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ON ONE ELASTIC SHELL DEFORMATION MODEL

Summary. The model is designated for calculation of the stress-deformed state of nonhomogeneous shell which makes it possible to takeinto account turnings of fibres, their distortion, geodesy torsion and change of their lengths under deformation. The approach to constructions a theory of shells was found by using the method of weight residuals.

ОБ ОДНОЙ МОДЕЛИ ДЕФОРМИРОВАНИЯ УПРУГОЙ ОБОЛОЧКИ

Резюме. Предлагается модель расчета напряженно-деформированного состояния неоднородной оболочки, позволяющая учитывать повороты волокон, их искривления, геодезическое кручение и изменение длины в процессе деформации. Подход к построению теории основан на использовании метода взвешенных невязок.

O PEWNYM MODELU DEFORMOWANIA SPRĘŻYSTEJ POWŁOKI

Streszczenie. Proponuje się model obliczenia stanu naprężeń i odkształceń niejednorodnej powłoki, który pozwala uwzględniać kąty obrotów włókien, ich skrzywienie, geodezyczne skręcanie i zmianę długości w procesie deformowania. Podejście do skonstruowania teorii powłok opiera się na metodzie zważonych odchyleń.

1. INTRODUCTION

The solution of the boundary of the three-dimensional elastic theory for shells with any geometrical and mechanical parameters and with any boundary conditions is considered to be a complicated problem. Therefore one can understand a lot of reseachers who by using special properties of shells tried to lowes the dimension of unitiable boundary problem. At the present time we have some well-known deformation models of shells. Further this model is suggested and it allows to take into account not only turnings of fibres but also their distortion, geodesy torsion and change of their lengths under deformation [1].

2. THE STRESS-DEFORMED STATE OF ELASTIC SHELLS

2.1. The model for calculation of the deformed stste of orthotropic plates

The approach proposed for constructing the theory shells was based on the method of weight residuals [2]. We can explain its work at the example of constructing a theory for orthotropic plates with constant thickness and with the given static boundary conditions on the lateral surfaces ($z = \mp h/2$)

$$\sigma^{\pm}_{i3} = q^{\pm}_{i} \tag{1}$$

We assume the following expressions for the components of the vector of the displacement

$$u_i = u_i^{(0)} P_0 + \gamma_i P_1 + \theta_i P_2 + \varphi_i P_3, \quad i=1,2; \qquad u_3 = wP_0 + \gamma_3 P_1 + \theta_3 P_2 + \varphi_3 P_3 \;\; \psi_3 P_4, \tag{2}$$

where $\bar{u}^{(0)} = (u, v, w)$, $P_k(\xi)$ - Legander's polinoms, $\xi = 2z/h$.

Six of the thirteen unknown functions depending of the coordinates x and y can by expressed through remaining seven functions using the boundary conditions (1). When substituting obtained the received formulus in the equations of equilibrium of elasticity theory in displacements and composing seven conditions of orthogonal residuals and weight functions by using Legander's polynomes we will receive the necessary system of differentional equations in partial derivatives of a smaller dimensions. The received system of differentional equations of the common fourteenth order is divided into some systems of a lower order which describe symmetrical and antisymmetrical stress-deformed states (SDS).

Thus conditions (1) give that

$$\theta_1 = \frac{q_1 h}{12G_{13}} - \frac{h}{6}(\gamma_{3,1} + \underline{\varphi_{3,1}}), \quad \varphi_1 = \frac{m_1}{12G_{13}} - \frac{h}{12}(\varepsilon_{13} + \theta_{3,1} + \underline{\psi_{3,1}}), \tag{3}$$

where $\varepsilon_{13} = \omega_{,1} + (2/h)\gamma_1$, $q_1 = q_1^+ - q_1^-$, $m_1 = (h/2)(q_1^+ + q_1^-)$.

When the conditions (1) are employed and the integral values are introduced

$$T_0 = \int_{-h/2}^{h/2} \sigma_{33} dz \quad \text{and} \quad M_0 = \int_{-h/2}^{h/2} \sigma_{33} z dz,$$

we will find $\sigma_{33} = \frac{T_0}{h} P_0 + \frac{6M_0}{h^2} P_1 + \frac{m_3 - T_0}{h} P_2 + \left(\frac{q_3}{2} - \frac{6M_0}{h^2}\right)$.

