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APPLICATION OF A TRANSFER MATRIX OF PROBLEMS OF DYNAMICS OF THIN - WALLED BARS

Summary. The paper deals with the solution of problems of free vibrations of thin - walled bars with open profile and variable section under any initial and boundary conditions when appling a transfer matrix.

ZASTOSOWANIE MACIERZY PRZENIESIENIA DO ZAGADNIEŃ DYNAMICZNYCH PRĘTÓW CIENKOŚCIENNYCH

Streszczenie. W pracy przedstawiono rozwiązanie problemów drgań swobodnych prętów cienkościennych o profilu otwartym i zmiennym przekroju przy dowolnych warunkach początkowo-brzegowych metodą macierzy przeniesienia.

ПРИМЕНЕНИЕ МАТРИЦЫ ПЕРЕНОСА К ДИНАМИЧЕСКИМ ВОПРОСАМ ТОНКОСТЕННЫХ ПРУТКОВ

Резюме. В статье приводится решение проблем колебаний свободных тонкостенных прутков открытого профиля и переменного цечения в любых начало - крайных ысловиях методом матрицы переноса.

1. INTRODUCTION

A theory of thin - walled bars evolved by И.Wagnera [17] and developed by A. Ostenfeld [13], F. Bleich i H. Bleich [1], R. Kappus [7], E.E. Lindquist i C.M. Fligg [10], and then systematized and generalized by

W.Z. Własow [19] is useful for many practical applications[6, 18, 8, 2]. Most cases being subject of elaborations cover certain specific geometrical features of a section [11, 16] or special states of load [2, 9]. Problems under consideration relate in general to bars with constant section or to bars with a predetermined function of changes in geometrical features of the bar. Numerical methods [12, 14, 4, 5, 6, 3], are in the main applicable for bars with any change in the sections, the method of finite elements being here used most often. The application of the method of finite elements for a theory of thin - walled bars needs elaborating of a method for discretization of such a bar and for determining of a shape function Calculations of this type are in general laborious and of rather small use from the practical point of view. A method of a transfer matrix has found much wider application for stereomechanical calculations of thin - walled bars [15]. The method of a transfer matrix consist in determining the matrix called the transfer matrix and this is a product of a span matrix and of section matrix. The span matrix is being defined on the basis of the solution of a given problem concerning the bar with constant or variable section. Whereas the conditions of equilibrium and of inseparability of displacement are the basis for determining the section matrix. The algorithm of calculations presented in the paper has been used to determine charakteristic parameters of a thin - walled bar with a wariable section. It can be generalized with regard to other stereomechanical problems.

2. CHARACTERISTIC VALUES AND EIGENFUNCTIONS OF THIN - WALLED BARS

2.1. Introduction

In case of thin walled bars critical forces P_{kr} and frequencies of free vibrations ω_n are charakteristic values. Functions of displacements of bending axes corresponding to these values are called eigenfunctions.

The problem of free vibrations is a homogeneous problem and thus it resolves itself into defining a form a characteristic determinant and then into determining of its charaktrristic values.

When solving the problem of free vibrations within the present work it has been assumed that the diagram is that of a bar under load of the force P acting centrally. Bars with any change in the section are modelled by means of bars with section being segmentally constant.

The method of a transfer matrix has found a wide application in stereomechanical calculations for such bars.

3. DETERMINING OF A SPAN MATRIX

The basic differential equations for the problem of free vibrations of a thin - walled bar with open profile and constant section being under load of the force P acting centrally here the form :

$$EA \frac{\partial \zeta}{\partial x^2} - \frac{\gamma A}{g} \frac{\partial \zeta}{\partial t^2} = 0$$

$$EJz \frac{\partial^4 \eta}{\partial x^4} - \frac{\gamma Jz}{g} \frac{\partial^4 \eta}{\partial x^2 \partial t^2} + \frac{\gamma A}{g} \frac{\partial^2 \eta}{\partial t^2} + P \frac{\partial^2 \eta}{\partial x^2} + \frac{\gamma A\ z\alpha}{g} \frac{\partial^2 \varphi}{\partial t^2} -$$

$$+ P\ z\alpha \frac{\partial^2 \varphi}{\partial x^2} = 0$$

$$EJy \frac{\partial^4 \xi}{\partial x^4} - \frac{\gamma Jy}{g} \frac{\partial^4 \xi}{\partial x^2 \partial t^2} + \frac{\gamma A}{g} \frac{\partial^2 \xi}{\partial t^2} + P \frac{\partial^2 \xi}{\partial x^2} + \frac{\gamma A\ y\alpha}{g} \frac{\partial^2 \varphi}{\partial t^2} -$$

