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CALCULATION OF THE STRESSED STATE OF NONHOMOGENEOUS PLATES WITH NONRIGID CONJUGATION OF LAYERS

Summary. Themodel is designated for calculation of the stressed state of orthotropic layer plates with nonrigid conjugation of layers. This model is based on solution of the three dimensional problem of the elasticitytheory. In accordance with the model transversal stresses turn to zero in the region of layers slicing and tangential displace ments under take breaking off crossing the surface of the conjugation. Conditions of continuity for normal stresses and displacements are carried out.

OBLICZANIE STANU NAPRĘŻENIA PŁYT WARSTWOWYCH O NIESZTYWNYM STYKU WARSTW

Streszczenie. W pracy zaproponowano model obliczeń stanu naprężeń warstwowych płyt ortotropowych o niesztywnym styku warstw na podstawie równań zadania przestrzennego teorii sprężystości. Odpowiednio do zaproponowanego modelu, w strefach rozwarstwienia i gładkości powierzchni styku naprężenia styczne zerują się. W wyniku powyższego przemieszczenia styczne przy przejściu przez powierzchnię sprzężenia warstw doznają nieciągłości. Przy tym dla składowych naprężeń normalnych i przemieszczeń odpowiadajacych im spełnione są warunki ciągłości.

РАСЧЕТ НАПРЯЖЕННОГО СОСТОЯНИЯ НЕОДНОРОДНЫХ ПЛИТ ПРИ НЕЖЕСТКОМ СОЕДИНЕНИИ СЛОЕВ

Резюме. Предлагается модель росчета напряженного состояния слоистых ортотропных плит при нежестком контакте слоев на основе решения уравнений пространственной задачи теории упругости. В соответствии с указанной моделью в зонах расслоения вследствие гладкости поверхности контакта касательные напряжения обращаются в нуль, вследствие чего касательные перемещения терпят разрыв при переходе через поверхность сопряжения слоев. При этом для нормалных составляющих напряжений и перемещений выполняются условия напрерывности.

1. INTRODUCTION

Problems of stress distribution in the nonhomogeneous layer environment, in particular, in multilayer plates with nonhomogeneous layers are the subject of numerous investigations. Substantial interest to such a group of problems can be explained, on the one hand, by the necessity of taking into consideration nonhomogeneous of the material and the solution of the problem on the basis of three dimensional elasticity equations and, on the other hand their application to a some practical problems. Cases when the layers in the considered orthotropic environment work jointly with out separation and slip were investigated in more detail. Considerations of the nonrigid conjugation of layers for the nonhomogeneous anisotropic environment essentially complicated the problem with its further numerical realisation. Here an approach to definition of the stress-deformed state of layer plates with nonrigid layer conjugation on the basis of the solution of equations of the threedimensional problem of theory of elasticity is proposed.

2. THE STRESS-DEFORMED STATE OF A NONHOMOGENEOUS PLATE

2.1. The approach to calculation of layers plate

The solution of the problems in the Decart system of coordinates x,y,z is found in the region $0 \le x \le \alpha$; $0 \le y \le \beta$; $z_i \le z \le z_{i+1}$, i=1,2,...,N, N-a number of layers. The butt-ends of the plate x=0, x=a; y=0, y=b are fixed in definite way. Different boundary conditions may be set on surfaces z=0, z=h. The conditions when loads act on the indicated surfaces are the most extended these

$$\sigma_z^0 = -q_z^-(x,y),\quad \tau_{xz}^0 = -q_x^-(x,y),\quad \tau_{yz}^0 = -q_y^-(x,y)$$
$$\sigma_z^N = q_z^+(x,y),\quad \tau_{xz}^N = q_x^+(x,y),\quad \tau_{yz}^N = q_y^+(x,y). \tag{1}$$

The stressed state of the nonhomogeneous plate is defined by normal $\sigma_x,\sigma_y,\sigma_z$ and tangential $\tau_{xy},\tau_{xz},\tau_{yz}$ stresses, displacements of the plate u_x,u_y,u_z, acting in directions x,y,z. In the general case elastic characteristics of the plate correspond orthotropic materials. Some times zones of layer separation appear when compatible contacts of layers are broken. If the forces of friction are small and can be neglected it is necessary to apply the model of ideal slipping of layers along the entire surface of the contact. In accordance with this model in the zones of layer seperation be came of the smoothness of contact surface tangential displacements suffer breaking off during crossing through the surface of layer contacts. For normal components of stresses σ_z^i and displacement u_z^i the conditions of continuity are fulfiled during crossing through the surface of conjugation

$$\sigma_z^i = \sigma_z^{i+1}, \quad u_z^i = u_z^{i+1}; \quad \tau_{xz}^i = \tau_{xz}^{i+1} = \tau_{yz}^i = \tau_{yz}^{i+1} = 0. \tag{2}$$