From $e_{33} = \sigma_{33}/E_3^* - \nu_{31}^* e_{11} - \nu_{32}^* e_{22}$ it followes

$$\gamma_3 = \frac{T_0}{2E_3^*} - \frac{h}{2}(\nu_{31}^*\varepsilon_1 + \nu_{32}^*\varepsilon_2) - \varphi_3, \quad \theta_3 = \frac{M_0}{E_3^* h} - \frac{h}{6}(\nu_{31}^*\kappa_1 + \nu_{32}^*\kappa_2) - \psi_3,$$

$$\varphi_3 = \frac{m_3 - T}{10E_3^*} - \frac{h}{10}(\nu_{31}^*\theta_{1,1} + \nu_{32}^*\theta_{2,2}), \quad \psi_3 = \frac{q_3 h}{28E_3^*} - \frac{3M_0}{7E_3^* h} - \frac{h}{14}(\nu_{31}^*\varphi_{1,1} + \nu_{32}^*\varphi_{2,2}), \tag{4}$$

where $\varepsilon_1 = u_{,1}$, $\varepsilon_2 = v_{,2}$, $\kappa_1 = \gamma_{1,1}$, $\kappa_2 = \gamma_{2,2}$.

From the summary one can see that the functions $u, v, w, \gamma_1, \gamma_2, \varphi_3, \psi_3$ should be used as seven unknown functions dependent on the coordinates x and y.

We will find equlibrium equations of an element of the plate when the orthogonal conditions for residuals of the first two equations of equlibrium from the three-dimensional elastic theory and Legander's polynomes P_0 and P_1 are constructed and residuals of the third equation of equlibrium and Legander's polynomes P_0, P_1 and P_2 are constructed

$$T_{1,1} + S_{,2} + q_1 = 0, \quad S_{,1} + T_{2,2} + q_2 = 0, \quad Q_{1,1} + Q_{2,2} + q_3 = 0$$
$$M_{1,1} + H_{,2} - Q_1 + m_1 = 0, \quad H_{,1} + M_{2,2} - Q_2 + m_2 = 0 \tag{5}$$

$$(h^2/6)(\sigma_{13,1}^{(1)} + \sigma_{23,2}^{(1)}) + m_3 - T_0 = 0, \quad (h^2/60)(\sigma_{13,1}^{(2)} + \sigma_{23,2}^{(2)}) + (h^2/12)q_3 - M_0 = 0 \tag{6}$$

The boundary at the edges can by easily received the orthogonal conditions for residuals of the boundary conditions on lateral surface of the plate and Legander's polynomes are constructed. We will find the following expression for transverse stresses by using correlations (3)

$$\sigma_{13}=\frac{Q_1}{h}P_0+\frac{q_1}{2}P_1\mp\frac{m_1-Q_1}{h}P_2+\underline{G_{13}[\varphi_{3,1}(P_3-P_1)+\psi_{3,1}(P_4-P_2)]} \tag{7}$$

where

$$Q_1=\frac{5}{6}G_{13}h\varepsilon_{13}+\frac{m_1}{6}-\frac{G_{13}}{6}(\theta_{3,1}+\underline{\psi_{3,1}}). \tag{8}$$

In the work [1] it is shown that functions Q_i and V_i (i=1,2) characterise the normal curvature of fibres which are orthogonal before deformation to the middle flat plate. It can be easily seen that throwing aside the underlined members in (3) means neglegt of the influence of changes on the coordinates x and y for the components

$$e_{33}^{(2)}=(10/h)\varphi_3 \quad \text{and} \quad e_{33}^{(3)}=(14/h)\psi_3, \quad e_{33}\left(e_{33}=\sum_{k=0}^{3}e_{33}^{(k)}P_k\right)$$

deformation on distortion of these fibres. This leads to throwing aside the underlined members in (7), i.e. to V.V.Novogilov's and E.Ressner's static hypotheses. Here the general order of the system of differentional equations (5-6) lowers up to ten and the equation (6) becomes the correlation for the integral values T_0 and M_0. Employing elastic correlations for an orthotropics body in the

$$\sigma_{11}=E_1^0(e_{11}\ \nu_{12}e_{22})+\nu_{31}^{*}\sigma_{33} \quad (1\Leftrightarrow2), \quad \sigma_{12}=G_{12}e_{12},$$

where $E_1^0=E_1/(1-\nu_{12}\nu_{21}),\nu_{31}^{*}=(\nu_{31}+\nu_{21}\nu_{32})/(1-\nu_{12}\nu_{21})$, we will find elastic correlations for the plate $T_1=E_1^0h(\varepsilon_1+\nu_{12}\varepsilon_2\ \ \nu_{31}^{*}T_0$,

$$M_1=E_1^0(h^2/6)(\kappa_1+\nu_{12}\kappa_2)+\nu_{31}^{*}M_0, \quad (1\Leftrightarrow2), \quad S=G_{12}h\omega, \quad H=G_{12}(h^2/6)\tau.$$

