на уравнениях теории упругости неоднородного анизотропного тела, аналитических преобразованих н методах численного анализа. При этом рассматривается вид анизотропии, когда материал имеет в каждой точке тела одну плоскость упругой симметрии, перпендикулярнуюобрщзующим поверхноцти конуса. Проведенные исследоваиия позволили выявить эффекты, обусробленные неоднородностью и анизотропией упругих свойств материала, видами нагружений.

1. INTRODUCTION

When determining stress level and deformability of conical structural elements of composite materials allowance should be made for their characteristics, as for example, anisotropy and non-homogeneity of mechanical parameters, the manner of layers conjugation, types of loading, etc. Among the available methods of solving the problems we should distinguish those on deformation of elements of anisotropic materials having one plane of elastic symmetry. Of practical significance is the fact that the solution obtained for those problems can be used for the calculation of hollow conical structural elements of an anisotropic materials when the main directions of elasticity do not coincide with those of coordinates. This takes place for structural elements made by winding. For thick- walled conical elements it is important to find a rational scheme of reinforcement. One can purposefully influence the strength and stiffness of structure changing the scheme of reinforcement by angular and radial coordinates. On the basis of the developed approach to the determination of the stress-strain state of anisotropic cone, the author studies the influence of reinforcements angle on the stress level and deformability of a specific conical structural.

2. INVESTIGATION OF THE INFLUENCE OF THE WINDING ANGLE ON THE STRESS-STRAIN OF A HOLLOW LAMINATED CONE

2.1. A calculation model for anisotropic cones

A model for stress-strain state calculation an anisotropic infinite multilayered cone is proposed. Bounding surfaces and surfaces of the conjugation layers are coaxial conical having a single vertex. The author studies the influence of the winding angle on stress and displacements through the thickness of laminated anisotropic cones in the context of a three-dimensional problem of the elasticity theory. The cone is assigned to spherical coordinate system φ, r, θ whereas the solution is sought in the area $\varphi \in [\varphi_O, \varphi_n]$, $r \in [O, 2\pi]$, where φ_O, φ_n are conical bounding surfaces. Consider the case when in each point of the given layer of non-homogeneous cone there is one plane of elastic symmetry perpendicular to the conical surface generating lines.

Let us represent the equations of the generalized Hookers law for the i-oh layer as [1]

$$\bar{e}^i = B^i \bar{\sigma}^i + \bar{f}^i, \quad \bar{e}^i = \{e^i_\varphi, e^i_\theta, e^i_r, e^i_{r\varphi}, e^i_{r\theta}, e^i_{\varphi\theta}\};$$
$$B^i = \|b^i_{lk}(\varphi)\|, \; b^i_{lk} = b^i_{kl} = (k, l = 1, 2, ..., 6), \tag{1}$$
$$b_{m5} = b_{m6} = b_{5m} = 6m = 0 \quad (m = 1, 2, 3, 4);$$
$$\bar{\sigma}^i = \{\sigma^i_\varphi, \sigma^i_r, \sigma^i_\theta, \tau^i_{r\varphi}, \tau^i_{r\theta}, \tau^i_{\varphi\theta}\}; \quad \bar{f}^i = \{\alpha^i_\varphi T, \alpha^i_\theta T, \alpha^i_r T, \alpha^i_{r\varphi} T, \alpha^i_{r\theta} T, \alpha^i_{\varphi\theta} T\}.$$

Here $e_\varphi{}^i$, $e_\theta{}^i$,..., $e_\varphi\theta^i$ - are the strain tensor components, $\sigma_\varphi{}^i$, $\sigma_\theta{}^i$,..., $\tau_{\varphi\theta}{}^i$ - are the stress tensor components. Elastic constants $b_{lk}{}^i$, coefficients of linear thermal expansion $\alpha_\varphi{}^i$, $\alpha_\theta{}^i$, $\alpha_r{}^i$ in the direction φ, θ, r,coefficients of temperature shift $\alpha_{r\varphi}{}^i$, $\alpha_{r\theta}{}^i$, $\alpha_{\varphi\theta}{}^i$ are the functions of the coordinate φ, this makes it possible to take into account arbitrary variation of material properties,through the cone thickness.