By taking the resolving functions as basis with the help of which we can formulate the conditions on limiting surfaces z=0, z=h and surfaces of conjugation and fullfiling transformations in the initial equations of the theory of elasticity the resolving system of equations in partial derivatives is received [1]. Solution of the received system is to satisfy conditions (1) and conditions of conjugation of layers which in the general case are expressed

$$T_i\bar{\sigma}^i(z_i) - S_i\bar{\sigma}^{i+1}(z_{i+1}) = 0, \tag{3}$$

where T_i, S_i - the given matrixies of size 6x6, elements of them take different values in dependence from the way of conjugation of layers. When the contact of layers is rigid $T_i = S_i = E$, where E - single matrix.

For this contact of layers a choice of resolving functions permit automatically continuously to spend problem solution on the entire interval of integration. Presentation of all factors of the stress-deformed state in double trigonometrical rows allows to strictly divide variables and for each pair of values m,n for i layer to receive a system of 6 ordinary differential equations with variable coefficients [2].

$$\frac{d\bar{\sigma}^i_{mn}}{dz} = B(z)\bar{\sigma}^i_{mn} + \bar{f}^i, \ B(z)=\|b_{ij}(z_i)\|, \ i,j=1,2,\dots,6; \ \bar{f}=\{f_1,\dots,f_6\}, \tag{4}$$

$$\sigma^i_{mn} = \{\sigma^i_{z,mn}, \tau^i_{xz,mn}, \tau^i_{yz,mn}, u^i_{z,mn}, u^i_{x,mn}, u^i_{y,mn}\}.$$

Integration of equtions (4) taking into considerations (1), (2), (3) made by means of the steady numerical method that allows to receive the solution with high degree of precision. For the case of ideal slipping after division of variables conditions (3) are expressed

$$\sigma^i_{z,mn}(z_i) = \sigma^{i+1}_{z,mn}(z_i); \ u^i_{z,mn}(z_i) = u^{i+1}_{z,mn}(z_i); \tag{5}$$

$$\tau^{i}_{xz,mn}(z_i)\ \tau^{i+1}_{xz,mn}(z_i)=\tau^{i}_{yz,mn}(z_i)=\tau^{i+1}_{yz,mn}(z_i)=0 \quad (i=1,2,\dots,N-1). \tag{6}$$

Function $u^{i}_{x,mn}(z_i)$, $u^{i}_{y,mn}(z_i)$, which are solutions of the equations (4) in crossing the surface of the contact of the layers are to undergo a sudden jump of such a value

$$u^{i}_{x,mn} - u^{i+1}_{x,mn} = \Delta u^{i}_{x,mn}; \quad u^{i}_{y,mn} - u^{i+1}_{y,mn} = \Delta u^{i}_{y,mn}, \tag{7}$$

to fullfil conditions (6) with respect to tangential stresses. This formulation leads to a multipoints boundary problem for the solution which transformations that can complement the generally acknowledged driving methods may be used. Here the solution of the considered class of problems to spend by reducing them to the solution of a number of two-point problems that allows to use the previously worked out apparatus of numerical solution.

In connection with that conditions (5)-(7) are expessed

$$\sigma^{i+1}_{mn}(z_i) = \bar{\sigma}^{i}_{mn}(z_i) + \bar{\sigma}^{i}_{0,mn}; \qquad \bar{\sigma}^{i}_{0,mn}=\{0,0,0,0,\Delta u^{i}_{x,mn},\Delta u^{i}_{y,mn}\} \tag{8}$$

where $\Delta u_{x,mn}$, $\Delta u_{y,mn}$ - are the unknown values so far.

Interating the one dimensional boundary problem for the system of the equations (4) taking into consideration of the conditions on the limiting surfaces (1) and the conditions of conjugation of the layers (8) a solution of the considered class problems is received as a linear function of values $\Delta u^{i}_{x,mn}$, $\Delta u^{i}_{y,mn}$ which are difined from the solution of two systems of linear algebraic equations in conformity with conditions (6)

$$\begin{gathered}\tau^{i}_{yz,mn}(z_i) + \sum_{j=1}^{N-1}(\Delta u^{j}_{y,mn}\tau^{i}_{yz,mn}(z_i,\Delta u^{j}_{y,mn})+\Delta u^{j}_{x,mn}\tau^{i}_{yz,mn}(z_i,\Delta u^{j}_{x,mn}))=0\\ \tau^{i}_{xz,mn}(z_i) + \sum_{j=1}^{N-1}(\Delta u^{j}_{x,mn}\tau^{i}_{xz,mn}(z_i,\Delta u^{j}_{x,mn})+\Delta u^{j}_{y,mn}\tau^{i}_{xz,mn}(z_i,\Delta u^{j}_{y,mn}))=0\ .\end{gathered} \tag{9}$$

After determing values $\Delta u^{i}_{x,mn}$, $\Delta u^{i}_{y,mn}$ from (9) the final solution of the problem is received.