$$- P\ y\alpha \frac{\partial^2 \varphi}{\partial x^2} = 0$$

$$\frac{\gamma A\ z\alpha^2}{g} \frac{\partial \eta}{\partial t^2} + P\ z\alpha^2 \frac{\partial \eta}{\partial x^2} - \frac{\gamma A\ y\alpha^2}{g} \frac{\partial \xi}{\partial t^2} - P\ y\alpha^2 \frac{\partial \xi}{\partial x^2} +$$

$$+ EI\omega - \frac{\partial^4 \varphi}{\partial x^4} - \frac{\gamma J\omega}{g} \frac{\partial^4 \varphi}{\partial x^2 \partial t^2} + \frac{\gamma A r^2}{g} \frac{\partial^2 \varphi}{\partial t^2} - GJx \frac{\partial^2 \varphi}{\partial x^2} + P\ r^2 \frac{\partial^2 \varphi}{\partial x^2} = 0$$

(3.1)

where:

$$r^2 = \frac{Jo}{A} + y\alpha^2 + z\alpha^2, \qquad Jo = Jz + Jy$$

The first equation of the system (3.1) represents a differential equation of free longitudinal vibrations of a bar. Three successive equations of system (3.1) are coupled differential equations of free transverse and torsional vibrations. As result of the solution of three equations the functions of displacements $\eta(x,t)$, $\xi(x,t)$, $\varphi(x,t)$ will be defined.

Due to applying the method of Poissona, distribution the functions of displacements $\eta(x,t)$, $\xi(x,t)$, $\varphi(x,t)$ - in case of free vibrations - will be expresed in the following form:

$$\begin{aligned}
\eta(x,t) &= \sum_n \eta_n(x)\sin\omega_n t\ ,\\
\eta(x,t) &= \sum_n \xi_n(x)\sin\omega_n t, \\
\varphi(x,t) &= \sum_n \varphi_n(x)\sin\omega_n t\ ,
\end{aligned} \tag{3.2}$$

where : ω_n - n-th angular frequency of free vibrations of the bar (charakteristic value)

$\eta_n(x)$, $\xi_n(x)$, $\varphi_n(x)$ - n-th functions of displacements of a deformed axis of the bar, called hereinafter eigenfunctions

The expansion of the function of displacements $\eta_n(x,t)$, $\xi_n(x,t)$, $\varphi_n(x,t)$ to power series having a form:

$$\begin{aligned}
\eta_n(x) &= \sum_{j=0}^{j=r} \Theta_j^n x^j\ ,\\
\xi_n(x) &= \sum_{j=0}^{j=r} \Psi_j^n x^j\ ,\\
\varphi_n(x) &= \sum_{j=0}^{j=r} K_j^n x^j\ ,
\end{aligned} \tag{3.3}$$

has been used to solve the system of equations (3.1).

The first four factors of each series (3.3) are boundary values of corresponding functions $\eta_n(x)$, $\xi_n(x)$, $\varphi_n(x)$ multiplied by numbers : one, two and six respectivelly.

After a number of transformations have been made, it is possible to express the functions of displacements in the form :

$$\begin{aligned}
\eta_n(x) &= S_1^n\Theta_0^n + S_2^n\Theta_1^n + S_3^n\Theta_2^n + S_4^n\Theta_3^n + S_5^n\Psi_0^n + S_6^n\Psi_1^n + S_7^n\Psi_2^n + S_8^n\Psi_3^n + \\
&\quad + S_9^nK_0^n + S_{10}^nK_1^n + S_{11}^nK_2^n + S_{12}^nK_3^n\ ,\\
\xi_n(x) &= S_{13}^n\Theta_0^n + S_{14}^n\Theta_1^n + S_{15}^n\Theta_2^n + S_{16}^n\Theta_3^n + S_{17}^n\Psi_0^n + S_{18}^n\Psi_1^n + S_{19}^n\Psi_2^n + \\
&\quad + S_{20}^n\Psi_3^n + S_{21}^nK_0^n + S_{220}^nK_1^n + S_{23}^nK_2^n + S_{24}^nK_3^n\ ,\\
\varphi_n(x) &= S_{25}^n\Theta_0^n + S_{26}^n\Theta_1^n + S_{27}^n\Theta_2^n + S_{28}^n\Theta_3^n + S_{29}^n\Psi_0^n + S_{30}^n\Psi_1^n + S_{31}^n\Psi_2^n + \\
&\quad + S_{32}^n\Psi_3^n + S_{33}^nK_0^n + S_{340}^nK_1^n + S_{35}^nK_2^n + S_{36}^nK_3^n\ ,
\end{aligned} \tag{3.4}$$