Relations (1) also hold for an orthotropic cone whose main directions of elasticity are turned around the normal to the surface r = const by the angle ß. In this case elastic constants $b_{lk}{}^i$ are defined from corresponding characteristics of an anisotropic material by equations [1].

$$\sigma^i_\varphi = \sigma^{i+1}_\varphi, \quad \tau^i_{r\varphi} = \tau^{i+1}_{r\varphi}, \quad \tau^i_{\varphi\theta} = \tau^{i+1}_{\varphi\theta},$$
$$u^i_\varphi = u^{i+1}_\varphi, \quad u^i_r = u^{i+1}_r, \quad u^i_\theta = u^{i+1}_\theta. \tag{2}$$

Two models of conjugation layers into a single packet are considered. In most cases there is a rigid contact when the cone layers are deformed without slipping and tearing.

In many cases those conditions may be violated and desalination zones may appear. When friction forces are small not taken into account it is possible to formulate a model for the ideal slippage of layers in those zones

$$\sigma_\varphi^i = \sigma_\varphi^{i+1}, \quad \tau_{r\varphi}^i = \tau_{r\varphi}^{i+1} = \tau_{\varphi\theta}^i = \tau_{\varphi\theta}^{i+1} = 0$$
$$u_\varphi^i = u_\varphi^{i+1}, \quad u_r^i \neq u_r^{i+1}, \quad u_\theta^i \neq u_\theta^{i+1}. \tag{3}$$

On conic bounding surfaces the loads applied are given by polynomial laws

$$\sigma_\varphi^p(\varphi_p, r, e) = r^{n-1}\sigma_\varphi^p(\varphi_p), \quad \tau_{r\varphi}^p(\varphi_p, r, e) = r^{n-1}\tau_{r\varphi}^p(\varphi_p),$$
$$\tau_{\varphi\theta}^p(\varphi_p, r, \theta) = r^{n-1}\tau_{\varphi\theta}^p(\varphi_p) \quad (p = 0, N;\ n = 1, 2, \ldots) \tag{4}$$

Take into consideration the equations of equilibrium, the expressions for deformation by displacements, the Hook law for non-homogeneity anisotropy body the resolving system of differential equations for definite of stressed state of laminated hollow cones is received. The resolving functions are taking as basis with the help of which we can formulate the conditions on limiting surfaces $\varphi=\varphi_0$, $\varphi=\varphi_n$ and surfaces of conjugation of layers (2-4). Making a series transformation a system of resolving equations in partial derivatives for i-oh cone layer is obtained

$$\frac{\partial \bar{\sigma}^i}{\partial \varphi} = \sum_{m=1}^{6} C_m^i \bar{\sigma}_m^i + \bar{g}^i, \quad C_m^i = \|c_{p,m}^i(\varphi)\|,$$
$$\bar{\sigma}^i = \{\sigma_\varphi^i, \tau_{r\varphi}^i, \tau_{\varphi\theta}^i, u_\varphi^i, u_r^i, u_\theta^i\}, \quad \bar{g}^i = \{g_1^i, g_2^i, \ldots, g_6^i\},$$
$$\bar{\sigma}_1^i = \bar{\sigma}^i, \bar{\sigma}_2^i = \frac{\partial \bar{\sigma}^i}{\partial r}, \quad \bar{\sigma}_3^i = \frac{\partial \bar{\sigma}^i}{\partial \theta}, \quad \bar{\sigma}_4^i = \frac{\partial^2 \bar{\sigma}^i}{\partial r \partial \theta}, \quad \bar{\sigma}_5^i = \frac{\partial^2 \bar{\sigma}^i}{\partial r^2}, \quad \bar{\sigma}_6^i = \frac{\partial^2 \bar{\sigma}^i}{\partial \theta^2}$$
$$(\varphi_{i-1} \le \varphi \le \varphi_{i+1};\ i = 1, 2, \ldots, N;\ p, q, m = 1, 2, \ldots, 6) \tag{5}$$

The vector components g^i are defined by the temperature field and depend on the properties of the layers material as well.

By representing the unknown functions and acting loads as expansion in terms of orthogonal trigonometric functions, on separating the variable in (5) we get a system of ordinary differential equations for each layer i and the magnitude of the harmonic number for the case of axially symmetric deformation

$$\frac{d\bar{\sigma}}{d\varphi} = A(\varphi)\bar{\sigma} + \bar{f}, \quad \bar{\sigma} = \{\sigma_{\varphi}, \tau_{r\varphi}, \tau_{\varphi\theta}, u_{\varphi}, u_r, u_{\theta}\},$$

$$\bar{f} = \{f_1, f_2,..., f_6\}, \quad A(\varphi) = \|a_{pq}(\varphi)\|, \quad p, q = 1, 2,..., 6 \tag{6}$$

Here nonzero elements of matrix A and vector f take the form [2].