We have expressions for transverse stresses from the correlations (8)

$$Q_1=(5/6)G_{13}h\varepsilon_{13}+m_1/6-\underline{(G_{13}/6)\theta_{3,1}} \tag{9}$$

Correlations between deformations and displacements will be found by direct calculation from respective correlations of the three-dimensional elastic theory. If me exlude the underlined members in (9) we will have the same resolving system of differential equations for the definition of the five unknown functions as in S.P.Timoshenko's shear model.

After defining these functions we can find Q_i and V_i by using of correlations (3) and (4), from (4) - G_3, Q_3, V_3, J_3, from (2)- displacements u_1,u_2,u_3, all SDS plate's components can be expressed.

2.2.Calculation of the cylindrical shell

As an example the problem of the closed cylindrical shell which is freely fixed at the edges and loaded by inner pressure $\bar{q}_3=-q_0 SIN(nps/l)$ [3] is being examined. In the table the results comparison for the exact solution (I) and approximate solutions are found on the basis of the classical theory (II), S.P.Timoshenko's model (III) and the proposed model (IV) are produced for the shell with such parametres : $R=60R_0$, $l=120R_0$, $h=3R_0$, $n=20$, $\nu=0,3$.

Table 1

The rezults comparison of calculation for the different theories

	I	II	III	IV
$\bar{\sigma}^+_{22}$	0,861	0,815(5,3%)	0,823(4,4%)	0,877(1,8%)
$\bar{\sigma}^-_{22}$	-0,960	-0,636(33,8%)	-0,568(40,8%)	-0,979(2,0%)
$\bar{\sigma}^+_{11}$	2,,47	2,42 (2,0%)	2,39 (3,2%)	2,54 (2,8%)
$\bar{\sigma}^-_{11}$	-2,71	-2,42 (10,7%)	-2,39 (11,8%)	-2,75 (1,5%)
$\bar{u}^+_3$	7,43	2,42 (67,4%)	8,84 (19,0%)	7,45 (0,3%)
$\bar{u}_3(0)$	8,38	2,42 (71,1%)	8,84 (5,5%)	8,25 (1,6%)
$\bar{u}^-_3$	8,89	2,42 (72,8%)	8,84 (0,6%)	8,82 (0,8%)
$\bar{\sigma}=\sigma/q_0$, $\bar{u}_3=E_0 u_3/q_0 R_0$				

In the conclusion we should like to note that t he boundary conditions accommodate exatly on the lateral surfaces of the plate and in the Sen-Venan's sense of the edges of the shell in accordance with the constructed theory, i.e. non-selflevel components of stresses and displacements are taken into account. We will be able to take into account the selflevel components of the order which depends on increasing number of the refrained members if the number of the members in introduction increases. In such a case supplementary members of the frontier layer type appear in the solution.

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Streszczenie

W celu jasności zaproponowano podejście opisane na przykładzie konstrukcji teorii dla ortotropowych płyt, na których bocznych powierzchniach zadaje się statyczne warunki brzegowe (1). Przyjmujac wyrazy dla komponentów wektora przemieszczeń w postaci (2), wyrażamy 6 z 13 niewiadomych funkcji za pośrednictwem pozostałych 7 funkcji, korzystając z warunków brzegowych (1). Wstawiając otrzymane wyrażenia w przemieszczeniowe równania równowagi teorii sprężystości i zostawiajac 7 umów ortogonalności odchyleń i wagowych funkcji, otrzymamy poszukiwany układ równań rożniczkowych mniejszego wymiaru.

Otrzymany układ równań cząstkowych 14 rzędu w omawianym przypadku rozpada się na układ niższego rzedu, który opisuje symetryczny i antysymetryczny stan naprężeń i odkształceń. Możliwość realizacji modelu i wiarygodność otrzymanych rezultatów zostały zilustrowane w tablicy, w której są zestawione rezultaty rozważanego zagadnienia w przestrzennym podejściu (I), na podstawie klasycznej teorii (II) , modelu S.P. Timoszenko (III), jak rownież zaproponowanego modelu (IV). Zestawienie rezultatów pozwala sądzić o wysokiej precyzji rezultatów, które otrzymano na podstawie zaproponowanego modelu.