where:

$$\eta_n(0) = \Theta_0^n \quad , \quad \xi_n(0) = \Psi_0^n \quad , \quad \varphi_n(0) = K_0^n \quad ,$$
$$\eta_n'(0) = \Theta_1^n \quad , \quad \xi_n'(0) = \Psi_1^n \quad , \quad \varphi_n'(0) = K_1^n \quad , \tag{3.5}$$
$$\eta_n''(0) = 2\,\Theta_2^n \quad , \quad \xi_n''(0) = 2\,\Psi_2^n \quad , \quad \varphi_n''(0) = 2\,K_2^n \quad ,$$
$$\eta_n'''(0) = 6\,\Theta_3^n \quad , \quad \xi_n'''(0) = 6\,\Psi_3^n \quad , \quad \varphi_n'''(0) = 6\,K_3^n \quad ,$$

n - n-th form of vibrations.

The functions of displacements (3.10), $\eta_n(x)$, $\xi_n(x)$, $\varphi_n(x)$ as well as their derivatives written down in a matrix form constitute a square matrix.

We shall state this matrix by $\mathbb{H}^s = \mathbb{H}^s(x)$, and the column matrices formed of values of the functions η_n, ξ_n, φ_n and of boundary values of these functions forx = 0, by $\mathbb{Y}^n(x)$ and $\mathbb{Y}_0^n$ respectivelly.

On the ground of the relationship (3.11) it is possible to write down the relation between the column matrix $\mathbb{Y}_0^n$ and the coplumn matrix formed of factors of power series as expressed for x = 0.

If we state the column matrix formed of factors of power series by $\mathbb{Y}_w^n$ then the relationship between $\mathbb{Y}_0^n$ i $\mathbb{Y}_w^n$ will take the form :

$$\mathbb{Y}_0^n = \mathbb{B} \cdot \mathbb{Y}_w^n \, , \tag{3.6}$$

where:

$$\mathbb{B} = \left\lceil \; 1 \;\; 1 \;\; \frac{1}{2} \;\; \frac{1}{6} \;\; 1 \;\; 1 \;\; \frac{1}{2} \;\; \frac{1}{6} \;\; 1 \;\; 1 \;\; \frac{1}{2} \;\; \frac{1}{6} \; \right\rfloor \tag{3.7}$$

is a diagonal matrix.

Accepting the above introduced designations we shell receive :

$$\mathbb{Y}^n(x) = \mathbb{H}^s(x) \cdot \mathbb{B} \cdot \mathbb{Y}_0^n \, , \tag{3.8}$$

where:

$\mathbb{Y}^n(x)$ - column matrix in the section determined by the coordinate x.

Product of matrices

$$\mathbb{H}^s(x) \cdot \mathbb{B} = \mathbb{H}^n(x) \tag{3.9}$$

will be called as a span matrix and then

$$\mathbb{Y}^n(x) = \mathbb{H}^n(x) \cdot \mathbb{Y}_0^n \, . \tag{3.10}$$

4. DETERMINATION OF A SECTION MATRIX AND OF A TRANSFER ONE

4.1. A bar without intermediate conditions

We shall formulate a section matrix for a bar without intermediate conditions in which principal central axes of inertia of the section in particular segments are uniplanar and the bar axis is a straight line.

Let us assume that the bar of $L = \sum_{i=1}^{m} l_i$ in lenght consist of segments with constant sections.

In order to determine characteristic parameters of thim - walled bars and then eigenfunctions it is necessary to express boyndary functions of displacements $\eta(x)$, $\xi(x)$, $\varphi(x)$ (for x = l) by means of boundary values of these functions - for x = 0.

This relationship represented in a matrix form is as follows :

$$\mathbb{Y}_m = \mathbb{H} \cdot \mathbb{Y}_0 \qquad (4.1.1)$$

where:

$\mathbb{Y}_0$ - column matrix formed of values of the functions and of their derivatives $\eta(x)$, $\xi(x)$, $\varphi(x)$ (for x = 0),

$\mathbb{Y}_m$ - column matrix formed of boundary values of functions $\eta(x)$, $\xi(x)$, $\varphi(x)$ and their derivatives in the point (x = l),

$\mathbb{H}$ - square matrix determining the relationship between the matrix $\mathbb{Y}_0$ and $\mathbb{Y}_m$.