The boundary problem described by a system of differential equations (6) is solved by a stable numerical method that makes it possible to solve one-dimensional boundary problems with a feasible range of accuracy.

2.2. **Calculation of the stressed state of a laminated hollow cone**

The above approach to handling a problem in three- dimensional formulation is used to study the influence of change in the winding angle through thickness on the stress level and deformability of a conical element made of material with elastic constants $E_{\varphi}=1.63E_0$, $E_{\theta}=1.6E_0$, $E_{\varphi}=20.1E_O$, $\nu_{\varphi\theta}=0.543$, $\nu_{r\varphi}=0.324$, $\nu_{\theta r}=0.024$, $G_{\varphi r}=G_{\varphi\theta}=0.0878E_0$.

A hollow cone is subjected to axially symmetric pressure $\sigma=\sigma_0 r^n$ applied to the surface $\varphi=\varphi_0=\pi/18$.

Consideration is being given to single-, two-, three-, five- layers cones whose layers are reinforced with fibers at equal and opposite angles with the axis r, i. e. the main elasticity directions are turned relative to the coordinate lines φ and θ by the angle β in layer 1, 3, 5 and by the angle $-\beta$ in layers 2 and 4.

The calculations were performed with the following initial data $\pi/2 \leq \varphi_N \leq \pi/4$, $\beta_0=\pi/12$, $\beta_N=\pi/6$, $\pi/4$, $\pi/3$, $5\pi/12$, $\pi/2$; n=2, 7.

Figure 1 gives the distribution of stress in the vicinity of conjugation surfaces of the first and second layers (by solid lines) and on the surface of applied loads (by one-dashed lines) of five layers cone for various values of parameters β, which characterizes the variability of winding. As evident from the solution as the thickness φ_N and the angle β increase so does the contribution of stress and displacements induced by the material anisotropy whereas for winding angles considered above this is due to the discrepancy between the main directions of elasticity and the directions of coordinate lines. When the angle β is close to zero and $\pi/2$

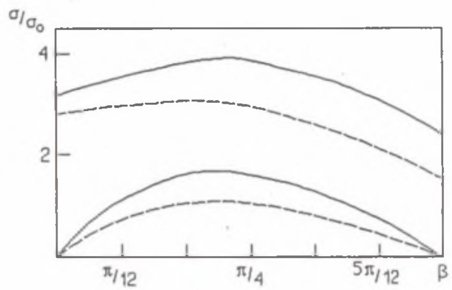

Fig.1. Distribution of stress
Rys.1. Rozkład naprężeń

the influence of u_θ, $\tau_{r\theta}$, $\tau_{\varphi\theta}$ on other factors of the strain-stress state is insignificant. A change in the angle ß through the thickness of the cone may lead to of stress qualitatively different distribution of the stress-strain state.

The investigation performed revealed that when determining the stress-strain state in cones of advanced materials account must be taken of specific features due to material non-homogeneity and anisotropy.

REFERENCES

[1] Lehnitskij S.G.: Theory of elasticity of anisotropy body. Nauka, Moskwa, 1977. In Russia.

[2] Pankratova N.D.: The solution of problems of the stressed state of anisotropic laminated cone. Dokladu AS Ukraine, N10, 1988. In Russia.

[3] Grigorenko Y.N., Vasilenko A.T., Pankratova N.D.: Problems of elasticity theory of non-homogeneity bodies. Nauk. Dumka. Kiev, 1991. In Russia.

Recenzent: Prof. dr hab. inż. Eugeniusz Świtoński

Wpłynęło do Redakcji w grudniu 1993 r.

Streszczenie

Motywacją techniczną do opracowania metod rozwiązania problemów deformacji elementów wykonanych z materiałów anizotropowych o jednej płaszczyźnie symetrii sprężystej jest potrzeba dokonywania obliczeń konstrukcyjnych elementów wytworzonych z materiałów ortotropowych metodą nawijania.

W przypadku grubościennych elementów stożkowych ważnym zagadnieniem jest znalezienie racjonalnego schematu zbrojenia. Sterując tym schematem w kierunku promieniowym i obwodowym można wpływać na wytrzymałość i sztywność konstrukcji. Zaproponowana metoda rozwiązywania zagadnień dotyczących stanu naprężeniowo-odkształceniowego elementów stożkowych wykonanych z materiału, który w każdym punkcie posiada jedną płaszczyznę symetrii sprężystej prostopadłą do powierzchni stożka, może służyć jako podstawa badań wpływu kąta zbrojenia na naprężenia i odkształcenia.