2.2. Calculation of two-layers plate

On the basis the worked out method the influence of layer connection types with due account of nonhomogenity and anisotropy of elastic properties of materials on the stressed state of the two-layers plate is studied. The case is investigated when the butt-ends of the plate x=0, x=α; y=o, y=β do not displace in this plane and free from normal load. The problem on the definition of stressed state of the nonhomogenity plate the internal layer of which is made from orthotropic material with elastic constants

$a_{11}=E_0^{-1}/138$; $a_{22}=a_{33}=E_0^{-1}/9.7$; $a_{12}=a_{13}= -0.31E_0^{-1}/138$; $a_{23}= -0.5E_0^{-1}/9.7$;

$a_{44}=E_0^{-1}/3.2$; $a_{55}=a_{66}=E_0^{-1}/6.9$;

E_0 - const is considered.

The external layer of the plate is made from the material turned to 90^0 to the material of the internal layer with elastic materials

$a_{11}=E_0^{-1}/9.7$; $a_{22}=E_0^{-1}/138$; $a_{33}=E_0^{-1}/9.7$; $a_{12}=-0.31E_0^{-1}/9.7$; $a_{13}=-0.5E_0^{-1}/9.7$;

$a_{23}=-0.31E_0^{-1}/138$; $a_{44}=a_{66}=E_0^{-1}/6.9$; $a_{55}=E_0^{-1}/3.2$.

The plate is under localised load that changes with the law

$$\sigma_z = \sigma_0 Sin\frac{N\ \pi x}{\alpha} \cdot Sin\frac{N\ \pi y}{\beta} . \tag{10}$$

Calculations are fullfiled for $\alpha=\beta=20h_0$; $h^{(i)}=h^{(e)}=2h_0$; h_0= const; N=9. All linear sizes are refered to unit of length egual to h_0. The some results of the solution of this problem is diven in fig.1. Here the values corresponding the nonrigid contact are shown by solid lines, the rigid one--dashed by the lines. Distributions of tangential stresses of the transverse shear τ_{xz} in the section x=9, $9h_0$, $y=10h_0$ and τ_{yz} in the section $x=10h_0$, y= $9,9h_0$ on the thickness of the plate are given. Here lines with number 1 are related to τ_{xz} and with number 2 - to τ_{yz}. The given rezults display the necessity to take in tò consideration effects preconditioned by the mode of the layers conjugation with due account of nonhomogenity and anisotropy of elastic properties of materials and types of here applied loads.

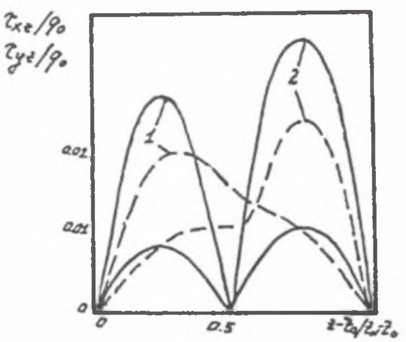

Fig. 1. Distribution of stress
Rys. 1. Rozkład naprężeń

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Streszczenie

W pracy zaproponowano model obliczeń warstwowych płyt ortotropowych w stanie napreżeniowo-odkształconym przy niesztywnym styku warstw. Celem uwzględnienia dowolnej zmienności niejednorodności sprężystych własności materiału na grubości płyty warstwowej w charakterze wyjściowym obiera się równania trojwymiarowej teorii spreżystości niejednorodnego ciała anizotropowego.

W wielu przypadkach, gdy spójność warstw jest zaburzona,powstają strefy rozwarstwienia. Gdy przy tym siły tarcia są małe i można je pominąć, to należy wykorzystać model idealnego poślizgu warstw po całej powierzchni styku. Odpowiedno do modelu rozpatrywane są warunki ciągłości naprężeń i przemieszczeń normalnych przy przejściu przez powierzchnie styku przemieszczeń stycznych. Takie podejście prowadzi do wielopunktowego zagadnienia brzegowego, do którego rozwiazania można wykorzystać przekształcenia wprowadzane w zwykłe metody brzegowe. W tym przypadku rozwiązanie klasy zadań sprowadza się do rozwiązania dwupunktowego szeregu, którego całkowanie wykonuje się stabilną metodą analizy numerycznej. Przedstawione obliczenia niejednorodnych płyt ortotropowych dają jakościową i ilościową interpretację wyników w zależności od rodzaju obciążenia, materiału warstw i ich sposobu połączenia.