The square matrix $\mathbb{H}$ is called a transfer matrix. The section matrix is being stated in sections of the rod, in which a sudden change of geometric characteristics or another discontinuity factor occurs. The form of the section matrix defines the relationship between values of the functions $\eta(x)$, $\xi(x)$, $\varphi(x)$ and their derivatives from the left side of the section (in the point of discontinuity) and values of these functions and their derivatives from the right side of the section. In order to determine the above relationships we shall express particular internal forces by corresponding geometrical characteristics of the section mof the bar and by derivatives of functions of displacements.

We shall write down the relationships existing between functions of displacements $\eta(x)$, $\xi(x)$, $\varphi(x)$ and their derivatives on both sides of the section, in which only a discontinuous change in geometrical features of the section takes place, on the ground of conditions of inseparability of displacements and of conditions of dynamic forces.

These relationships take the form :

$$\eta(x)_p = \eta(x)_l - (z\alpha_p - z\alpha_l) \cdot \varphi(x)_l \quad , \qquad (4.1.2)_1$$

$$\xi(x)_p = \xi(x)_l - (y\alpha_p - y\alpha_l) \cdot \varphi(x)_l \quad , \qquad (4.1.2)_2$$

$$\eta'(x)_p = \eta'(x)_l - (z\alpha_p - z\alpha_l) \cdot \varphi(x)'_l \quad , \qquad (4.1.2)_3$$

$$\xi'(x)_p = \xi'(x)_l - (y\alpha_p - y\alpha_l) \cdot \varphi(x)'_l \quad , \qquad (4.1.2)_4$$

$$\varphi(x)_p = \varphi(x)_l \quad , \qquad (4.1.2)_5$$

$$\varphi'(x)_p = W_l \cdot \varphi'(x)_l \quad , \qquad (4.1.2)_6$$

$$\varphi'''(x)_p = \frac{J_{zl}}{J\omega_p}(z\alpha_p - z\alpha_l) \cdot \eta'''(x)_l - \frac{J_{yl}}{J\omega_p}(y\alpha_p - y\alpha_l) +$$

$$+ \frac{GJ_{xp}W_l - GJ_{xl}}{EJ\omega_p}\varphi'(x)_l + \frac{J\omega_l}{J\omega_p}\varphi'''(x)_l \qquad (4.1.2)_7$$

$$\xi''(x)_p = \frac{J_{yl}}{J_{yp}}\xi''(x)_l \quad , \qquad (4.1.2)_8$$

$$\eta''(x)_p = \frac{J_{zl}}{J_{zp}}\eta''(x)_l \quad , \qquad (4.1.2)_9$$

$$\varphi''(x)_p = \frac{J\omega_l}{J\omega_p}\varphi''(x)_l \quad , \qquad (4.1.2)_{10}$$

$$\eta'''(x)_p = \frac{J_{zl}}{J_{zp}}\eta'''(x)_l \quad , \qquad (4.1.2)_{11}$$

$$\xi'''(x)_p = \frac{J_{yl}}{J_{yp}}\xi'''(x)_l \quad , \qquad (4.1.2)_{12}$$

where:

$$W_k = \frac{(z\alpha_p - z\alpha_l)y_k - (y\alpha_p - y\alpha_l) + \omega_l^k}{\omega_p^k}$$

k - point of contact of both sections.

The symbols with the index "l"presented in the expression (4.1.2) stand for functions of displacements along with derlwatives and for geometrical features referring to the left side of the section i under consideration. The symbols with the index "p" have been used to designate corresponding values referring tob the right side of the section i.

The relationships (4.1.2), written down in a matrix form constitute a square matrix which we call a section matrix or a node matrix.

If we state the matrix of i-th span with $\mathbb{H}_i$ and the matrix of i-th section (a sudden change) with $\mathbb{F}_i$, then the transfer matrix $\mathbb{H}$ for a given bar divided into m segments has a form:

$$\mathbb{H} = \mathbb{H}_m \cdot \mathbb{F}_{m-1} \cdot \mathbb{H}_{m-1} \cdot \mathbb{F}_{m-2} \cdot \ldots \cdot \mathbb{F}_i \cdot \mathbb{H}_{i-1} \cdot \ldots \cdot \mathbb{H}_1 \cdot \mathbb{F}_0 \qquad (4.1.3)$$

or

$$\overline{\mathbb{H}} = \mathbb{H}_m \prod_{j=1}^{j=m-1} \mathbb{F}_j \, \mathbb{H}_j \qquad (4.1.4)$$

4.2. A bar with intermediate conditions

Let as assume, that in the section of the bar, in which we are willing to build a section matrix aspring support and concentrated mass will take place apart from the change in geometrical features.

If we designate constant spring actions towards the axes y, z and around the axis x by Cy, Cz, $C\varphi$ respectiwely and the concentrated mass as well as its mass moment of inertia around the axis ζ by m and $J\zeta$ we shall receive :

$$\eta_p'' = \frac{m\omega_n^2 - Cy}{EJz_p}\eta_l + \frac{Jz_l}{Jz_p}\eta_l'' + \frac{m\omega_n^2 z\alpha_l - Cy\, z\alpha_l}{EJz_p}\varphi_l ,$$

$$\xi_p'' = \frac{m\omega_n^2 - Cz}{EJy_p}\xi_l + \frac{Jy_l}{Jy_p}\xi_l'' + \frac{C_z^2\, yz\alpha_l - m\omega_n^2\, y\alpha_l}{EJy_p}\varphi_l , \qquad (4.2.1)$$

$$\eta_p'' = \frac{J\zeta\, \omega_n^2 - Cy}{EJ\omega_p}\varphi_l + \frac{GJx_p W_l - GJx_l}{EJ\omega_p}\varphi_l + \frac{J\omega_l}{J\omega_p}\varphi_l'' +$$

$$+ \frac{Jz_l}{J\omega_p}(z\alpha_p - z\alpha_l)\eta_l'' - \frac{Jy_l}{J\omega_p}(y\alpha_p - y\alpha_l)\xi_l''$$

The other relationships are the same as those for a bar without intermediate conditions.

5. DETERMINATION OF CHARAKTERISTIC VALUES AND OF EIGENFUNCTIONS

Charakteristic values (frequencies of free vibrations or critical forces) are determined from the condition of equating a proper minor of the transfer matrix $\mathbb{H}$ (4.1.3) - i. e. a characteristic determinant depends on boundary conditions of rod under consideration.

From the midst of twelve boundary values of the functions η, ξ, φ, forming a column matrix in the initial section, six values are determined on the basis of boundary conditions for x = 0 and the remaining six values from a system of homogenous equations defining the boyndary conditions for x = l. Aiming at the simplicity of a notation we shall omit the index n signifying n -th form of vibrations in the continuation of our considerations when keeping in view the fact that a column matrix and a span matrix depend on the form of vibrations. If we designate a vector of the state on the right side of the section "i-i", by $\mathbb{Y}^{P}_{i-1}$ then we shall write down the column matrix in any point of the span "i" determined by the relationship (3.10), in a form :

$$\mathbb{Y}_i(x) = \mathbb{H}_i(x) \cdot \mathbb{Y}^{P}_{i-1} \quad , \tag{5.1}$$

where:

$\mathbb{H}_i(x)$ - represents the matrix for i -th segment determined by the relationsip (3.20),

$\mathbb{Y}_i(x)$ - column matrix defining eigenfunctions of the span under consideration.

When accepting the previously assumed designations the column matrix $\mathbb{Y}^{P}_{i-1}$ isdefined as follows :

$$\mathbb{Y}^{P}_{i-1} = \mathbb{F}_{i-1}\mathbb{H}_{i-1} \cdot \ldots \cdot \mathbb{F}_2\mathbb{H}_2 \cdot \mathbb{F}_1\mathbb{H}_1 \cdot \mathbb{Y}_0 \tag{5.2}$$

or

$$\mathbb{Y}^{P}_{i-1} = \left[\prod_{j=1}^{j=i-1} \mathbb{F}_j \, \mathbb{H}_j \right] \cdot \mathbb{Y}_0 \tag{5.3}$$

Due to introducing the relationship (5.3) to (5.1) we obtain the expression for eigenfunctions formulated as fallows :

$$\mathbb{Y}_i(x) = \mathbb{H}_i(x) \cdot \left[\prod_{j=1}^{j=i-1} \mathbb{F}_j \, \mathbb{H}_j \right] \cdot \mathbb{Y}_0 \tag{5.4}$$

It is necessary to emphasizethat the matrices $\mathbb{F}$ and $\mathbb{H}$ are functions of design features of the bar and of frequencies of free vibrations ω_n. Each frequency ω_n hasa different corresponding column matrix $\mathbb{Y}_i(x)$.

6. EXAMPLE EXPRESSO IN NUMBERS

The application of the solution presented above has been illustrated on the example of frequencies of free vibrations of a bar with sudden variable section the geometrical characteristic of which are as fallows :

segment 1

- crossd section area $A = 30,72\ cm^2$
- moment of inertia of section at clear torsion $J_x = 7,21\ cm^4$
- moment of inertia of section in relation to the axis y $J_y = 976,8\ cm^4$
- moment of inertia of section in relation to the axis z $J_z = 383,8\ cm^4$
- fragmentary moment of inertia of section $J_\omega = 4829\ cm^6$
- coordinate of bending centre along the axis z $y_\alpha = 4,25\ cm$
- coordinate of bending centre along the axis y $z_\alpha = 2,48\ cm$

segment 2

$A = 24,1\ cm^2$

$J_x = 3,72\ cm^4$

$J_y = 682,69\ cm^4$

$J_z = 364,8\ cm^4$

$J_\omega = 3068,48\ cm^6$

$y_\alpha = -\ 0,m4\ cm$

$z_\alpha = -3,45\ cm$

- moduls of elasticiti $E = 2,1 \cdot 10^7\ N/cm^2$ and $G = 0,84 \cdot 10^7\ N/cm^2$.
- weight by volume of material of which the bars are made $\gamma = 7,8 \cdot 10^{-2} N/cm^2$
- acceleration of gravity $g = 981\ cm/s^2$
- lenght of each segment (three cases) $l = 200\ cm,\ 400\ cm,\ 600\ cm$

The following boyndary conditions were taken into consideration:

$$
\begin{aligned}
&x = 0 \quad && \eta = 0, \quad \xi = 0, \quad \varphi = 0 \\
& && \eta' = 0, \quad \xi' = 0, \quad \varphi' = 0 \\
&x = 2l \quad && \eta = 0, \quad \xi = 0, \quad \varphi = 0 \\
& && \eta' = 0, \quad \xi' = 0, \quad \varphi' = 0
\end{aligned}
\tag{6.1}
$$

$$
\begin{aligned}
&x = 0 \quad && \eta = 0, \quad \xi = 0, \quad \varphi = 0 \\
& && \eta' = 0, \quad \xi' = 0, \quad \varphi' = 0 \\
&x = 2l \quad && \eta = 0, \quad \xi = 0, \quad \varphi = 0 \\
& && \eta'' = 0, \quad \xi'' = 0, \quad \varphi'' = 0
\end{aligned}
\tag{6.2}
$$

η - displacement of points of the axis of bending centres towards the axis y,

ξ - displacement of points of the axis of bending centres towards the axis z,

φ - angular displacement of section.

The calculated frequencies of free vibrations are:

a) for boundary conditions (6.1)

	l = 400 cm	l = 800 cm	l = 1200cm
ω_{min}	35,5 s^{-1}	9,9 s^{-1}	3,6 s^{-1}

b) for boundary conditions (6.2)

	l = 400 cm	l = 800 cm	l = 1200cm
ω_{min}	26,1 s^{-1}	7,9 s^{-1}	3,0 s^{-1}

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Streszczenie

W pracy sformułowano jedno z zagadnień dotyczące dynamiki prętów cienkościennych. Opracowane algorytmy stanowią rozwiązanie zagadnienia drgań swobodnych prętów cienkościennych o profilu otwartym i zmiennym przekroju. Zagadnienia analizowano w ujęciu liniowo sprężystym przy założeniach W.Z.Własowa [19].

W rozwiązaniu tego zagadnienia dla prętów o stałym przekroju zastosowano rozwinięcie funkcji przemieszczeń $\eta(x)$, $\xi(x)$, $\varphi(x)$ w szeregi potęgowe.

W przypadku prętów o zmiennym przekroju w rozwiązaniu zastosowano metodę macierzy przeniesienia, w której macierz przęsła została określona w oparciu orozwiązanie jak dla pręta o stałym przekroju.

Zastosowanie macierzy przeniesienia, w której, jak wiadomo , nie występują układy równań algebraicznych o dużej liczbie niewiadomych, umożliwia wykonanie obliczeń numerycznych nie ograniczając przaktycznie liczby przęseł pręta.

Na podstawie opracowanych algorytmów uruchomiono programy komputerowe liczące elementy macierzy przęsła, macierzy przekroju pręta bez warunków pośrednich, macierzy przekroju pręta z warunkami pośrednimi oraz program wyznaczający wartości własne i wektory własne prętów cienkościennych dla dowolnych warunków brzegowych.